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* Publication of closing discussion pending.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

THE SUSPENSION BRIDGE TOWER CANTILEVER PROBLEM

By Blair Birdsall,[1] Assoc. M. Am. Soc. C. E.

Synopsis

A generalized theory is offered in this paper for the study of the behavior of suspension bridge towers under the action of forces in a plane parallel to the axis of the bridge.[2] Reduced to its essentials, the structure under consideration is a vertical cantilever, of variable cross section, with a fixed base. The top of the cantilever is free to rotate, and is deflected by means of a horizontal and a vertical load. The usual problem is to find the horizontal tower-top force required to produce a certain deflection in conjunction with a given vertical tower-top load. As soon as the horizontal force has been found, it is a simple matter to find any other properties of the deflected tower, such as the elastic curve or the stresses.

The theory for the so-called "General Case" includes the effect of the weight of the tower as a producer of bending moment in the deflected tower, as well as the effect of an eccentrically placed tower-top load and eccentric loads at the roadway level. (In a mathematical sense, the true general case has not been used in the body of this paper, as the emphasis is on practical application. Thus, the so-called "General Case" presents the tools by means of which the true general case may be approached as nearly as necessary to produce what the writer considers to be a sufficient degree of accuracy for the most complicated case ordinarily encountered in practice.) Simplifications of the "General Case" for many special cases are either given in detail or outlined.

The results of applying the theory to three tower shafts are used as a means of indicating the accuracy of several approximate methods which may be used as practical tools for solving this problem if the precision of the mathematical treatment is not required.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by August 15,1941.

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[2] The more detailed study from which this paper was prepared has been placed on file for reference at Engineering Societies Library, 33 West 39th Street, New York, N. Y.

Introduction

The mathematical principles underlying the development of this theory are not new. In 1882 the late S. W. Robinson,[3] M. Am. Soc. C. E., published the general solution of the differential equation for bending moment in a beam or column caused by combined transverse and axial loads. The method of solution for this type of equation (linear differential equation with constant coefficients) may be found in any book on differential equations. The salient features of the application of the general solution to the problem of flexure and direct stress have been summarized by Francis P. Witmer,[4] M. Am. Soc. C. E. The applications of the general solution to some of the cases included in this paper have been published[5] in a standard textbook.

In this paper, the writer has attempted to make a comprehensive presentation of the subject, combining the fundamental theory for handling the problem under any combination of loads with a description of some practical short-cut methods, for the use of both the student and the practicing engineer.

Outline of the Problem and Definition of Symbols

The problem consists of finding the horizontal load required to produce a desired deflection at the top of a vertical, fixed-base cantilever which is acted upon by certain known vertical loads. The conditions of the problem are illustrated by Fig. 1. All loads are expressed in pounds, all dimensions in feet, and all moments in pound-feet. Other definitions are:

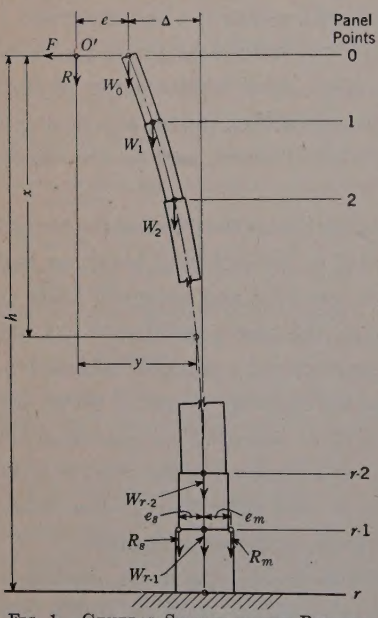

FIG. 1.—GENERAL SKETCH OF THE PROBLEM

F = desired horizontal tower-top load;

R = known vertical external load (cable reaction) on tower top;

e = eccentricity of R with respect to center line of top of tower;

Δ = known required deflection of tower top;

$0, 1, 2 \cdots (r-1), r$ = panel points along the center line of tower at which there are changes in moment of inertia, or known concentrated loads, or both;

$W_0, W_1, \cdots W_{r-1}$ = parts of tower weight assumed to be concentrated at the panel points indicated by the subscripts;

R_s, R_m = reactions on tower at roadway level;

[3] "Strength of Wrought Iron Bridge Members," by S. W. Robinson, Van Nostrand Science Series, No. 60, D. Van Nostrand, Publisher.

[4] "Basic Formulas for Combined Flexure and Direct Stress," by Francis P. Witmer, *Civil Engineering*, December, 1937, p. 855.

[5] "Modern Framed Structures," by the late J. B. Johnson, C. W. Bryan, Members, Am. Soc. C. E., and F. E. Turneaure, Hon. M. Am. Soc. C. E., Pt. II, 10th Ed., John Wiley & Sons, Inc., p. 278.

e_s, e_m = eccentricity of roadway reactions with respect to center line of tower at that point;

$x_1, x_2, \cdots x_{r-1}, x_r$ = known vertical distances from tower top to panel points;

$y_1, y_2 \cdots y_{r-1}, y_r$ = ordinates to panel points from line of action of R;

S_a = length of panel between panel points $a - 1$ and a;

h = total height of tower;

I_a = moment of inertia (in²-ft²) of the tower shaft between panel points $a - 1$ and a;

E = modulus of elasticity of tower material (pounds per square inch);

M = bending moment;

A, B, D, and C = constants determined from the loading conditions; and

G_a, K_a = integration constants for the panel between points $a - 1$ and a.

Basic Assumptions.—Six basic assumptions should be kept in mind:

1. All vertical distances remain constant;
2. Shear deformation is zero;
3. The tower base is rigidly fixed;
4. The tower weight is applied as a series of concentrated vertical loads;
5. The tower shaft is made up of a series of panels in each of which the moment of inertia is constant; and
6. The moment arm of R does not change due to rotation of the tower top.

The deflections and deformations are so small in comparison with the vertical dimensions that the error introduced by Assumption 1 is negligible. The heaviest concrete pier will yield to a certain extent, but the deflecting load and tower moments based on Assumption 3 will be on the side of safety. In Assumption 4 the concentrations are placed at the points of change in moment of inertia. However, the weights can be applied at any points without affecting the method of solution. The labor of computation would be increased, however, as each concentration as well as each change in moment of inertia requires an additional pair of integration constants unless they occur at the same point.

The Problem.—

Given: All external loads except deflecting load F; all physical characteristics of the tower shaft; and all dimensions, including the deflection Δ, except the ordinates to the elastic curve.

To Find: The deflecting load F, the ordinates to the elastic curve, and any other desired information, such as the bending moments, the stresses, or the tangents to the elastic curve.

The Differential Equation

From elementary mechanics, the general differential equation for bending moment is

$$\frac{d^2y}{dx^2} = \frac{M}{E\,I} \qquad \text{(1)}$$

The moment due to the type of loading contemplated in this paper may be

expressed as follows:

$$\frac{M}{E\,I} = \pm A\,y \pm B\,x \pm D\,x^2 \pm C \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which the sign depends on the direction of rotation. The function of x represents the moment due to transverse loads or any other moment which is independent of the ordinate y, and no combination of transverse loads can lead to a higher power of x than the second. The product $A\,y$ represents the moment due to axial loads. There will be no higher power of y than the first as long as concentrated loads only, and not distributed loads, have moment arms that depend on the ordinate y.

In order to determine the proper signs, assume that the cantilever of Fig. 1 is rotated 90° in a counterclockwise direction, and place the origin of coordinates at O'. By the usual convention, it is apparent that the column carries negative moment. Any load to the left of a section, which has a counterclockwise direction of rotation about the neutral axis of the section, helps to produce the negative moment.

Thus, since the predominating axial loads have counterclockwise moments about sections of the tower to the right of the points of application of these loads, the resulting sign of $A\,y$ must be negative. Since y is positive, however, A must be negative, and the term, as it appears in Eq. 2, is $-A\,y$. Since x is always positive, the same reasoning applies to all other terms, and Eq. 2 becomes:

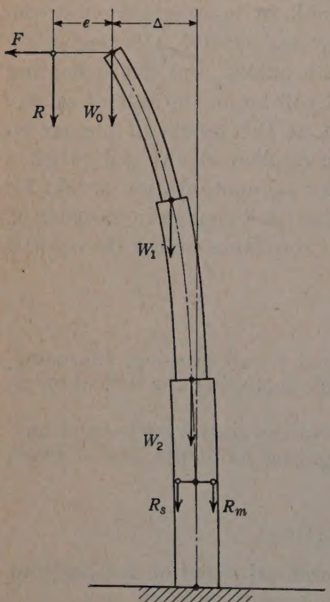

Fig. 2.—The "General" Case

$$\frac{d^2y}{dx^2} = \frac{M}{E\,I} = -A\,y - B\,x - D\,x^2 - C \ldots (3)$$

It is shown elsewhere[4] that, in general, the term for the axial loads is negative for loads producing compression, and positive for loads producing tension, and that the signs of the terms for the transverse loads depend on the direction of rotation. The general solution of Eq. 3 is:

$$y = -\frac{K}{A}\sin x\,\sqrt{A} - \frac{G}{A}\cos x\,\sqrt{A}$$
$$-\frac{D}{A}x^2 - \frac{B}{A}x + \frac{2\,D}{A^2} - \frac{C}{A} \ldots\ldots (4)$$

The formal derivation of this solution is given in Appendix I of the complete paper.[2] Expressions for slope, bending moment, and shear may be found in terms of the independent variable x by taking successive derivatives of Eq. 4; thus—

Slope of the elastic curve:

$$\frac{dy}{dx} = -\frac{K}{\sqrt{A}}\cos x\,\sqrt{A} + \frac{G}{\sqrt{A}}\sin x\,\sqrt{A} - \frac{2\,D}{A}x - \frac{B}{A} \ldots\ldots\ldots (5)$$

bending moment:

$$E\,I\,\frac{d^2y}{dx^2} = E\,I\left(K \sin x\,\sqrt{A} + G \cos x\,\sqrt{A} - \frac{2\,D}{A}\right) \ldots\ldots\ldots (6)$$

shear:

$$E\,I\,\frac{d^3y}{dx^3} = E\,I\,(K\,\sqrt{A} \cos x\,\sqrt{A} - G\,\sqrt{A} \sin x\,\sqrt{A}) \ldots\ldots\ldots (7)$$

Theory

The "General" Case; Variable Moment of Inertia, Eccentricity at Tower Top, Tower Weight and Eccentric Roadway Loads Considered.—To simplify the presentation, a tower shaft having three different values of I is used. The roadway loads are placed at a section that lies within the lowest of the three tower divisions. The problem is illustrated by Fig. 2.

For this case, four equations of the following form are obtained from Eqs. 3 and 6:

$$\frac{d^2y}{dx^2} = -\,n_a^2\,y - B_a\,x - C_a = K_a \sin n_a\,x + G_a \cos n_a\,x \ldots\ldots\ldots (8)$$

The conditions of the problem yield the following values for the loading constants of Eq. 3: With $D = 0$ in all panels, let $A = n^2$; then,

$$\left.\begin{aligned}
A_1 &= \frac{R + W_0}{E\,I_1} = n_1^2, \quad B_1 = \frac{F}{E\,I_1}\\
A_2 &= \frac{R + W_0 + W_1}{E\,I_2} = n_2^2, \quad B_2 = \frac{F}{E\,I_2}\\
A_3 &= \frac{R + W_0 + W_1 + W_2}{E\,I_3} = n_3^2, \quad B_3 = \frac{F}{E\,I_3}\\
A_4 &= \frac{R + W_0 + W_1 + W_2 + R_s + R_m}{E\,I_3} = n_4^2, \quad B_4 = B_3\\
C_1 &= -\,\frac{W_0\,e}{E\,I_1}\\
C_2 &= -\,\frac{W_0\,e + W_1\,y_1}{E\,I_2}\\
C_3 &= -\,\frac{W_0\,e + W_1\,y_1 + W_2\,y_2}{E\,I_3}\\
&\text{and}\\
C_4 &= -\,\frac{W_0\,e + W_1\,y_1 + W_2\,y_2 + R_s\,y_3 - R_s\,e_s + R_m\,y_3 + R_m\,e_m}{E\,I_3}
\end{aligned}\right\} \ldots (9)$$

By Eq. 5, for this case, there are four equations of the following form for the slope of the elastic curve:

$$\frac{dy}{dx} = -\,\frac{B_a}{n_a^2} - \frac{K_a}{n_a} \cos n_a\,x + \frac{G_a}{n_a} \sin n_a\,x \ldots\ldots\ldots\ldots (10)$$

The eight equations of the form of Eqs. 8 and 10 (one formula for moment and one for slope in each panel) contain twelve unknowns, including the eight

constants of integration K_1, K_2, K_3, K_4, G_1, G_2, G_3, and G_4; the load F; and the ordinates y_1, y_2, and y_3. Force F is included in the B-terms, and the ordinates are included in the C-terms, as may be noted in Eqs. 9. They can all be found by applying the following known conditions (the number after the dash in the last column indicates the value of a for the particular panel in question):

Condition No.	Statement of condition	Applies to Eq.:
I	When $x = 0$, $y = e$	(8–1)
II	When $x = x_1$, $y = y_1$	(8–1)
III	When $x = x_2$, $y = y_2$	(8–2)
IV	When $x = x_3$, $y = y_3$	(8–3)
V	When $x = h$, $y = e + \Delta$	(8–4)
VI	When $x = h$, $\frac{dy}{dx} = 0$	(10–4)
VII	When $x = x_1$, $M_{(0-1)} = M_{(1-2)}$	(8–1) and (8–2)
VIII	When $x = x_2$, $M_{(1-2)} = M_{(2-3)}$	(8–2) and (8–3)
IX	When $x = x_3$, $M_{(3-4)} = M_{(2-3)} - R_s e_s + R_m e_m$	(8–3) and (8–4)
X	When $x = x_1$, $\frac{dy}{dx}(0\text{–}1) = \frac{dy}{dx}(1\text{–}2)$	(10–1) and (10–2)
XI	When $x = x_2$, $\frac{dy}{dx}(1\text{–}2) = \frac{dy}{dx}(2\text{–}3)$	(10–2) and (10–3)
XII	When $x = x_3$, $\frac{dy}{dx}(2\text{–}3) = \frac{dy}{dx}(3\text{–}4)$	(10–3) and (10–4)

(It should be noted that Eq. 8 must be multiplied by the proper $E\,I$-values in order to obtain the bending moment when applying Conditions VII, VIII, and IX.)

The twelve equations obtained by applying these conditions may be solved to obtain the twelve unknowns.

Special Case I; Variable Moment of Inertia, Eccentricity at Tower Top, Tower Weight Considered, but Not Eccentric Roadway Loads.—Use the same procedure as for the "General" Case. Referring to Fig. 1, R_s and R_m now disappear and the problem reduces to that for a tower with three panels. There are nine unknowns: The six constants of integration K_1, K_2, K_3, G_1, G_2, and G_3; the load F; and the ordinates y_1 and y_2. They may all be found by applying nine known conditions which are identical with Conditions I to III, V to VIII, X, and XI of the "General" Case. (In any case that excludes eccentric roadway loads, a concentric load placed at the roadway level may be handled as if it were a tower weight concentration.)

Special Case II; Variable Moment of Inertia, Eccentricity at Tower Top, Tower Weight and Roadway Loads Not Considered.—Eqs. 9 are now greatly modified, inasmuch as R_s, R_m, and all values of W disappear. Since the shear is now continuous through the panel points, shears instead of slopes may be equated at these points. The basic equations then become—

For moment:

$$\frac{d^2y}{dx^2} = -n_a^2\,y - B_a\,x = K_a \sin n_a\,x + G_a \cos n_a\,x \,\ldots\ldots\ldots (11)$$

for shear:

$$\frac{d^3y}{dx^3} = -n_a^2\frac{dy}{dx} - B_a = K_a\,n_a\cos n_a\,x - G_a\,n_a\sin n_a\,x \ldots\ldots(12)$$

The solution is obtained by finding the seven unknowns F, K_1, G_1, K_2, G_2, K_3, G_3. Four equations are obtained by equating moments and shears at the panel points. (Eqs. 11 and 12 must be multiplied by the proper $E\,I$-value to obtain moment and shear.) The other three equations are obtained by applying the following known conditions: In Eq. 11, for $a = 1$, $y = e$ when $x = 0$. In Eq. 11, for $a = 3$, $y = e + \Delta$ when $x = h$. In Eq. 12, for $a = 3$, $\frac{dy}{dx} = 0$ when $x = h$.

Special Case III; Variable Moment of Inertia, No Eccentricity at Tower Top, Tower Weight Considered, but Not Eccentric Roadway Loads.—This case is identical with Special Case I, except that $e = 0$.

Special Case IV; Variable Moment of Inertia, No Eccentricity at Tower Top, No Roadway Loads, and Tower Weight Neglected.—This case is identical with Special Case II, except that $e = 0$. In both cases, the labor required may be reduced somewhat by following the procedure outlined in a standard textbook.[5] The value of A is removed from the denominator of the first two terms of the general solution, Eq. 4, and incorporated in the constants K and G. By following through the development of the problem from this point, it may be seen that the resulting simultaneous equations which express equality of moments and shears at the panel points do not require the $E\,I$ coefficients.

Special Case V; Constant Moment of Inertia, Eccentricity at Tower Top, Tower Weight Considered, but No Eccentric Roadway Loads.—The equations required for solving this case are obtained from those of Special Case I by substituting the constant value of I for I_1, I_2, and I_3. Note that this substitution must also be made to obtain the values of n_1, n_2, and n_3. (It is obvious that the same substitution in the equations for the General Case will take the roadway loads into account.)

Special Case VI; Constant Moment of Inertia, Eccentricity at Tower Top, Tower Weight Neglected, No Roadway Loads.—Since the tower consists of only one panel, the problem reduces to the following two equations—

Moment:

$$E\,I\frac{d^2y}{dx^2} = -R\,y - F\,x = E\,I\,(K\sin n\,x + G\cos n\,x)\ldots\ldots(13)$$

shear:

$$E\,I\frac{d^3y}{dx^3} = -R\frac{dy}{dx} - F = E\,I\,(K\,n\cos n\,x - G\,n\sin n\,x)\ldots\ldots(14)$$

In Eq. 13, when $x = 0$, $y = e$:

$$-R\,e = E\,I\,G\ldots\ldots(15)$$

In Eq. 13, when $x = h$ and $y = e + \Delta$:

$$-R\,(e + \Delta) - F\,h = E\,I\,(K\sin n\,h + G\cos n\,h)\ldots\ldots(16)$$

In Eq. 14, when $x = h$ and $\frac{dy}{dx} = 0$:

$$-F = E\,I\,(K\,n \cos n\,h - G\,n \sin n\,h) \qquad (17)$$

Solving Eqs. 15, 16, and 17 for F:

$$F = \frac{R\,n\,[e\,(1 - \sec n\,h) + \Delta]}{\tan n\,h - n\,h} \qquad (18)$$

Special Case VII; Constant Moment of Inertia, No Eccentricity at Tower Top, Tower Weight Considered, No Eccentric Roadway Loads.—Equations may be found for this case by substituting the constant value of I in the equations found for Special Case III.

Special Case VIII; Constant Moment of Inertia, No Eccentricity at Tower Top, Tower Weight Neglected, No Roadway Loads.—This case is identical with Special Case VI when $e = 0$. Thus, Eq. 18 becomes:[6]

$$F = \frac{R\,n\,\Delta}{\tan n\,h - n\,h} \qquad (19)$$

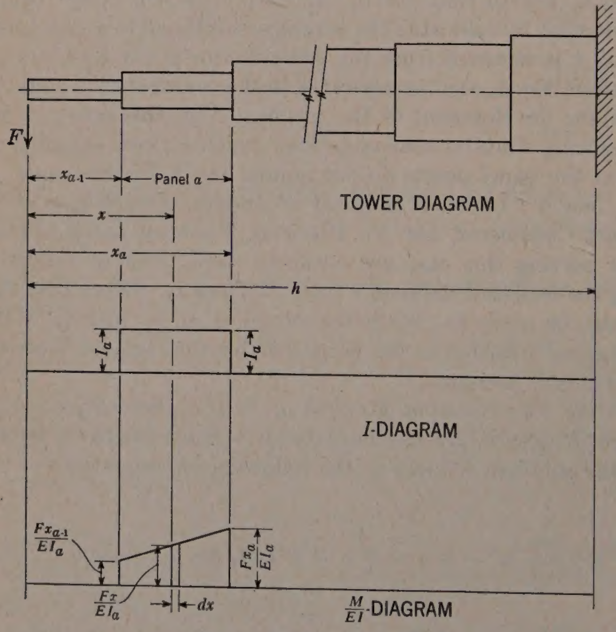

FIG. 3.—SPECIAL CASE IX

Special Case IX; Variable Moment of Inertia, Tower Weight Neglected, No Roadway Loads, and $R = 0$.—The problem (see Fig. 3) is now reduced to that of a simple cantilever of variable cross section. To find the deflection Δ by the moment-area method, find the moment of the $\frac{M}{E\,I}$-diagram about

[6] This case is also outlined in Pt. II of "Modern Framed Structures," by J. B. Johnson, C. W. Bryan, and F. E. Turneaure, 10th Ed., pp. 278 and 279.

point 0. The moment of the small part of the diagram included in dx, in panel a, is: $dM_a = \frac{F\,x^2}{E\,I_a}\,dx$; integrating for panel a: $M_a = \frac{F}{E\,I_a}\int_{x_{a-1}}^{x_a} x^2\,dx$ $= \frac{F}{E\,I_a}\left(\frac{x_a^3 - x_{a-1}^3}{3}\right)$. Summing up for all panels:

$$\Delta = \sum M = \frac{F}{E}\sum_0^h \frac{x_a^3 - x_{a-1}^3}{3\,I_a};$$

or

$$F = \frac{3\,\Delta\,E}{\sum_0^h \frac{x_a^3 - x_{a-1}^3}{I_a}} \qquad (20)$$

Special Case IX, Applied to the Condition in Which the Moment of Inertia Cannot Be Considered Constant Over Any Part of the Tower Leg.—If I can be expressed as a continuous function of x, the deflection may be found as the

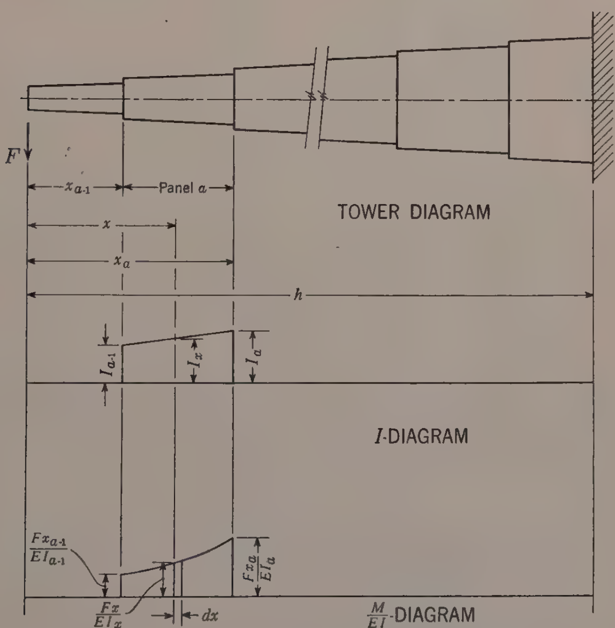

FIG. 4.—LAYOUT FOR FINDING Δ WHEN $R = 0$ AND I VARIES AS LINEAR FUNCTION OF x IN EACH PANEL

moment of the $\frac{M}{E\,I}$-diagram in the following manner:

$$\Delta = \int_0^h \frac{F\,x^2\,dx}{E\,I_x} = \frac{F}{E}\int_0^h \frac{x^2\,dx}{I_x} \qquad (21)$$

More often the moment of inertia is not a continuous function of x, but can be expressed as a series of linear functions of x. From Fig. 4, the moment

of the diagram for panel a is:

$$M = \frac{F}{E} \int_{x_{a-1}}^{x_a} \frac{x^2}{I_x}\, dx \quad \text{(22)}$$

From Fig. 4, the equation of the I-diagram for panel a is:

$$\frac{I_x - I_{a-1}}{x - x_{a-1}} = \frac{I_a - I_{a-1}}{x_a - x_{a-1}} \quad \text{(23)}$$

Solving Eq. 23 for I_x, substituting in Eq. 22 and integrating:

$$M = \frac{F}{E\,(I_a - I_{a-1})^3} \Big[\tfrac{1}{2}\,(x_a - x_{a-1})^3\,(I_a^2 - I_{a-1}^2) - 2\,(I_{a-1}\,x_a - I_a\,x_{a-1})\,(I_a - I_{a-1})\,(x_a - x_{a-1})^2 + (x_a - x_{a-1})\,(I_{a-1}\,x_a - I_a\,x_{a-1})^2 \log_e \frac{I_a}{I_{a-1}} \Big] \quad \text{(24)}$$

Summing all segments:

$$\Delta = \sum_0^h M = \frac{F}{E} \sum_0^h \frac{1}{(I_a - I_{a-1})^3} \Big[\tfrac{1}{2}\,(x_a - x_{a-1})^3\,(I_a^2 - I_{a-1}^2) - 2\,(I_{a-1}\,x_a - I_a\,x_{a-1})\,(I_a - I_{a-1})\,(x_a - x_{a-1})^2 + (x_a - x_{a-1})\,(I_{a-1}\,x_a - I_a\,x_{a-1})^2 \log_e \frac{I_a}{I_{a-1}} \Big] \quad \text{(25)}$$

Special Case X; Constant Moment of Inertia, $R = 0$, Tower Weights Neglected.—This is the case of the simple cantilever, in which

$$F = \frac{3\,E\,I\,\Delta}{h^3} \quad \text{(26)}$$

It is interesting to note that this can be obtained as a special application of Special Case VIII, in the following manner: Expand the tangent in Eq. 19 as an infinite series: $F = \dfrac{R\,n\,\Delta}{\left(n\,h + \dfrac{n^3\,h^3}{3} + \dfrac{2\,n^5\,h^5}{15} + \cdots\right) - n\,h}$. Simplify: $F = \dfrac{R\,\Delta}{\dfrac{n^2\,h^3}{3}\left(1 + \dfrac{2\,n^2\,h^2}{5} + \cdots\right)}$; but $n^2 = \dfrac{R}{E\,I}$. Therefore:

$$F = \frac{3\,E\,I\,\Delta}{h^3\left(1 + \dfrac{2\,R\,h^2}{5\,E\,I} + \cdots\right)} \quad \text{(27)}$$

Then, when $R = 0$, Eq. 27 becomes Eq. 26.

Special Case XI; Constant Moment of Inertia, Eccentricity at Tower Top, Tower Weight Neglected, $F = 0$.—In Eq. 18 of Special Case VI, let $F = 0$:

$$R = \frac{E\,I}{h^2}\left(\sec^{-1}\frac{e + \Delta}{e}\right)^2 \quad \text{(28)}$$

Special Case XII; Constant Moment of Inertia, No Eccentricity at Tower Top, Tower Weight Neglected, No Roadway Loads, F = 0.—In Eq. 28, let $e = 0$. Then, since $\sec^{-1} \infty = \frac{\pi}{2}$:

$$R = \frac{E I \pi^2}{4 h^2} \dots\dots\dots\dots (29)$$

Thus, in this case, the vertical load R is independent of the deflection Δ. This is Euler's formula for a column with fixed base and top free to move laterally. It yields the value of R for which the tower will be in unstable equilibrium. If, for any reason, the vertical load becomes slightly eccentric with respect to the center line of tower at its base, the tower will bend continuously until failure occurs.

For values of R greater than $\frac{E I \pi^2}{4 h^2}$ or values of $n h$ greater than $\frac{\pi}{2}$ a negative F is required to hold the tower in equilibrium for any desired deflection. This may be verified by inspection of Eq. 19.

General Observations on Theory

Although not more than two changes in moment of inertia have been used in any case, and only four tower panels were used in the so-called "General Case," the same method may be applied for any number of changes in moment of inertia, and any number of tower panels. A more complex case simply increases the labor required. No matter what the conditions may be, this analysis requires two constants of integration for each section of tower in which there is no change in moment of inertia and no point at which a load is assumed to act. If the concentrated loads along the shaft are considered, the ordinate to each load point is another unknown that must be found. The remaining unknown is the horizontal tower-top load F. As soon as the setup of the problem is known, the number of simultaneous equations required may thus be determined by inspection.

If the effect of wind loads is desired in combination with any or all of the other loads, the wind can be considered as a series of concentrated horizontal loads acting at convenient points along the shaft. This will add to the complexity of the function of x in the expression for moment and may or may not introduce additional constants of integration, depending on whether or not the tower is thereby divided into a larger number of panels.

Since the lateral bracing of the tower adds somewhat to the effective moment of inertia, some designers find it advisable to add from 3% to 5% to the calculated moments of inertia of the tower leg to take account of this effect.

All of the simultaneous equations for solving any case are linear. Therefore, it is evident that, for a given value of R, F will be a linear function of Δ. For the same reason, the ordinate to any point on the elastic curve is a linear function of Δ.

As will be shown in the numerical examples, when R is plotted against F for a given Δ, the resulting curve is so nearly a straight line that it may be considered so for all practical purposes. This approximate linear relationship is demonstrated mathematically in the complete unabridged paper.[2]

These characteristics are a great aid in determining the flexure curve for a tower. For a given Δ, which may conveniently be taken as unity, the curve of applied vertical load plotted against F may be determined by finding only two points. Furthermore, for any given value of vertical load R along the curve, the F corresponding to a different Δ may be found by direct proportion if R is concentric. If R is eccentric, the value of F for various values of Δ may be found by a simple linear relationship only at the points where F has been calculated.

Numerical Applications

The results of a few numerical applications are given herein to support some of the foregoing general observations and to indicate the error to be expected if certain approximate short cuts are used. The dimensions and physical properties of the tower shafts are given in the unabridged paper.[2]

Tower A (Height = 188 Ft, One Change in Moment of Inertia, Equivalent I = 1,061.7 In.²-Ft²).—The results for this tower are shown in Fig. 5(a), where values of F in terms of Δ are plotted against values of R. Curve 1 is based on Special Case III.

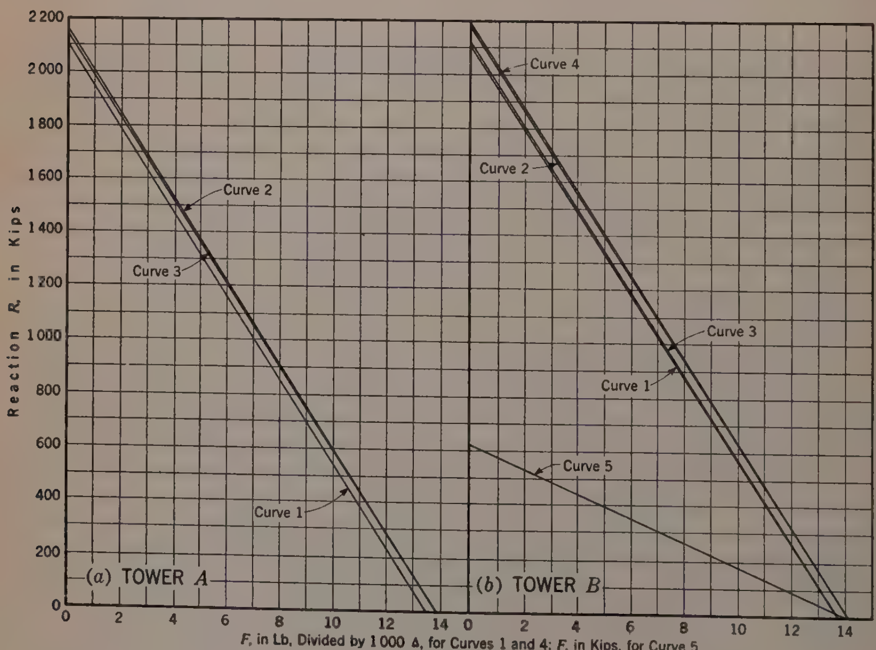

Fig. 5.—Flexure Lines

In finding curve 2, the weight of the tower was neglected. The F-intercept was found by using Special Case IX. The other two points were found by means of Special Case IV.

In finding curve 3, the weight of the tower was neglected and a constant moment of inertia was used. Curves that were established by finding more

than two points indicated that, for all practical purposes, F is a linear function of R. This is also confirmed by the discussion in Appendix II of the unabridged paper.[2] For these reasons, only two points were found on curve 3. The F-intercept was found by using Special Case X, and the R-intercept by using Special Case XII. The constant I was found by equating the value of F on curve 2 for $R = 0$ to $\frac{3\,E\,I\,\Delta}{h^3}$. This equation was then solved for I to obtain an equivalent moment of inertia.

Tower B (Height = 188 Ft, Two Changes in Moment of Inertia, Equivalent I = 1,078.2 In.²-Ft²).—The results for this tower are shown in Fig. 5(*b*). Curve 1 is based on Special Case III and curve 2 is based on a weightless tower. The F-intercept for curve 2 was found by using Special Case IX. The other three points were found by means of Special Case IV.

Curve 3 is also based on Special Case IV, but the following correction was applied to take account of the weight of the tower. For each of two of the points found on curve 2, the ordinates to the elastic curve were found at the points at which tower-weight loads were applied in finding curve 1. The tower-weight loads were then placed at these points. The sum of their moments about the center line of tower at the tower base, divided by the total height of tower, represents the approximate relief of F as found for curve 2, caused by the presence of the tower-weight loads.

Curve 4 is based on a weightless tower with constant I. The F-intercept was found by using Special Case X, and the R-intercept by using Special Case XII. The constant moment of inertia for this curve was found by equating the value of F on curve 2 for $R = 0$ to $\frac{3\,E\,I\,\Delta}{h^3}$. This equation was then solved for I to obtain an equivalent I.

Curve 5 is also based on a weightless tower with constant moment of a inertia equal to that used for curve 4. In this case, however, the vertical load R was given an eccentricity of 2 ft at the tower top, and all points were found for a tower-top deflection of 1 ft. Special Case XI was required to find the R-intercept, and Special Case VI to find the other points.

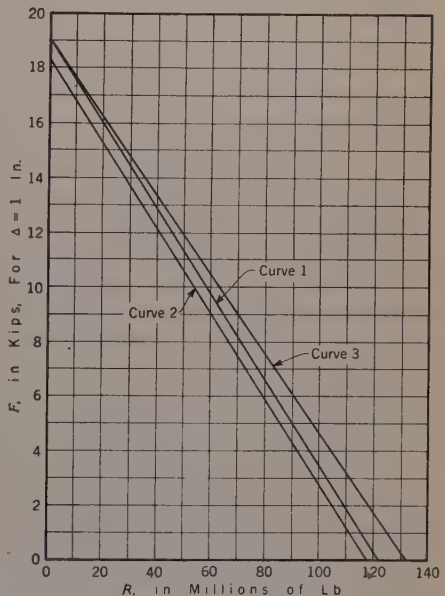

FIG. 6.—FLEXURE LINES FOR TOWER C

Tower C (Height = 702 Ft, Thirteen Changes in Moment of Inertia, Equivalent I = 906,740 In.²-Ft²).—The results for this tower are shown in Fig. 6. In this case the ordinates represent values of F for a deflection of 1 in. Curve 1 is based on a weightless tower and required the use of Special Cases IV and IX.

Curve 2 is also based on Special Case IV, but the following correction was applied to take account of the effect of the tower weight. Using the tower elastic curve for the point on curve 1 where $R = 52{,}000{,}000$ lb, a reduction in F was found by the method used in finding the points of curve 3 for tower B. For the point where $R = 0$, the same numerical reduction in F was used.

Curve 3 is based on a weightless tower with a constant moment of inertia, and required the use of Special Cases VIII, X, and XII. The constant I was found by equating the value of F on curve 1 for $R = 0$ to $\frac{3\,E\,I\,\Delta}{h^3}$. This equation was solved for I to obtain an equivalent moment of inertia.

Discussion of Numerical Applications

The results of this numerical work clearly indicate that, as nearly as it can be plotted, the horizontal load required to produce a given deflection is a linear function of the vertical load. This is confirmed by the discussion in Appendix II of the unabridged paper,[2] wherein the deviation from a straight line is discovered algebraically. It appears that for many purposes of design and estimate, it would be sufficiently accurate to use the following method of obtaining the flexure characteristics of a tower:

Using Special Case IX, find F for the weightless tower when $R = 0$. Equate this value to $\frac{3\,E\,I\,\Delta}{h^3}$ (Special Case X) and solve for I. Using this as an equivalent uniform moment of inertia, find the value of R when $F = 0$ by using the formula $R = \frac{E\,I\,\pi^2}{4\,h^2}$. This is from Special Case XII, and is also Euler's formula. The line joining these two intercepts is a close approximation of the true flexure line for the weightless tower. The worst case arising in the foregoing numerical applications occurs for tower C. Curve 3 is found as in the foregoing, and curve 1 is the true flexure line for the weightless tower. In the vicinity of the dead-load tower-top reaction (52,000,000 lb), there is an error in F of 650 lb in 11,000 lb, or approximately 6%. This can be brought to within about 6% of the true value for the weighted tower by the following method: Divide the tower into convenient panels. Find the weight of each section and apply it at the proper panel point as a concentrated load. Find the ordinates to these points on the elastic curve of the weightless tower by using Special Case IX, or simply by assuming the tower to take the shape of some well-known curve such as the parabola. Find the sum of the moments of the weight loads about the center line of the tower at its base. Divide this by the total height of the tower and reduce all values of F found on the approximate line for the weightless tower by the amount of this quotient.

Finally, when it is desired to obtain the greatest accuracy which is possible without following the rigorous solution for the weighted tower, the following method may be used: Depending on whether or not the tower-top reaction is eccentric, use Special Cases II and IX, or Special Cases IV and IX, to find two points, the F-intercept and one other, on the flexure line for the weightless tower. For one of these conditions, find the ordinates to the elastic curve, and proceed as before to reduce F by taking moments of the tower weights about

the tower base. The accuracy of this method for tower B may be seen by comparing curve 3, Fig. 5, which was found by this method, with curve 1, found by the exact method. It is doubtful, however, that this approximate method saves enough time, in comparison with the exact method, to make it worth while. In the exact method it is necessary to find only two points of the flexure line. It is the opinion of the writer that, if the degree of accuracy desired is such as to require the use of either the latter approximate method or the exact method, the exact method may as well be used unless the tower-weight concentrations cannot reasonably be placed at the points of change in moment of inertia. If they must be placed at intermediate points, additional integration constants will be required, and the last approximation described in the foregoing would be far less tedious than the rigorous solution.

All of this reasoning is based on the assumption that the principal objective of the solution is the deflecting force F required at the tower top. However, if the problem is to be solved in order to review the design of the tower shaft, and the principal objectives are the true elastic curve and the stresses in the tower sections, it appears to the writer that the only safe procedure is to use the rigorous solution.

In some design or erection problems, it may be desirable to set down a series of flexure lines for various values of deflection or eccentricity. The general disposition of various sets of curves for a few simplified cases is shown in Fig. 7. The following methods of attack are suggested by the arrangement of these lines.

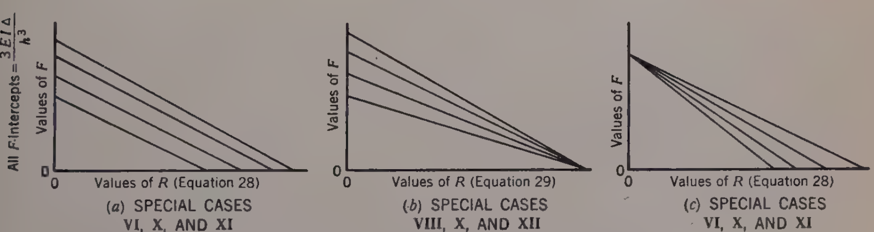

Fig. 7.—Arrangement of Flexure Lines for Various Cases

Special Cases VI, X, and XI.—In Fig. 7(*a*), $e > 0$ = constant, and each curve represents one value of Δ. The procedure is to find the F-intercept and one other value of F in terms of Δ. Any desired number of lines may then be found by applying various values of Δ to these expressions.

Special Cases VIII, X, and XII.—In Fig. 7(*b*), $e = 0$, and each curve represents one value of Δ. The procedure is the same as in Fig. 7(*a*) to find the F-intercept for each line. For the simple case of a constant moment of inertia and a weightless tower, the common R-intercept is found by Special Case XII. For other cases: Find a point on one line; produce this line from its F-intercept through the point just found to the R-axis; and draw all other lines through this common R-intercept.

Special Cases VI, X, and XI.—In Fig. 7(*c*), $e > 0$; Δ = constant; and each curve represents one value of e. The procedure is to find the common F-intercept and solve for one other point on each line.

Conclusion

The relation between horizontal and vertical tower-top loads for any tower-top deflection can be found by rigorous mathematical analysis, which includes the effects of the following influences:

(1) An eccentricity of the vertical tower-top load;
(2) The eccentricity of the weight of the tower (considered as a series of concentrated loads) caused by deflection;
(3) Reactions at the roadway level; and
(4) Wind loads (considered as a series of concentrated horizontal loads).

The analyses of this case and of several simpler cases have been developed in this paper. Many of the latter have been published elsewhere, but the writer feels that there is a definite advantage in having all these cases assembled for ready reference.

It is possible to obtain approximate values which are very close to the true values by means of short-cut methods, several of which are described herein.

It is mathematically correct to state that, for any given combination of other loads, the horizontal tower-top load is a linear function of the tower-top deflection. For all practical purposes, it is correct to state that, for a given deflection, the horizontal tower-top load is a linear function of the vertical tower-top load.

Acknowledgment

The writer wishes to acknowledge with gratitude the inspiration of the late George E. Beggs, M. Am. Soc. C. E., and his generous aid in the form of constructive criticism and advice. He also wishes to express his appreciation to the John A. Roebling's Sons Company for free access to its files and the use of its facilities, and in particular to Walter F. Weber, wire rope engineer of that firm, for his active help and useful advice; to Professors H. P. Robertson of Princeton University, Princeton, N. J., and M. S. Knebelman of Washington State College, Pullman, Wash., for their helpful advice in developing the mathematical theory which is included in the paper and the appendixes of the unabridged paper.[2]

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

SURFACE RUNOFF DETERMINATION FROM RAINFALL WITHOUT USING COEFFICIENTS

By W. W. Horner,[1] M. Am. Soc. C. E., and S. W. Jens,[2] Assoc. M. Am. Soc. C. E.

Synopsis

In hydraulic engineering practice, the relation between rainfall and runoff has generally been represented as a ratio or coefficient. It has been recognized that the form of this relationship should be "rainfall minus losses equals runoff." Heretofore the inadequacy of hydrologic data has discouraged attempts to evaluate losses as they occur during a storm period. In this paper the writers call attention to the recent improvement in hydrologic data with respect to precipitation and stream flow, and to the information with respect to infiltration that has developed from the research program of the U. S. Department of Agriculture; and they outline a method of applying this information to the evaluation of surface runoff from precipitation data without the use of a coefficient. The method is presented as being generally applicable to all drainage basins, and is described in detail as it would be used in urban storm drainage.

PART I.—GENERAL PRINCIPLES

Introduction

Fig. 1 shows two patterns of one-day storms, each producing a total precipitation of 6 in. On these storm patterns have been indorsed the values of infiltration capacity that might prevail in a midwestern drainage basin, under average agricultural land use, in midsummer. The two infiltration capacity curves are basically the same, but appear differently on the two diagrams as a result of the difference in precipitation pattern.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 15, 1941**.

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[2] Associate of W. W. Horner & H. Shifrin, Cons. Engr., St. Louis, Mo.

The shaded area represents, by hourly mean values, the excess of precipitation rate over infiltration capacity rate, and therefore the hourly production of excess rainfall. With some allowance for interception and depression storage, this will become an evaluation of surface runoff, as it is produced.

The mass values of surface runoff are materially different on the two diagrams. Obviously these two storms on the same area and in the same season will produce materially different values of flood flow. Engineers have been accustomed to attempt to express the volume of surface runoff from a storm of particular duration as a percentage of the total rainfall. Fig. 1 indicates why no single percentage or coefficient can be highly significant. It indicates further why these coefficients might be expected to have a wide range of values, even in a single season of the year.

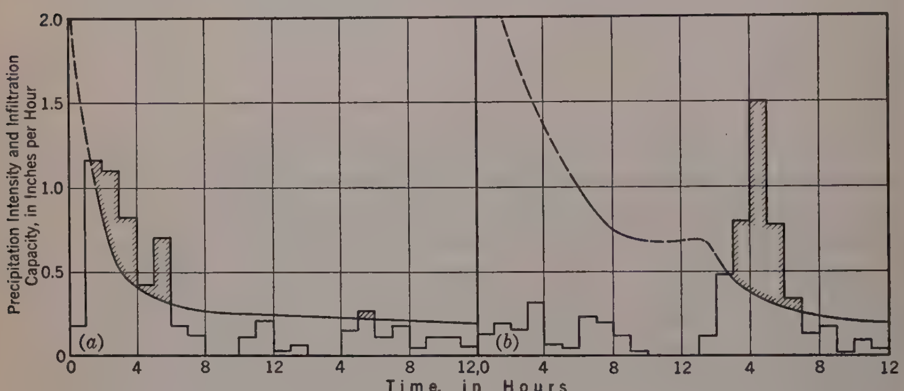

FIG. 1

The hourly variation in surface runoff production is quite different in the two cases; it is readily imaginable that the peak rates of surface runoff which these storms would produce in any stream system would be quite different, and it would follow that any attempt to express peak or crest runoff rate as a percentage or coefficient of the rainfall rate, for some critical time, would also result in a wide range of coefficients even in a single season of the year.

HYDROLOGIC DATA

The efforts of the engineering profession to develop, from available data, coefficients of runoff, either by volume or rate, have been due largely to the fact that, until recently, insufficient hydrologic data have been available to justify an attempt at more refined procedure.

Precipitation information was not formerly collected or published with any great consideration for the needs of engineering practice. For large areas in the United States, the only rainfall data available have consisted of the published records of daily totals. Recording rain gages existed at the first-order stations of the U. S. Weather Bureau, but the records were not published in detail.

Where excess rainfall is to be determined by subtracting infiltration rates from precipitation rates, the engineer must know the pattern of precipitation occurrence; if he is to utilize hourly rainfall rates, he must do it with an understanding of the manner in which precipitation actually occurs, on the average within the hour. The result of the study by Erwin R. Breihan,[3] Jun. Am. Soc. C. E., reproduced as Fig. 2, may be startling to the engineer who has been accustomed to using hourly mean rates. Such an engineer will have some difficulty in assimilating the idea that, on the average, half of the rainfall occurring within an hour actually falls at intensities equal to, or exceeding, twice the mean hourly rate.

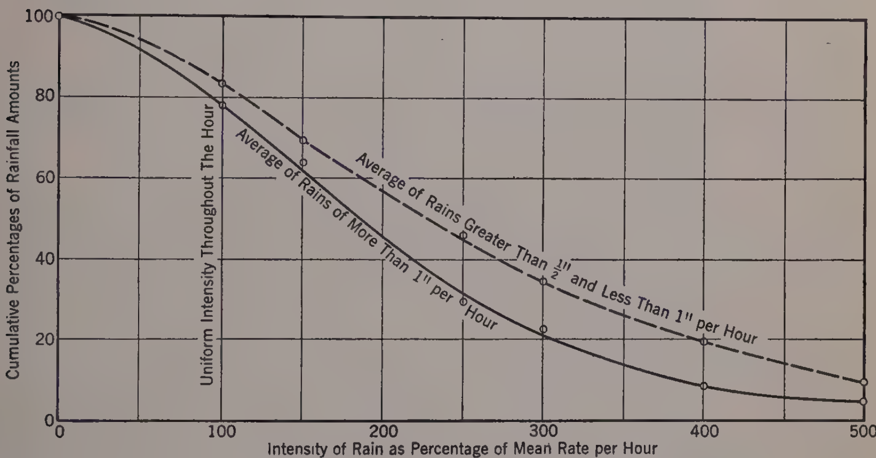

Fig. 2.—Relation of Precipitation Intensity Within the Hour, to Mean Hourly Rate

The tremendous expansion of both public and private work in the hydraulic field, and the corresponding expansion of the engineering practice in this field, have produced a continuing pressure for better information, which has finally resulted in an extension of the basic data-collection services and installations. The expansion of the network of recording rain gages by the Weather Bureau and other agencies, and of the stream gaging work of the U. S. Geological Survey and of the states, has multiplied hydrologic stations and has resulted in a great increase in the number with recording equipment. With the wealth of accurate information at hand, it is to be expected that engineering practice itself will be reorganized, and that hereafter the design of important hydraulic structures should not, and need not, be based on crude over-all general relationships in the hydrologic field.

Infiltration

Hydraulic practice has recognized that flood flows represent a less volume of water than the related precipitation, because a considerable part of the

[3] *Civil Engineering*, May, 1940, p. 303.

precipitated water infiltrates into the soil. Prior to 1920 there was no appreciable body of derived data that gave quantitative values of infiltration "loss" in detail.

The research of the Miami Valley Conservancy District, particularly that related to the small control plots, revealed the possibilities of securing new information and better ideas of rainfall-runoff relationships through studies on such small segregated areas.

Under the inspiration of the Miami example, one of the writers, in 1921, arranged for a series of small plot studies on plastic clay soils on the Washington University campus at St. Louis,[4] Mo., and about 1930 conducted similar studies on the Texas black soils, at Dallas.[5] As early as 1922, the Forest Service of the U. S. Department of Agriculture installed a number of control plots, and about 1933 extended this type of research installation. In the same year the Soil Erosion Service (later the Soil Conservation Service), in cooperation with State Agricultural Experiment Stations, began its research program on a nation-wide basis, using both plots and small watersheds.

As to nearly all of these projects, the values of precipitation and runoff were originally analyzed in an effort to determine coefficients of runoff, and comparatively little progress was made toward a better understanding of infiltration characteristics. It remained for Robert E. Horton,[6] M. Am. Soc. C. E., after reviewing the results of some of this earlier research work, to make a definite statement with regard to the relation between infiltration and these values that appear to control it. At that time, he developed more specifically the hypothesis of a limiting infiltration capacity, and defined infiltration capacity as "the maximum rate at which the soil, when in a given condition, can absorb falling rain." Later, Mr. Horton[7] presented his initial development of the theory of overland flow as related to surface detention. It is interesting that as late as 1935 he assumed that infiltration capacity might be satisfactorily approximated as having a constant uniform value during the first hour or two of a precipitation period.

During the four years (1937–1941), the basic information needed for a better understanding of infiltration capacity became available out of the extended research program of the Department of Agriculture, and values of infiltration capacity are derivable as secondary data from possibly as many as 100,000 controlled experiments. Although a tremendous amount of analytical work must yet be done before this mass of secondary data can be produced and published so as to cover conditions, reasonably, in the United States, it is possible at this time to secure and analyze such parts of this information as may be pertinent to a particular problem.

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[4] Under agreement between the Engineering Dept. of the City of St. Louis and the Engineering School of Washington Univ. Original in the thesis file, Washington Univ. Published in part in *Municipal and County Engineering*, December, 1922.

[5] Rept. to City Council, Dallas, Tex. (unpublished); also "Surface Runoff Phenomena," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y., 1935, pp. 17–18.

[6] "Role of Infiltration in the Hydrologic Cycle," by Robert E. Horton, *Transactions*, Am. Geophysical Union, 1933, p. 446.

[7] "Surface Runoff Phenomena," by Robert E. Horton, *Publication 101*, Horton Hydrological Lab., Voorheesville, N. Y., 1935.

Character of Infiltration-Capacity Curves

Characteristic curves of infiltration capacity, such as have been found to be derivable from different types of basic data,[8] are shown in Fig. 1. Fig. 3(*a*) shows a graph of infiltration capacity under artificially applied precipitation on grass, on an old hydraulic fill area near Washington, D. C., one day after this plot received a rain of 3.50 in. per hr. Fig. 3(*b*), similarly, was prepared from one of the plots tested at Washington University in 1923.

Fig. 4 also shows infiltration capacities at Edwardsville, Ill., for the storms of March 30, 1938. Antecedent rainfall consisted of 0.66 in. which fell on March 28, with a gross duration of 11.75 hr (6.5 hr net). The 49.95-acre watershed was a poor pasture with grass and weeds 6 in. to 8 in. high; 61% of the soil is Bogota silt loam; surface slopes vary between 0.8% and 30%.

The efforts of the engineering profession to express the relationship between rainfall and surface runoff in terms of coefficients of runoff, involving either volumes or rates, have been due to the fact that until recently sufficient hydrologic data have not been available to permit visualization of their relationships in more detail.

It will be noted that these curves all have characteristically similar shapes, the value of infiltration capacity being relatively high in the beginning of precipitation, decreasing rapidly as precipitation continues, and tending to reach rather definite minimum values, for a particular precipitation period, within a time of 2 to 20 hr. The shape of the early segment of these curves is particularly important when this information is to be applied to the calculation of runoff from small areas under intense precipitation. Where the problem involves high intensity storms of several hours' duration, relatively high initial values may not greatly affect the volume of total runoff. Nevertheless, in the Edwardsville storm of July 17, 1938, the first 8 hr of rainfall, amounting to 1.5 in., produced no runoff.

The production of such characteristic curves as those in Figs. 3 and 4 represents a long advance over the earlier conception of uniform infiltration capacity. Hydrologists are not as yet fully agreed as to the mechanics of infiltration that result in the production of high initial values and the steady reduction toward a constant minimum, and this matter is being investigated intensively.

The results of research indicate that: (1) Infiltration capacity varies little with surface slope; (2) it probably varies materially with soil porosity and with soil moisture, possibly with soil moisture deficiency below field capacity; (3) it may change rapidly with an alteration of soil surface condition such as may occur under the puddling action of rain impact, or under erosion and inworking of fines where the soil is not protected by good vegetal cover;

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[8] For methodology see:

For Plats—"Analysis of Runoff Plat Experiments With Varying Infiltration Capacity," by R. E. Horton, *Transactions*, Am. Geophysical Union, 1939, p. 693.

"Sprinkled Plat—Infiltration and Runoff Experiments on Arizona Desert Soils," by E. L. Beutner, R. R. Gaebe, Jun. Am. Soc. C. E., and R. E. Horton, *loc. cit.*, 1940, Pt. I, p. 550.

"A Graphical Method of Analysis of Sprinkled-Plot Hydrographs," by A. L. Sharp and H. N. Holtan, *loc. cit.*, p. 558.

For Small Watersheds—"Infiltration-Capacity Values Derived from Small Watershed Data—As Determined from a Study of an 18-Month Record at Edwardsville, Illinois," by W. W. Horner and C. L. Lloyd, *loc. cit.*, p. 522.

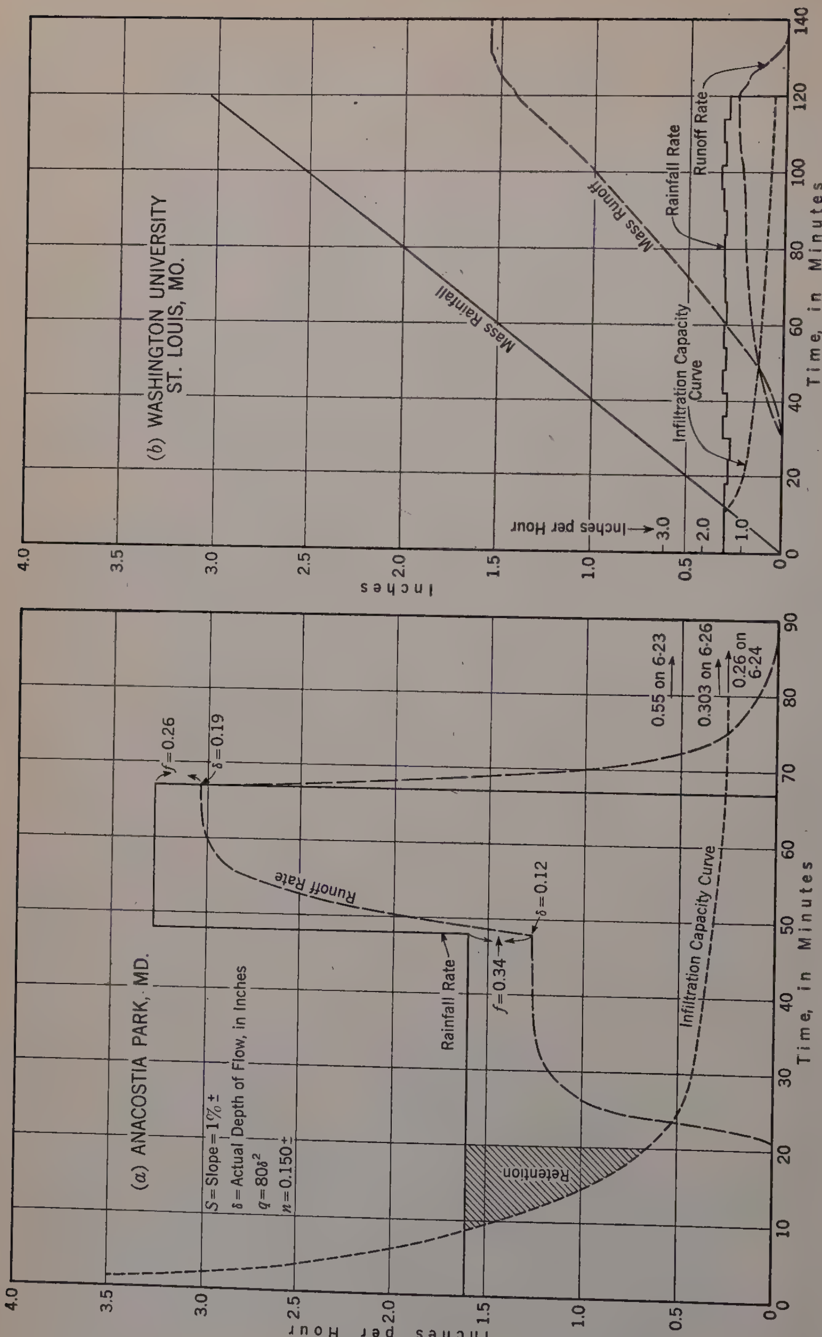


FIG. 3.—SPRINKLING TESTS FOR INFILTRATION CAPACITY

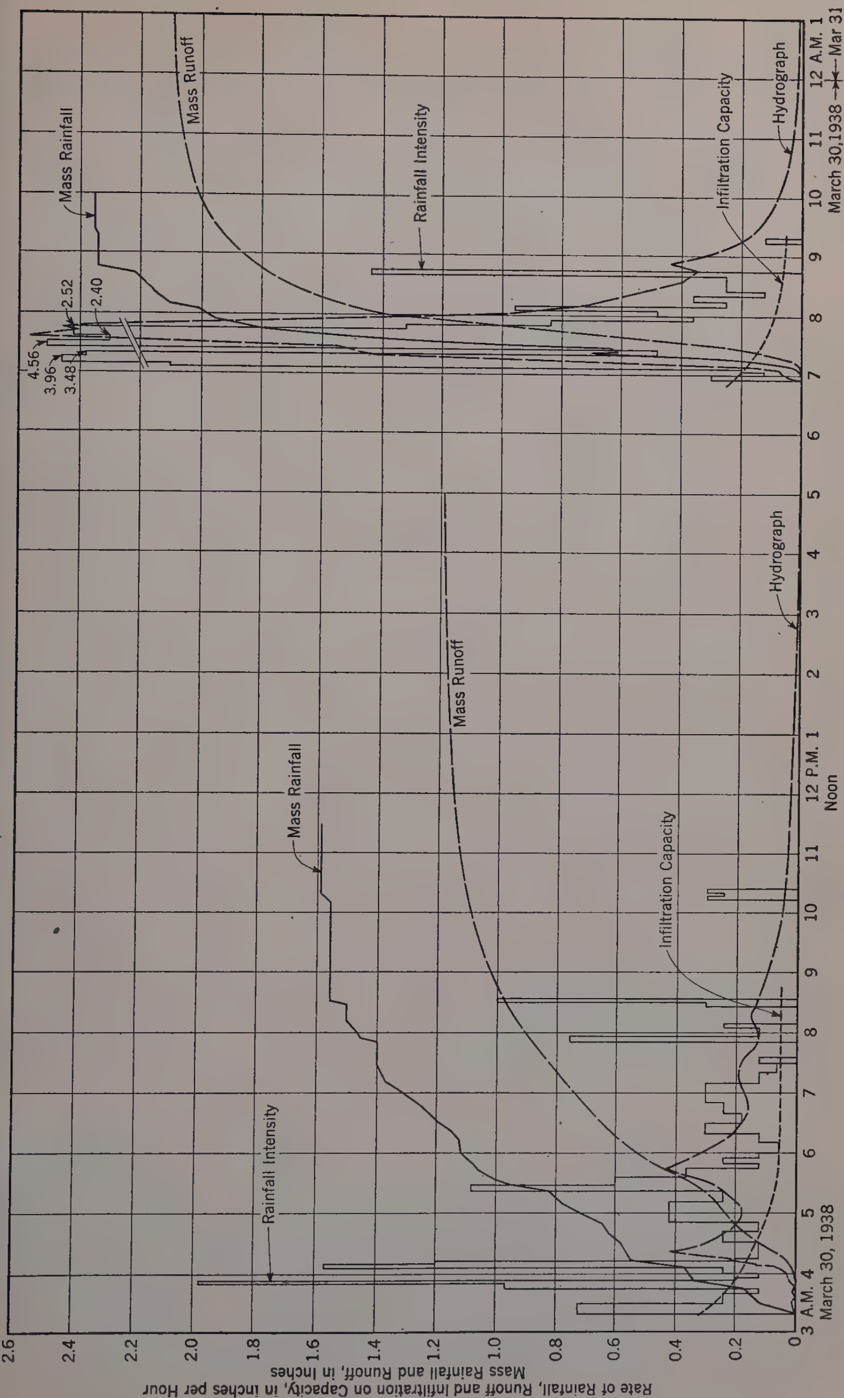


FIG. 4.—SPRINKLING TEST AT EDWARDSVILLE, ILL.

(4) it may be quite different for bare cultivated soils as compared with grass or other good vegetal cover; and (5) for bare soils it may vary with precipitation intensity, but under good vegetal cover it is relatively independent of intensity.

Much is yet to be learned from continuing research in this field, but it is important to engineering practice that characteristic values are becoming available which may be applied as representative of soil and cover conditions encountered in particular problems.

Relation of Precipitation Rate, Infiltration-Capacity Rate, and Excess Rainfall

Infiltration capacity is defined as "the maximum rate at which the soil, in a given condition, can absorb falling rain." Consequently, when infiltration-capacity values are applied to a precipitation-intensity diagram, the rate of infiltration is apparent at once. Referring to Fig. 4, it will be noted that between certain time periods, the rate of precipitation is less than the infiltration capacity. Consequently, for those periods, infiltration is limited by the precipitation rate. All the precipitation is infiltrated and no excess rainfall is produced. During certain other time periods where the rate of rainfall exceeds infiltration capacity, infiltration is limited to the capacity rate, and excess rainfall or supply is produced in amounts equal to the difference between precipitation rate and infiltration capacity.

When infiltration-capacity curves applicable to the soil and cover in a drainage basin are applied to precipitation-intensity diagrams, either for an actual storm or for a design storm, it is entirely practicable to compute for time increments, throughout the storm period, the rate of production of excess rainfall; or, in equivalent terms, to determine the rate at which excess rainfall will occur. This process removes entirely that phase of the old coefficient of runoff that was related to actual loss. For complete engineering application, this must be followed by supplementary technique through which the diagram indicating the rate of production of excess rainfall may be transformed into an actual flood flow hydrograph, through an application of the unit hydrograph, pluviograph, or channel storage methods.

Infiltration has ceased to be a vague phenomenon; it has been shown to have quantitative rate values with respect to any particular soil and cover condition; the manner in which these values change during a period of precipitation is determinable; therefore, infiltration expressed in terms of infiltration capacity may now be introduced into engineering practice.

Application to Hydraulic Engineering

During the ten years, 1931–1940, engineer-hydrologists have been working to develop a strictly rational method of approaching the rainfall-runoff relationship that can be based on sound theory throughout. This work has had to be conducted, until quite recently, with relatively meager data with which to test it. Much of the credit for the present development must be given to Mr. Horton, and probably the publication most nearly classic to the course of this development is his privately published *Bulletin 101*.[7] However, his work has been extensively complemented, criticized, and refined by others.

The results of research in this field have been presented in publications of the Department of Agriculture, or cooperating State Experiment Stations, of the U. S. Geological Survey, and in the *Transactions* of the American Geophysical Union, Section of Hydrology. A reading of selected papers from the latter for the period 1934 to 1940 gives an adequate perspective of the investigational work that has been done. As yet there has been no presentation to engineering practice of a definite technique through which the new principles and newly derived data may be applied to the evaluation of surface runoff under specific conditions.

The writers have recently had the opportunity to study and develop a methodology for such an application, first for small areas, conventional in character, at the Washington (D. C.) National Airport, and second for a natural drainage basin of about 800 sq miles. Out of the first experience there has been evolved a methodology presented in this paper as applicable to the determination of surface runoff from urban areas. Out of the second experience has come a clearer concept of a satisfactory procedure for large natural drainage basins.

The essential stages involved in this procedure are as follows:

I—Delineation of the precipitation pattern from which surface runoff is to be evaluated;
II—Choice of basic curve of infiltration capacity;
III—Adjustment of infiltration capacity values to antecedent conditions and precipitation pattern;
IV—Determination of the rate of production of excess rainfall;
V—Interception, depression storage, and infiltration out of surface detention; and
VI—Translation of mass surface runoff to hydrograph form.

I—Delineation of the Precipitation Pattern from Which Surface Runoff Is To Be Evaluated.—This pattern may be set up either in the form of a rainfall intensity diagram or of the mass curve. In the latter form the relations of precipitation and infiltration capacity are not so readily visible from hour to hour. The writers much prefer to show precipitation as an intensity diagram. For small drainage areas it is essential that this diagram be closely representative of the rainfall intensity as it actually occurs. In the illustration in Part II, a time unit of 10 min has been used and this appears to be sufficiently refined and generally satisfactory. For such small areas, retention and infiltration, out of surface detention reached significant amounts and must be evaluated separately.

For large natural drainage basins, the appropriate time unit will depend somewhat on the character of the available rainfall data. For such large areas precipitation pattern should be set up at normal Thiessen centers. At the points usually chosen for such centers generally only daily rainfall amounts are available, although often with observers' notes as to the beginning and end of heavy precipitation. It is necessary to transform this information into the most probable intensity pattern, and this must be done through comparison with the nearest available recording gage records. Obviously, if there are no

records of hourly rainfall available within a reasonable distance from such Thiessen centers, it would be useless to attempt preparation of a detailed pattern of precipitation intensity at those points, and time units as great as 3 to 6 hr may be the smallest that are justified. Where the data permit, however, the pattern should be set up in at least hourly amounts.

II—Choice of Basic Curve of Infiltration Capacity.—Information with respect to land use is now quite readily available for most parts of the United States in the form of aerial photographs. Using such photographic information, the percentage of the land under any particular type of vegetal cover, such as cultivated land, pasture land, woodland, etc., may be readily determined for any drainage basin or any part of such area. Soils maps are also available for a large part of the United States. Using both types of information, it is not a difficult matter, by well-devised sampling, to take an inventory of land use and soil for a complete drainage basin of considerable size. From the results of the research programs, basic infiltration capacity curves may be selected that will be satisfactorily representative of any particular combination of soil and cover under specific seasonal conditions.

III—Adjustment of Infiltration-Capacity Values to Antecedent Conditions and Precipitation Pattern.—Although, for any soil, cover, and seasonal condition the graph representing the march of infiltration-capacity values appears to have a quite definite form under continuous excess rainfall, nevertheless, the appearance of these values during a particular precipitation period will vary with the following:

(*a*) The initial soil moisture and soil condition will vary from storm to storm, and a wide variation of initial values of infiltration capacity may be expected. For very dry, open, or cracked condition of the soil, the initial values may exceed 4 in. per hr; for wet or plastic soils, it may be as low as 0.2 in. to 0.3 in. per hr. The writers have found that for any one soil these values are responsive to antecedent precipitation, and show a fair correlation with antecedent infiltration when expressed in amount and elapsed time since occurrence. A study of a number of infiltration capacity curves produced under different antecedent conditions makes possible a reasonable choice of the initial value to be used.

(*b*) Referring to Fig. 3 it will be noted that, except for the first ten minutes, precipitation exceeded infiltration capacity. The graphs of infiltration capacity are typical of those derived from infiltrometer test runs and are representative of the march of values, throughout time, in which infiltration is continually occurring at capacity rates. Very nearly the same curves occur under natural precipitation during compact storms starting at high intensity such as the first storm shown in Fig. 4. For storms in which the precipitation rate in the early part is less than the infiltration-capacity rate, the same values of infiltration capacity will occur, but at some later time in the course of the storm.

To permit a satisfactory relation of infiltration-capacity values to precipitation rates for any particular storm pattern requires a knowledge not only of

the normal capacity curve such as that in Fig. 3(*a*), but also as to where, in time, these values will appear under intermittent or varying precipitation. The required adjustment can be made from an examination of a good series of such curves derived from small watershed data under widely different rain patterns; or it may be approximated from infiltrometer tests designed for that purpose. For the latter tests, precipitation would not be applied continuously at excess rates; the tests would include several series having intervals at lower rates and also varied intervals during which no precipitation is applied.

For light open soils, the writers have found that an approximate adjustment of infiltration capacity rates to the time of occurrence may be made on the basis of mass infiltration as illustrated in Part II (see heading "Infiltration Capacity"). For soils of high clay content where the decrease in infiltration capacity rate is not so directly responsive to soil moisture change (but may be due in part to change in soil structure such as that resulting from colloidal swell) a different type of adjustment may be required.

The type of adjustment, both as to initial and continuing infiltration capacities suggested previously, is at this time the most difficult phase of the application of the proposed methodology. With the present knowledge of the mechanics of infiltration there is no definite rule that can be formulated in allocating these values. They must be applied to each storm pattern on something of a case system in which the judgment is guided by what actually happened under conditions for which infiltration-capacity curves have been derived. In addition to study of small watershed data, the guide to judgment can also be obtained from a well-devised series of infiltrometer tests.

For large drainage basins and long storms a considerable error in the choice of initial values of infiltration capacity will have little significance. For small areas, as illustrated in Part II, this item may be of major importance. This stage of the procedure requires a thorough knowledge of infiltration-capacity values for the soils and covers involved, and a complete study of all available data is a pre-essential to the undertaking of an evaluation of surface runoff.

The procedure outlined is usable at this time for any project in which the probable expenditures involved appear to justify an extended engineering analysis. Under these conditions it can be applied with a much greater assurance of accuracy, and with the use of much lower factors of safety than one involving a choice of a coefficient of runoff. Undoubtedly out of the continuing research programs, and out of the experience resulting from applications to large drainage basins, information will develop that may be expected to simplify and clarify this type of adjustment greatly. In that the method attempts to recognize the more important controlling variables and to allocate logical values to them, it will probably never be possible to reduce it to a simple routine.

IV—Determination of Rate of Production of Excess Rainfall.—With the precipitation diagram and the infiltration capacity values adjusted as outlined, a determination of the rate of production of excess rainfall is purely a matter of the subtraction of coincident values for each of the time units utilized. These values represent the gross quantity of excess rainfall produced at the

ground surface during each time unit. They are subject to some further modification as outlined in Section V.

V—Interception, Depression Storage, and Infiltration Out of Surface Detention.—Surface detention is the term now generally applied to water in transit on the ground surface. In its broadest use it includes also depression storage—that is, water required to fill depressions in the ground surface below their overflow levels. Expressed in another way, it is the depth of water that must exist on the ground surface to permit flow over the surface, toward the stream margin, inlet or other approach to the main drainage system. It is not intended to cover water actually in stream channels of appreciable size.

The effect of surface detention has long been recognized in the discussion of engineering practice, but has generally been assumed to be so highly variable and so intangible in its dimensional aspects that it was not capable of direct engineering evaluation. It remained again for Mr. Horton to develop, specifically, the hydraulics of overland flow, and to illustrate it with the meager data which he found available in 1935.[7] An evaluation of surface detention has now been found to be of extraordinary engineering importance in studies of rainfall-runoff relationships from small areas. As shown in detail in Part II, it is fundamental that at the end of a period of excess rainfall a part of the "supply" will be in the form of a layer of water in transit over the surface. So long as this persists on any part of the surface, infiltration will occur out of it at capacity rate, and the surface runoff will be less than the supply by the volume so extracted. The depth of surface detention will vary with slope, character of surface and cover, and with the rate of production of supply. For surfaces such as pasture or forest litter on flat slopes at the end of periods of high excess rainfall, surface detention will be very great; the time required to dispose of it will be considerable, and the abstraction through infiltration a significant quantity. For smooth surfaces, steep slopes, and small rates of excess rainfall, the quantity may be negligible.

The term "retention" is used herein to define that part of the supply which is abstracted permanently during the period of excess rainfall. It includes interception and depression storage. Its requirement must be satisfied out of the early precipitation; interception out of the first period of precipitation; and depression storage out of the first period of excess. The water quantities so abstracted will remain on the foliage or in depression storage at the end of the period.

For smooth, cultivated land, the value of retention appears to be on the order of 0.05 to 0.10. For contour cultivation it is considerably greater under good management. For good pasture it may be as high as 0.2, and for good forest litter up to 0.3. Abstractions in such amounts will occur during the early part of the storm. These quantities will be removed by evaporation and infiltration after the end of precipitation, or as to depression storage, after the end of excess rainfall. In large storms, if there are several hours of little or no precipitation, retention may be wholly or partly disposed of, and the requirements must again be filled during the subsequent part of the storm. In one long storm studied for a forested area, it appeared that retention actually

exceeded one inch of rainfall. For compact storms, except where the cover is good grass or forest litter, the values of retention may be negligible.

In the procedure outlined herein, the foregoing factors should be recognized and applied in the following manner:

Characteristic values of interception and of depression storage may be determined from an analysis of the small watershed hydrographs. After excess rainfall has been computed for each time unit, values of interception and depression storage should be deducted from such excess in the time units where either interception or the filling of depression storage may obviously occur.

For small drainage areas where the infiltration pattern is set up in 10-min units and where it reflects precipitation intensity accurately, an allowance for infiltration out of surface detention must be determined in the manner illustrated in Part II. For large drainage basins where precipitation is set up in hourly mean values, and where no recognition is given to the rates which may occur within the hour, the resulting net rainfall computed will probably be less than the true value. Under these conditions, infiltration out of surface detention may be neglected and the two errors will be considered as compensating.

VI—Translation of Mass Surface Runoff to Hydrograph Form.—The final stage of this application involves the translation of the excess rainfall diagram into stream flow, generally through unit hydrographs of excess rainfall, or through surface detention and channel storage computations. It is assumed herein that, for most basins, net excess rainfall produced on the land represents surface runoff, and that this in general will be substantially equivalent to stream flow volume for a storm period and related rise of the hydrograph. A reduction will occur between surface runoff and stream flow where a part of the runoff is lost as seepage into stream bed or bank or on flood plains. An increase will occur where a part of the infiltered water returns to the stream system, during the rise of the hydrograph. There are exceptional conditions, as for some basins, where the return flow through the ground is so rapid as to constitute a considerable part of the flood flow. For such basins, infiltration capacity may be used to evaluate the hydrograph of surface runoff, and a separate procedure is developed to determine the hydrograph of ground water return flow. For many basins the latter is an insignificant part of the flood hydrograph and may be neglected.

PART II.—APPLICATION OF DESIGN METHODS

Introduction

It is intended in this part of the paper to accomplish two objectives:

(*a*) To permit a better visualization of the various factors and processes discussed in Part I by actually applying them to a small restricted area of conventional type, such as a developed city block; and

(*b*) To show the character of the runoff that will occur from such a conventional area, since the hydrograph will differ because of differences in certain controlling variables. After producing the actual synthetic hydrograph for

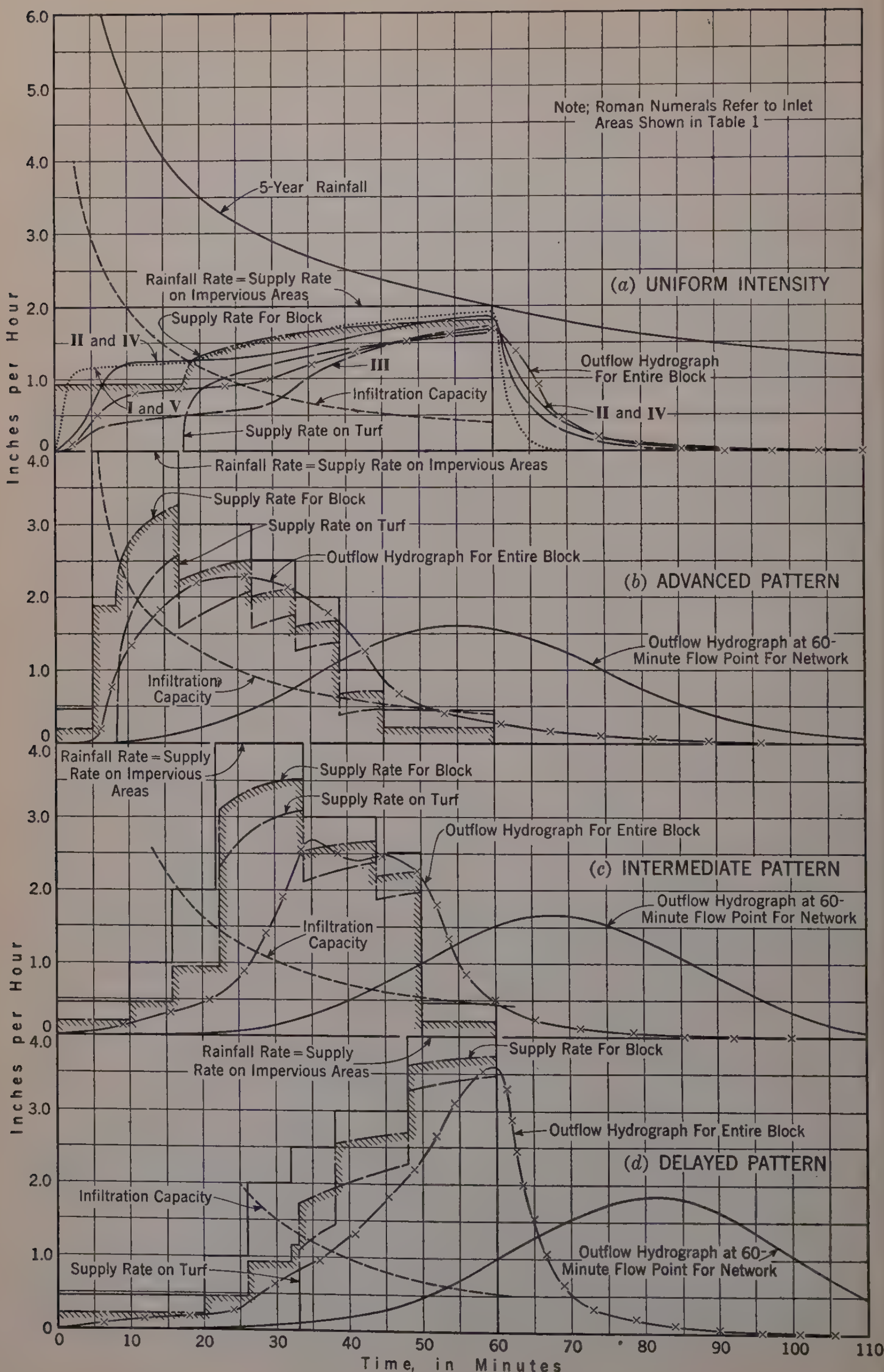


FIG. 5.—RAINFALL CURVES; 60-MIN RAIN

TABLE 1.—TRIBUTARY AREAS

No.	Description	Curves (See Figs. 5 and 14)					
		I	II	III	IV	V	Total
1	Tributary area, in acres......	1.3509	0.1222	2.4325	0.1223	1.3509	5.3788
	Percentage of Area:						
2	Pervious................	38.9	18.9	75.5	18.9	38.9	53.6
3	Impervious............	61.1	81.1	24.5	81.1	61.1	46.4

each combination of conditions, it is proposed to discuss the relation of the peak flow to the mean rainfall in terms of the accepted definition of the coefficient of runoff.

For the purpose of this demonstration, three precipitation periods have been chosen, having respective durations of 20, 40, and 60 min. The mean intensity for each of these durations is that taken from the rainfall curve shown in Fig. 5(*a*), which is substantially a 5-yr rainfall curve for St. Louis (see Table 1 for inlet areas corresponding to curves I to V). In the ordinary application of the rational method, precipitation intensity for specific duration periods is customarily considered as if it were uniform for the period. This is a condition that practically never occurs in nature. The probable occurrence of intensities for parts of the duration period has been investigated (see Fig. 2). For the purpose of this demonstration, the mean intensity for each period has been broken up into probable intensities and durations in approximate accord with the results of the analysis by Mr. Breihan.[3] For the 60-min precipitation period, these sub-intensities and duration units are shown on each of the diagrams, Figs. 5(*b*), 5(*c*), and 5(*d*). Mr. Breihan's investigation did not extend to the order in which these sub-intensity blocks might be expected to appear, and no other good investigation of this kind has been reported. The nearest approach to such a study is that described by D. I. Blumenstock,[9] but this relates only to variations in mean intensity, from hour to hour, within storms of considerable duration.

Experience has shown that each of the intensity patterns shown in Figs. 5(*b*), 5(*c*), and 5(*d*) may be expected to occur at some time, pattern 5(*b*) being the most common. The three patterns shown have been chosen arbitrarily for the purpose of this investigation and are referred to hereafter as the "advanced," "intermediate," and "delayed" patterns.

Mr. Breihan's investigations did not extend to a similar variation of intensities within precipitation periods of less than one hour. For the purpose of this investigation, the variations during the periods of 40 min and 20 min have been proportioned in a manner similar to that which was found for the one-hour period (see Figs. 6 and 7).

Physical Characteristics of the Area to Which Rainfall is Applied

For the purpose of this investigation, Fig. 8 shows a conventional city block bisected by a paved alley. There are twelve 50-ft lots on each street frontage. The actual areas devoted to roofs, sidewalks, driveways, street and alley

[9] *Technical Bulletin No. 698*, U. S. Dept. of Agriculture.

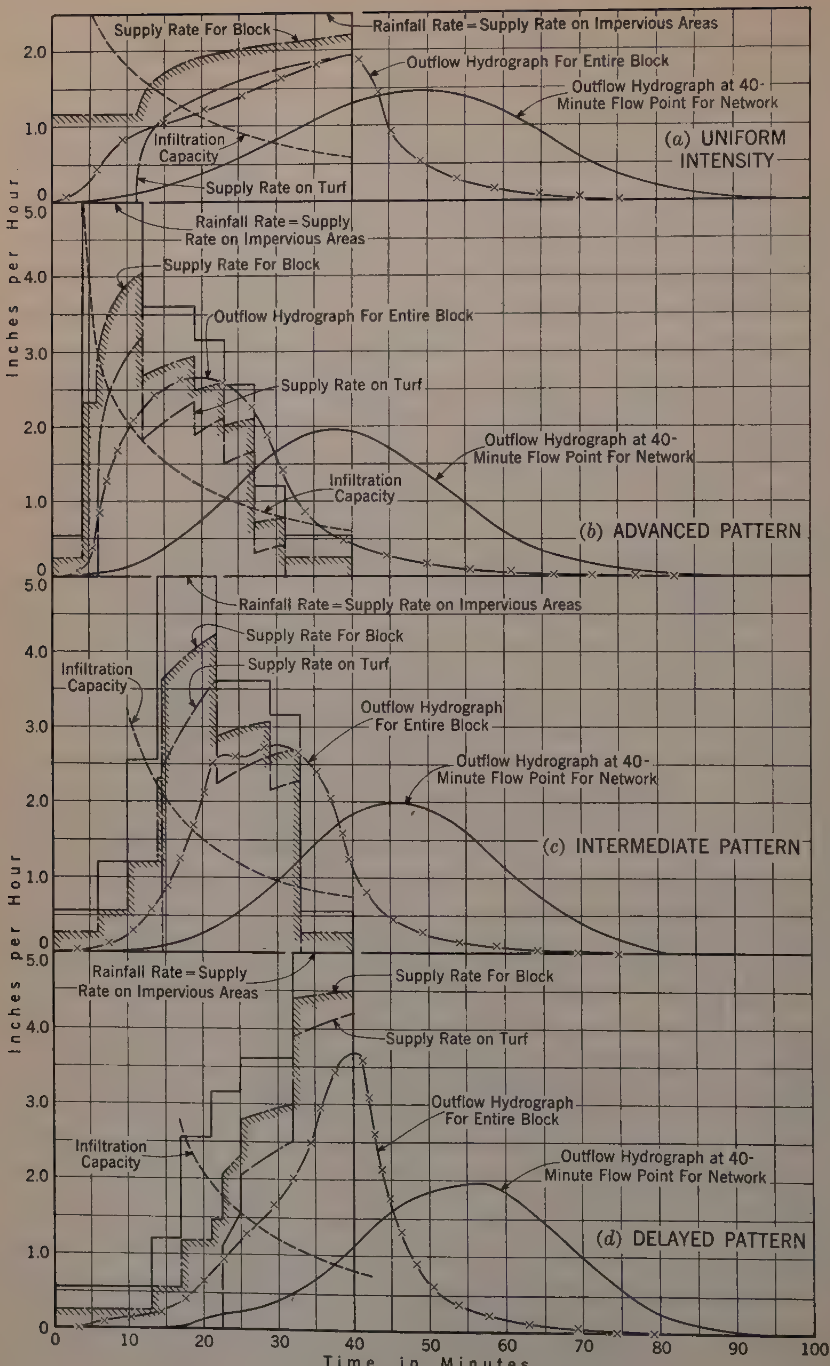


FIG. 6.—RAINFALL CURVES; 40-MIN RAIN

pavements, and lawns are shown in detail in Fig. 8(*b*) and Table 2. The general conditions correspond rather closely to one of the city blocks heretofore gaged in St. Louis. Not only is the percentage of the area of each of these

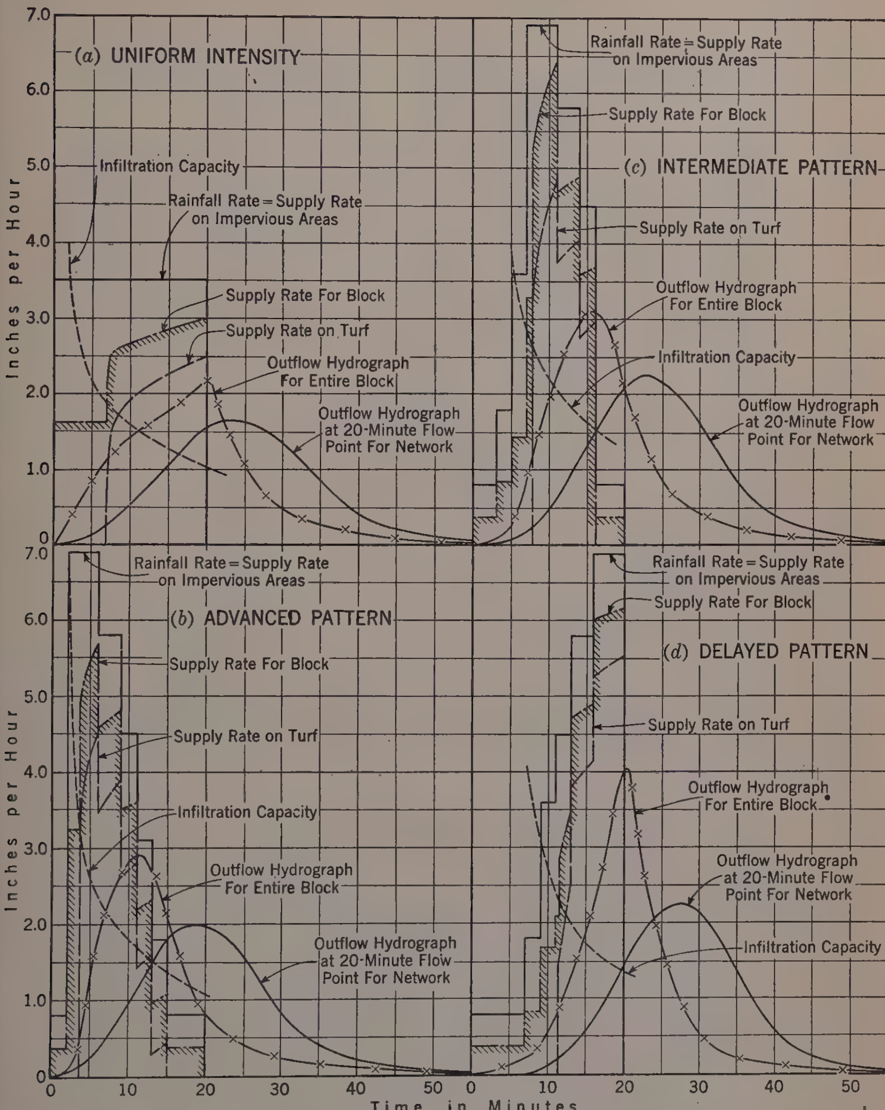
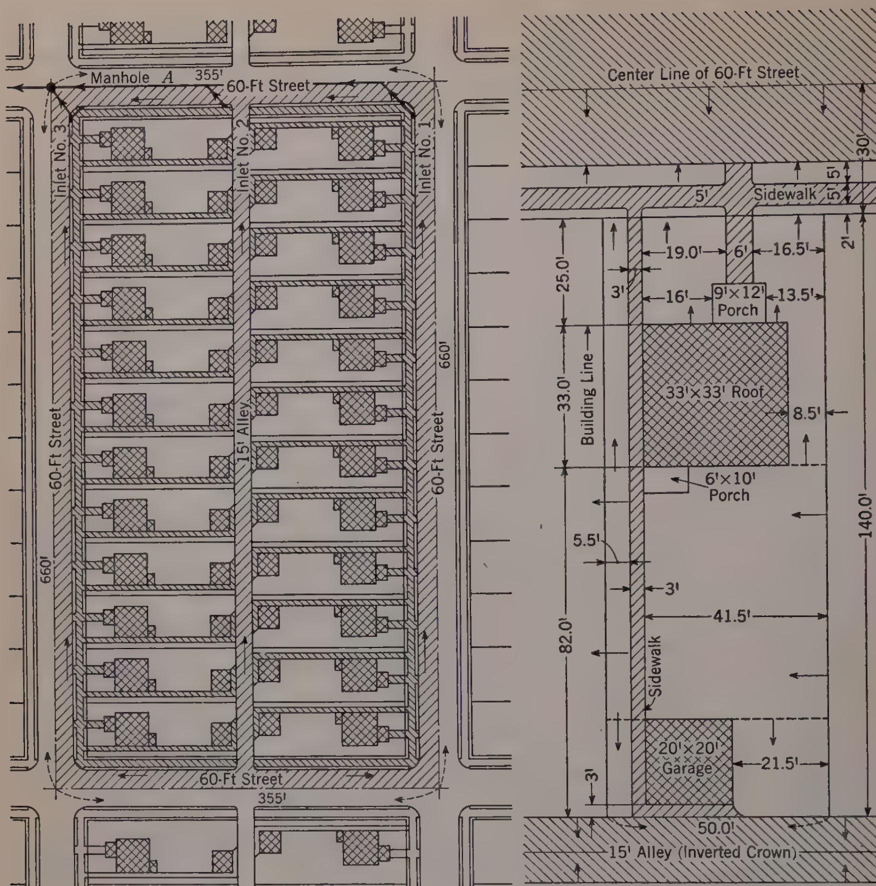


FIG. 7.—RAINFALL CURVES; 20-MIN RAIN

classes important (see Table 2), but the position of the area within the block significantly affects the character of its runoff hydrograph. Equally important are the surface slopes of each component part of the area. Figs. 5, 6, and 7 are based on overland flow slopes of 1% and alley gutter slopes of 0.5%.



(*a*) PLAN (*b*) DETAILS OF LOT

FIG. 8.—TYPICAL URBAN BLOCK

TABLE 2.—AREAS TRIBUTARY TO INLETS SHOWN IN FIG. 8

No.	Description	Inlets Nos. (See Fig. 8(*a*)): 1	2	3	Manhole *A*	Fig. 8(*b*)
1	Area, in acres	1.3509	2.5547	1.4731	5.3787	0.2038
	Percentage:					
2	Pervious	37.1	72.8	35.6	53.6	57.2
3	Impervious	62.9	27.2	64.4	46.4	42.8

It is assumed herein that the sewers to be designed are separate storm sewers, that there is no storm sewer abutting the lot frontage, and that therefore all drainage is routed through either the street gutters or the alley pavement. Roof drainage is presumed to be collected in house gutter systems and carried through downspouts and short sewers to outlets through the curbing into the

gutter. In this respect the conditions are different from those heretofore described for the St. Louis blocks gaged, where the sewers were of the combined type, and the roof drainage was connected directly to the sewers.

Routing roof water through the street gutters will increase, slightly, the time of its arrival at the collecting point manhole (manhole A, Fig. 8(a)) and give a slightly delayed runoff peak for the city block as a whole.

Infiltration Capacity

Fig. 5(a) shows a curve of infiltration capacities throughout the precipitation period. This curve is not intended to represent a specific condition accurately, but follows somewhat the curve of infiltration capacity under 2-in. rainfall rates, as computed from the small plots tested at Washington University in 1923 (see Fig. 3(b)). It may be characterized as representing, rather closely, the march of infiltration capacities that would exist for a turfed area; a soil condition consisting of about 4 in. of loam underlain with yellow clay, and a moisture condition that would exist in St. Louis during a midsummer period in which heavy rains occurred about twice a month, the last one approximately two weeks in advance of the conditions that this curve represents.

As far as is known, no sprinkling plot experiments have been made on well-developed city lawn grass. The nearest approach to such a situation, other than the Washington University tests, is in the runs made on the Anacostia Parkway, for which one graph has been shown (Fig. 3(a)). For that area, however, the soil is a plastic mud fill and not comparable to the ones referred to herein.

A preliminary study of rainfall-runoff relationships reported in 1934[10] indicates that both infiltration capacity and retention characteristics are higher for such lawns than for the sprinkling plots otherwise reported. This is undoubtedly due to the fact that such lawns are frequently mowed, are undisturbed by cultivation, and that it is possible, therefore, for extensive porosity to develop because of dead root growth and absence of compaction. Retention values are also increased because of the mat of grass-cutting litter that accumulates at the ground surface, and infiltration opportunity is extended for the same reason.

The calculations herein will involve somewhat lower infiltration capacities than may be expected under the same summer conditions on the city lawns, and somewhat less retention than will exist on such lawns; and therefore the resulting rates of runoff will be higher than would be expected for such conditions. This fact, however, does not affect the validity of the calculations with respect to the assumptions on which they are based.

In Fig. 5(a), the infiltration-capacity curve is shown in the position it would have held had it been actually derived from such a plot under a uniform 2-in. rainfall rate. For application to the precipitation patterns shown in Figs. 5(b), 5(c), and 5(d), the position of the curve has been adjusted slightly. For the purpose of this adjustment, it was assumed, the initial condition being

[10] "Relation Between Rainfall and Run-Off from Small Urban Areas," by W. W. Horner and F. L. Flynt, *Transactions*, Am. Soc. C. E., Vol. 101 (1936), p. 140 (first published in *Proceedings*, Am. Soc. C. E., October, 1934).

the same for each diagram, the infiltration capacity would arrive at any specific rate when the mass infiltration was the same as that which preceded the occurrence of such a rate in Fig. 5(*a*). Specifically, in the adjusted positions, the 2-in. infiltration-capacity rate is so placed in time that the mass infiltration occurring prior to that time will be the same for each of the diagrams; and a similar adjustment was made with respect to other critical rate points on the curve.

For the rates of 40 min and 20 min duration (Figs. 6 and 7) the infiltration capacity was adjusted in a similar manner. It is recognized that present knowledge of the mechanics of infiltration is not such as to validate this adjustment procedure fully, but a study of the St. Louis infiltration-capacity curves appears to indicate that for any initial condition, the march of infiltration-capacity rates is closely correlated with mass infiltration. There is good evidence of a similar correlation at the Edwardsville station of the Soil Conservation Service.

Precipitation, Infiltration Capacity, and Net Rainfall

The elemental surfaces shown on the city block fall into either the pervious or impervious class. The position, size of these surfaces, and the direction and length of overland flow are shown subsequently in Figs. 10(*b*) and 11(*b*).

For simplicity, it is assumed for the impervious surfaces that there is no permanent retention and no infiltration, although in an actual case a very small amount of each may exist. Therefore, net rainfall is equal to the precipitation, and the rate of production of net rainfall or "supply" (hereafter given a symbol σ) is represented by a precipitation intensity diagram.

For the pervious surfaces, it is assumed in all cases that the permanent retention is 0.1 in. It appears to be reasonably representative of the turfed areas that have been studied under sprinkling plot experiments. One half of this retention, or 0.05 in., is assumed to be of the character of interception, and is taken out of the first 0.05 in. of rainfall that occurs. The other 0.05 in. is treated as depression storage and is taken out as the first 0.05 in. of net rainfall after infiltration capacity drops below the precipitation rate. As stated, these values are undoubtedly somewhat lower than will actually exist on well-developed city lawns.

With this allowance for retention, the rate of production of net rainfall or "supply" is determined by subtracting the infiltration capacity rates from the precipitation intensity rates, and the result is shown in Figs. 5, 6, and 7; and, as so shown, represents the diagram of net rainfall or rate of production of surface runoff for the pervious area.

These two "supply" diagrams (that is, the precipitation intensity diagram as to impervious surfaces and the net rainfall diagram as to pervious surfaces) determine the mass values of surface runoff that are thereafter unchanged (except for a small amount of infiltration during the recession period), and appear in the same values under the varying rate diagrams for overland flow for each particular surface, for inflow into the street and alley gutters and for the final runoff hydrograph for the city block as a whole, at manhole *A* (Fig. 8(*a*)).

Overland Flow and Surface Detention

The characteristics of overland flow have been the subject of extensive investigation through the analysis of rainfall plots. It has been extensively discussed by Mr. Horton.[7, 11] The field investigations have shown that overland flow in thin sheets may be entirely turbulent or partly turbulent and partly laminar. For smooth surfaces, it appears that fully turbulent flow will be developed normally, and that the Manning formula is applicable. For subdivided flow through grass, most of the experimental data indicate that the flow is laminar to some extent and may be reasonably represented by a condition of 75% turbulent flow, in which case the profile of overland flow may be assumed to be parabolic and the flow to take place in accordance with the relation,

$$Q = k\,\delta^2 \qquad (1)$$

in which: δ is the depth of flow at any point; k is a constant; and Q is the rate of flow. Since the profile of the water surface is parabolic, in this case, the mean depth of surface detention may be taken as two thirds of the depth at the outflow margin of any strip, and the rate of outflow may be expressed either in terms of mean depth or of outflow depth. For the purpose of this demonstration, it is assumed that the flow from paved impervious surfaces will take place in accordance with the Manning formula, using an n of 0.015, and may be represented by the equation

$$V = 18.9\,\delta^{0.67}\,S^{0.5} \qquad (2a)$$

in which V is in feet per second and δ is in inches. It is assumed that the flow over turfed surfaces will be 75% turbulent. Coefficients have been adopted from a study of several sets of experimental data, and the flow formula used is

$$V = 0.96\,\delta\,S^{0.5} \qquad (2b)$$

The values of n derived from experimental studies of turfed plots and used in Eq. 2*b* are not of the same order as the Manning n, because the condition is actually one of subdivided flow. For evaluation of overland flow through turf, it seems advisable to avoid the use of values comparable to the Manning n and to discuss rather the velocity coefficients in Eq. 2*b*. The Anacostia plot (Fig. 3(*a*)) gives a coefficient of slightly more than 0.6. The aforementioned value of 0.96 was derived by the writers from experiments reported in 1938.[12] Unpublished values for flows 1 to 2 in. deep through Bermuda grass[13] indicate that these can be expressed by a coefficient of about 1.2, although the flow was turbulent. In general, these values over a considerable range of turf conditions seem to have less variation than the n-values for natural stream channels.

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[11] *Transactions*, Am. Geophysical Union, 1939, p. 693.

[12] Unpublished manuscript of the SCS entitled "Results of Studies Involving the Application of Rainfall at Uniform Rates to Control Plot Conditions," by C. M. Woodruff, D. D. Smith, and Darnell M. Whitt, June, 1938.

[13] "Some Experiments on Shallow Flows over Grass Slope," by W. O. Ree, Spartansburg Lab., SCS, *Transactions*, Am. Geophysical Union, 1939, Pt. IV, p. 653.

If any specific project is undertaken for the development of truly representative hydrographs of flow involving the turf on city lawns, the value of the coefficient in Eq. 2*b* should be given some further study. For the reasons heretofore discussed, it appears that on well-developed city lawns, this coefficient may be as low as that found for the Anacostia Parkway—that is, on the order of 0.6. This assumption is borne out in part from an examination of an actual hydrograph for one of the city blocks studied previously,[10] where the delayed peak and the considerable bulk of flow occurring after the end of rainfall indicate in part relatively low velocities of overland flow.

On the basis of the flow-depth relationships of the foregoing type, Mr. Horton has developed, for 75% turbulent flow, the equations for the rising side of the hydrograph, and for the depth of detention corresponding to any rate of flow.[14] These equations, when developed from Eqs. 2, become:

For turf—

$$q_s = \sigma \tanh^2 \left[\frac{1.5 \sqrt{3{,}520}\, \sigma^{0.5}\, S^{0.25}\, t}{60\, l} \right] \qquad (3a)$$

and

$$\delta = \frac{l^{0.5}\, \sigma^{0.25}}{S^{0.25} \sqrt{3{,}520}} \tanh \left[\frac{1.5 \sqrt{3{,}520}\, \sigma^{0.5}\, S^{0.25}\, t}{60\, l} \right] \qquad (3b)$$

Figs. 9(*b*) and 9(*c*) show families of curves for runoff across turf strips for different slopes and lengths.

For pavement, the writers have developed the following relationships:

$$q = \sigma \tanh^{1.67} \left[\frac{1.60\, (1{,}020)^{0.60}\, S^{0.30}\, \sigma^{0.60}\, t}{n^{0.60}\, l^{0.60}\, 60} \right] \qquad (4a)$$

and

$$\delta = \frac{\sigma^{0.60}\, n^{0.60}\, l^{0.60}}{(1{,}020)^{0.60}\, S^{0.30}} \tanh \left[\frac{1.60\, (1{,}020)^{0.60}\, S^{0.30}\, \sigma^{0.60}\, t}{n^{0.60}\, l^{0.60}\, 60} \right] \qquad (4b)$$

In Eqs. 4, q is expressed in inches per hour, σ = the rate of supply of excess rainfall in inches per hour, S = the absolute slope, l = the length of overland flow in feet, t = time in minutes, n = the coefficient of roughness, and δ = the depth of detention, in inches, at the outflow margin at any time t. The mean depth of detention is for turf $\frac{2}{3}\,\delta$ and for pavement approximately $\frac{5}{8}\,\delta$.

For any specific area of surface, such as those shown in Fig. 8(*a*), S, l, and n are inserted and Eqs. 4 then show the relation between the rate of overland flow or the marginal depth of overland flow and the rate of supply at any time t. For each condition of flow, Eqs. 4 have been graphed for overland flow over 10.5-ft and 7.5-ft lengths of pavement, for various rates of supply. (See Fig. 9*a*.)

In determining the hydrograph of overland flow, the q equations, Eqs. 3*a* and 4*a*, are applied to the rate of supply shown on the diagram, as for example, in Fig. 5(*a*). Where the rate of supply is varying, the procedure is simplified by reducing the rates to uniform rates for short periods. Where the rate of supply is uniform for a considerable time, the rising side of the hydrograph

[14] "The Interpretation and Application of Runoff Plat Experiments," by Robert E. Horton, *Proceedings*, Soils Science Soc. of America, Vol. 3, 1938.

is that represented by Eqs. 3a and 4a for any single rate. If the uniform rate of supply continues for a considerable time, equilibrium flow is established and the rate of overland runoff becomes equal to the rate of supply.

If the supply rate increases either before or after equilibrium flow has been established, the rising side of the hydrograph is continued as follows: The surface detention at a time when the supply rate increases is related to the

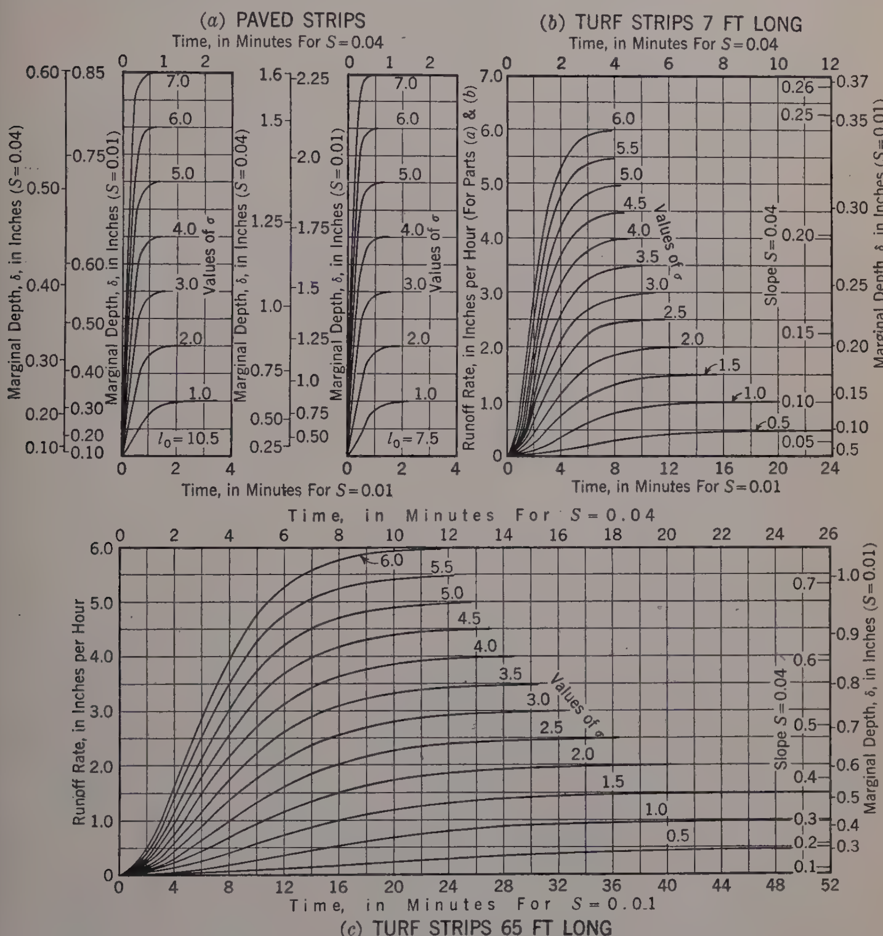


FIG. 9.—RUNOFF RATES, OVERLAND FLOW

value of Q by Eq. 1. This same value of detention would have been produced under the second supply rate at a different time, and the Q corresponding to such δ at that time has the same value as the Q resulting from the first supply rate. Accordingly, it is possible to join two different curves representative of the rising side of the hydrograph for different supply rates. For example, in Fig. 9(c), if the initial supply rate was 1.5 in. per hr and it lasted for 16 min,

the rate of runoff at that time would be 1 in. per hr. If at that time the supply rate changed to 2.5 in. per hr, the hydrograph can be continued by using the curve for the 2.5-in. supply rate at the point of equal flow which, for that curve, is found at 8 min. The rising side of the hydrograph will then develop along this curve until the supply rate is again changed.

When the supply rate is reduced from one value to another, the hydrograph takes the form of a recession curve, the rate at the lower end of which is the equilibrium rate of flow for the new supply rate. The hydrograph, therefore, continues along the recession curve, dropping to the new supply rate, and then uniformly along that supply rate value until another change takes place.

At the end of the precipitation period, or at the end of the supply period, the hydrograph continues as a recession curve. In this case the mass runoff under the recession curve must be equal to the mean detention at the time when the supply rate is terminated, minus the mass disposed of by infiltration during the recession period. This application of infiltration is necessary for the reason that, whereas the original supply rate was determined by subtracting infiltration capacity from precipitation rate for the period of rainfall, the infiltration subsequent to the end of precipitation has not otherwise been evaluated.

The character of the recession curve of overland flow raises interesting hydraulic questions with respect to which further research is desirable. In order to permit a mathematical analysis of this problem, it is necessary to know the relation during the recession period of the outfall depth at the control section to the mean depth of detention. Studies made by the writers indicate that the parabolic profile, which seems to be characteristic of the rising side of the hydrograph, cannot persist for any length of time during the recession period. An analysis of the relationships of the assumption that the water on the strip maintains a parabolic profile gives a very simple equation for the recession side, but this equation indicates a discharge of the detained water much more rapidly than the results of experiments indicate to be possible. It seems obvious that, at the end of the supply period, a drawdown occurs at the outlet, changing the relationship between the control depth and the mean depth of detention. An analysis of the recession curve for the Anacostia plot shown in Fig. 3(*a*) indicates that the relation of outflow rate to the simultaneous mean depth of detention may be expressed approximately as

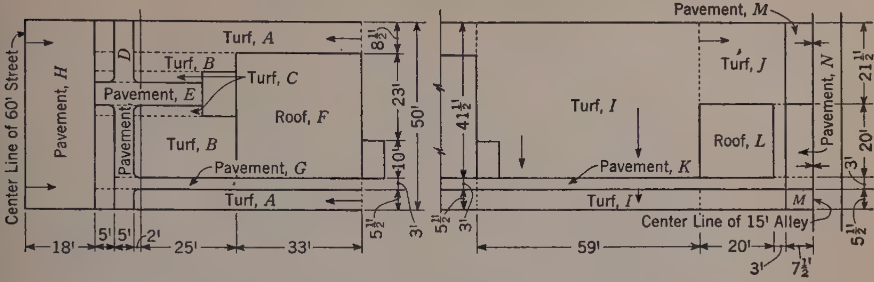
$$q = 18\,\delta_a \quad\quad (5)$$

in which δ_a represents the average depth, over the entire tributary area, of the residual detention at that time. Further study of a large number of such recession curves may give a better indication of how the water surface profile is modified during this period. For the purpose of the demonstration of this paper, the recession curves have been adjusted to conform generally to those of experimental plots that have been examined.[15]

The effect of this procedure in transforming the supply rate shown in Fig. 5(*a*) to a hydrograph of surface runoff, for each of the elemental areas

[15] See "A Graphical Analysis of Sprinkled Plat Hydrographs," by A. L. Sharp and H. N. Holtan, *Transactions*, Am. Geophysical Union, 1940, p. 558.

TABLE 3.—Areas Involved in Hydrographs of Figs. 10 and 11



(*a*) Lot to Street Corner

Unit (See Fig. 10)	Area, in acres	Percentage of total	Length of strip, l_0, in ft
A[a]	0.0209	20.40	65
B[a]	0.0158	15.40	32
C[a]	0.0032	3.10	23
D[b]	0.0047	4.58	5
E[b]	0.0038	3.81	28
F[c]	0.0288	28.10	....
G[b]	0.0045	4.39	65
H[b]	0.0207	20.22	18
Total	0.1024	100.00	....
Pervious	0.0399	39.00	....
Impervious	0.0625	61.00	....

(*b*) Lot to Alley

Unit (See Fig. 11)	Area, in acres	Percentage of total	Length of strip, l_0, in ft
I[a]	0.0023	61.30	47
J[a]	0.0143	14.03	23
K[b]	0.0062	6.09	89.5
L[c]	0.0092	9.05	....
M[b]	0.0046	4.57	7.5
N[b]	0.0048	4.76	10.5
Total	0.1014	100.00	....
Pervious	0.0766	75.5	....
Impervious	0.0248	24.5	....

[a] Turf. [b] Pavement. [c] Roofs.

shown in Table 3, is illustrated in detail in Figs. 10 to 14 which follow. In all of these computations, the runoff rates are maintained in inches per hour.

The rainfall chosen to illustrate the application of this method is the simple one presented in Fig. 5(*a*). This rainfall is reproduced in Fig. 10(*a*), showing the respective outflow graphs for each of the elemental areas draining to the street. It is noticeable that for the smoothly paved areas equilibrium flow is quickly established, and the outflow graph departs comparatively little from the precipitation or "supply" diagram. The hydrographs for the turfed areas reflect both the effect of infiltration and surface detention. It will be noted that runoff becomes perceptible at about 18 min, which is approximately the beginning of the supply rate diagram as shown in Fig. 5(*a*). The difference between the hydrographs for areas *A*, *B*, and *C* reflects the difference in the length of overland flow, all of them having the same supply rate and the same surface slope, for this illustration, of 1%. The effect of the longer overland flow for unit *A* is also apparent in the increased depth of surface detention which results in the greater mass under the recession curve *A*.

In Table 3(*a*), each of these unit areas is shown in its actual acreage and also as a percentage of the area of the lot draining to the street.

In Fig. 10(*b*), the ordinates of the hydrographs in Fig. 10(*a*) have been multiplied by their respective areal percentages, and therefore appear in terms of inches per hour for the entire tributary area shown on the small plat in

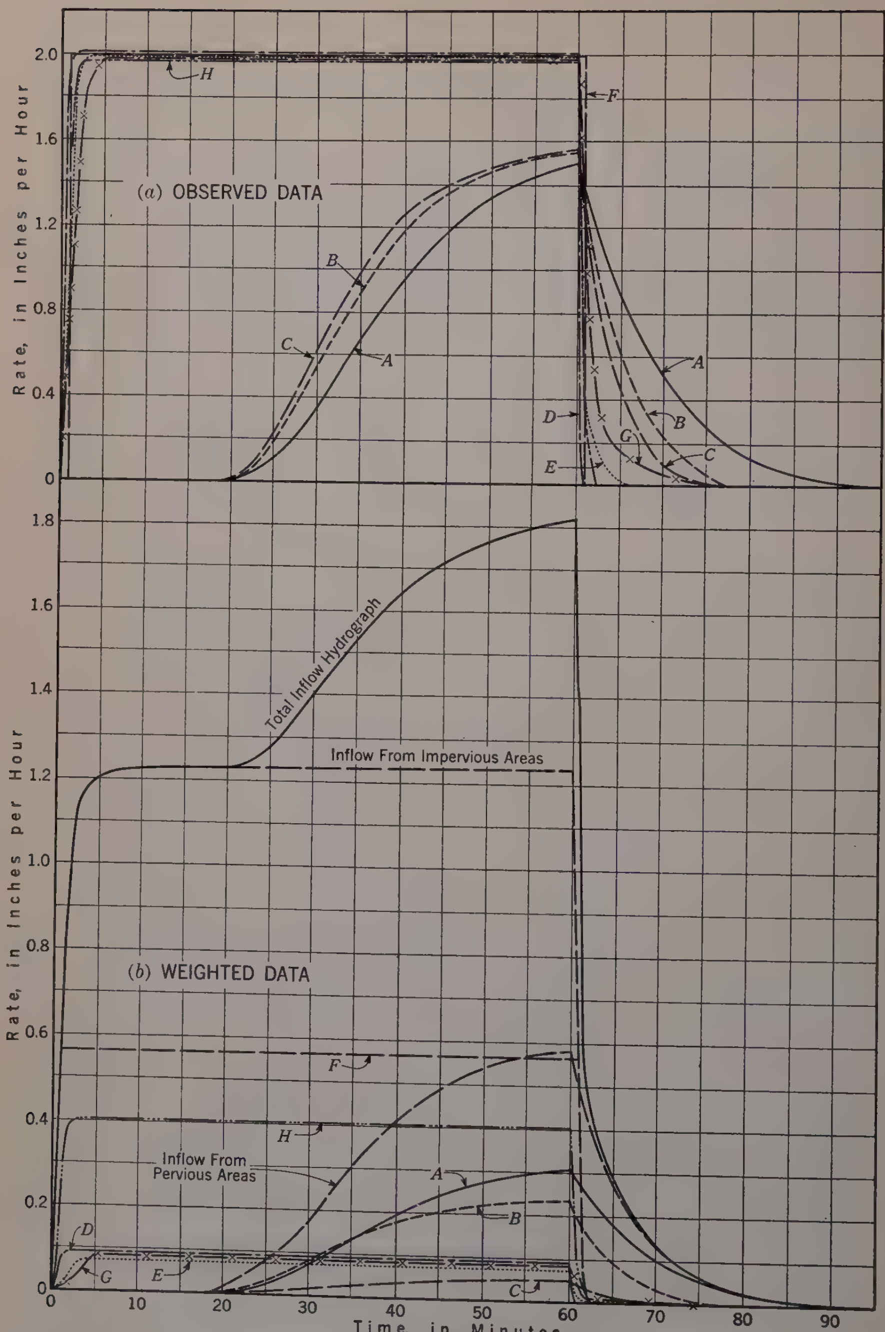


FIG. 10.—HYDROGRAPHS OF FLOW FROM A TYPICAL LOT TO A STREET GUTTER (RAIN, 2 IN. PER HR FOR 60 MIN)

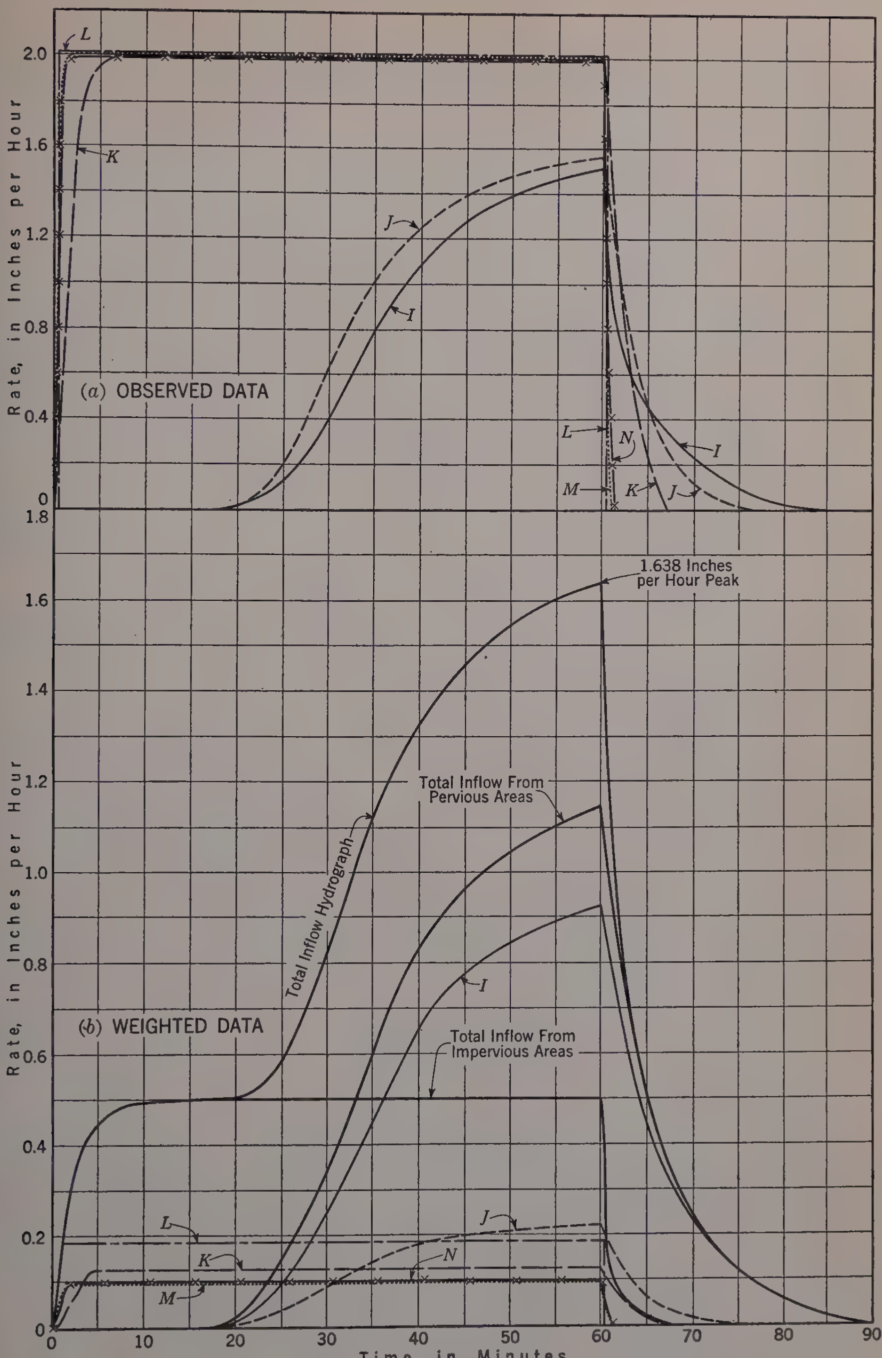


FIG. 11.—HYDROGRAPHS OF FLOW FROM A TYPICAL LOT TO AN ALLEY (RAIN, 2 IN. PER HR FOR 60 MIN)

Table 3(*a*). In order to visualize the relative effect of the pervious and impervious areas on the final outflow hydrograph from this entire part of the lot into the street gutter, the ordinates have been accumulated separately for the impervious and pervious units. These are then added to form the final hydrograph entitled "Total Inflow Hydrograph" (that is, inflow to the street gutter). This shows clearly how, until the time that the infiltration capacity became equal to the rainfall rate, plus the additional time required to satisfy retention, the hydrograph is developed entirely from the flow from impervious surfaces, and that equilibrium flow from these surfaces has been established before the flow from the pervious areas becomes effective.

At this point it is well to note again that the character of the hydrograph does not result from the relative percentages of pervious and impervious areas alone, even for any one slope. It is distinctly sensitive to the length of overland flow on the turf strips and to the relative position of the pervious and impervious areas with respect to the outflow channel. In this example all of the smooth impervious areas, with one minor exception, have been shown as having direct-flow access to the street gutter. If certain of these areas were so placed as to discharge their flow on to the turf, the character of the hydrograph would have been materially different.

Fig. 11 shows the development of the hydrographs from the rear of the lot draining to the inverted alley channel. The general effect of the pervious and impervious areas is the same as for the street system, but the shape of the hydrograph is materially different because of the greater percentage of pervious area and of the greater length of overland flow on it.

These illustrations do not reflect the effect of slope, as a uniform slope of 1% has been utilized for all areas within the lot. The effect of slope, however, is material, particularly on the turfed areas. Separate graphs, prepared for steeper slopes, are not presented at this time, but where the turfed areas have slopes of, say, 4%, the surface detention is materially reduced and the peak rate of runoff materially increased, even though the infiltration capacity rates are the same. The result of this application of hydraulics to the net rainfall or supply rate is the production of the actual hydrograph of overland flow that may thereafter be treated as the inflow hydrograph to the street gutter or the inflow hydrograph to the inverted alley channel.

Gutter Inflow, Gutter Storage, and Gutter Outflow to the Street Inlet

For this demonstration, inasmuch as it is assumed that each of the building lots has been laid out and developed in the same manner, the inflow to the gutter or to the alley is uniform from all lots. Therefore, the gutters represent hydraulic systems having a free outflow to the sewer inlet at the lower end and an inflow which may be taken as increasing from the upper to the lower end in proportion to length. It is proposed here to examine the effect of gutter-flow characteristics on the hydrograph by the use of the storage equation in which the outflow at the inlet Q is equal to the inflow represented by the hydrograph in Figs. 10(*b*) and 11(*b*), respectively, plus or minus the rate of change of storage in the gutter.

To permit this application to be made, it is necessary to develop, for any condition of equilibrium flow, the relation of the storage volume in the gutter to the discharge Q at the inlet. This storage volume is readily determinable when the character of the water surface profile is known. In this case, the relation between rate of flow to length of flow along the gutter is a linear one, in which

$$\frac{Q_x}{x} = \frac{Q_l}{l} \qquad (6)$$

l being the total length of the gutter, and x the distance along the gutter measured from the upper end. For the alley-channel gutter, the width of the water surface is great compared to the depth, and the hydraulic radius may be closely approximated as being equal to the mean depth.

In ordinary municipal practice, it has been the custom to assume that the water surface is parallel to the flow-line gradient, in which case the depth h_x at any point x becomes a direct exponential function of the rate of flow Q_x at that point. For the conditions: $A = 15\,h^2$; $v = \frac{1.486}{0.015} R^{0.67} S^{0.5}$; $R = \frac{h}{2}$; and $v = 62.4\,h^{0.67}\,S^{0.5}$

$$Q = A\,v = 936.1\,h^{2.67}\,S^{0.50} \qquad (7a)$$

and, assuming a gutter 600 ft long,

$$V = 5{,}142.9\,h^2 \qquad (7b)$$

For this condition of flow, in the alley gutter shown in Fig. 12(b), the equation

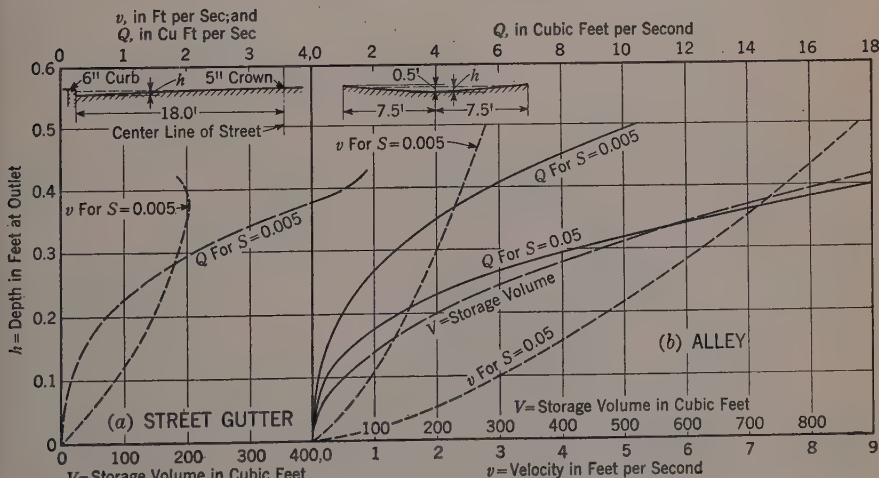


FIG. 12.—HYDRAULIC CHARACTERISTICS

between h_x and Q, the outfall discharge becomes

$$Q = \frac{936\,l}{x} S^{0.5}\,h_x^{2.67} \qquad (8)$$

The assumption involved in Eq. 7*a*, of course, is physically inconsistent, as the values of h cannot vary along the gutter without producing varying values of the water surface slope i.

In connection with this demonstration, considerable study was made with respect to the character of the genuine water surface profile for the alley gutter. It was found that a rigid analysis of this hydraulic problem is quite involved, and that the matter could be approached best by successive approximations. The water surface profile has been investigated in this manner with the following conclusions:

For the channel cross sections shown in Fig. 12 for rates of grade in excess of 0.004 and for the range of flows that would normally be encountered, the exponential formula, Eq. 7*a*, is reasonably representative of the water surface. A second approximation involving varying water slopes determined from the values of h growing out of Eq. 7*a* results in only a slight change in the profile in the upper 20% of the length and, of course, if computed accurately, a slight change at the outlet due to the discharge drawdown curve. These changes affect storage volume so slightly that the discharge-depth relationship may be determined from Eq. 7*a* within the limits of accuracy justified.

The limitations between which these relationships are sufficiently accurate should be noted carefully. For extremely flat grades (that is, those less than 0.0025 on smooth surfaces such as pavement), and for grades of less than 1% where the flow for such cross sections occurs over turf, the water profile should be determined carefully by successive approximations and the discharge-storage equation developed by summation. This has been found to be particularly important in studies made for the drainage of the National Airport at Washington, D. C., where the channel flow is entirely through flat turfed gutters. Under these conditions, storage volume reaches large quantitative values with respect to discharge Q, must be rather accurately determined, and affects, importantly, the relation between channel inflow and outflow.

The relationship between storage volume and depth at the outlet has been evaluated and is shown for the alley on the control graph, Fig. 12(*b*). It can also be expressed directly as a relation between discharge rate and storage volume.

This can be done by eliminating h between Eqs. 7*a* and 7*b* for the alley, which results in the following relationship:

$$V = k\,Q^{0.75} \qquad (9a)$$

in which V is in cubic feet; and Q in inches per hour for tributary area A in acres and, for this block,

$$k = \frac{60\,l}{7}\left(\frac{A}{936\,S^{0.5}}\right)^{0.75} \qquad (9b)$$

For a short finite Δt in which the hydrograph may be taken as a straight line, the insertion of these values in the storage equation, with inflow I in inches

per hour, gives

$$60\, I\, \Delta t - \frac{Q_1 + Q_2}{2}\, 60\, \Delta t - (k\, Q_2^{0.75} - k\, Q_1^{0.75}) = 0 \quad \text{.......(10)}$$

Rearranging the terms, this can be expressed as

$$fQ_2 = I + fQ_1 \quad \text{.......(11)}$$

in which (expressing I, Q_1 and Q_2 in inches per hour)

$$fQ_2 = \frac{2\, k\, Q_2^{0.75}\, A^{-1/4} + 60\, Q_2\, \Delta t}{120\, \Delta t} \quad \text{.......(12a)}$$

and

$$fQ_1 = \frac{2\, k\, Q_1^{0.75}\, A^{-1/4} - 60\, Q_1\, \Delta t}{120\, \Delta t} \quad \text{.......(12b)}$$

This relationship is shown graphically in Fig. 13. Similar diagrams were prepared for the street gutter and were used to translate the gutter inflow

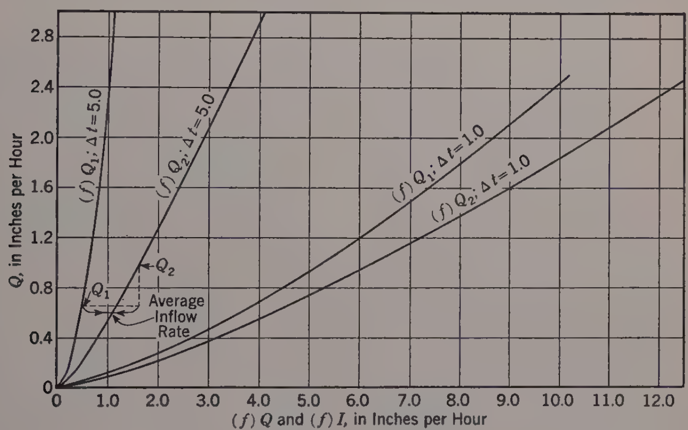


FIG. 13.—GRAPHS FOR THE SOLUTION OF THE STORAGE EQUATION FOR THE ALLEY

hydrograph to the gutter outflow hydrograph. Because of the character of the street gutter cross sections, the equations are quite involved, and the work was done graphically.

In this application, the value of I is the mean value for the period Δt; Q_1 is the value at the beginning of Δt; and Q_2 is the value at the end of period Δt in min; A is in acres. The graph in Fig. 13 has been prepared by inserting the value of k representative of the total tributary area to the alley for a length of 600 ft, and a slope of 0.005. The method of determining Q_2 from Q_1 is shown in an example by the dotted lines in this diagram.

For the purpose of this application, the inflow hydrographs of the type shown in Fig. 11(b) were modified by evaluating the rate of change of storage and adjusting the ordinates accordingly. The resulting outflow graph to the inlet from the alley channel is shown in Fig. 5(a).

It will be noted that for this case the storage volume is so small with respect to the mass of the flow quantity that the channel storage has no effect on the peak rate. However, it does modify the earlier part of the hydrograph somewhat. The earlier part of the hydrograph is important in translating these values to flow at a critical point on the sewer system, and therefore this modification also becomes important to the ultimate peak flow in the sewer.

The effect of gutter channel storage for the shorter and more intense rains is not apparent from the graphs. Actually, however, the reducing value of this gutter storage becomes highly important for the rains of 20-min duration. For example, the hydrograph shown in Fig. 7(*d*) for the delayed-pattern, 20-min rain, when compared to the inflow hydrograph to the gutters, shows that the gutter storage not only affects the rising side of the hydrograph, but actually reduces the peak rate about 30%. A similar but less drastic reduction is found for the other 20-min rains, and the effect is noticeable for the 40-min rains.

In Fig. 14, the outflow graphs to the inlets for each part of the block have been multiplied by the percentage that each part is of the total block area,

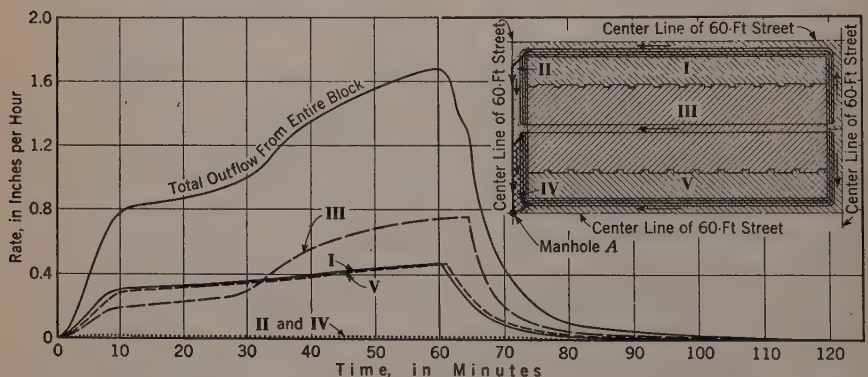


FIG. 14.—HYDROGRAPHS OF FLOW FROM A TYPICAL BLOCK (RAIN, 2 IN. PER HR FOR 60 MIN)

and therefore transformed to inches per hour for the block as a whole (see Table 1 for areas). It has been assumed that the inlets are discharging into a sewer on the cross street; for this purpose the velocity in the sewer is taken as 5 ft per sec, the time of flow between the inlet connections has been evaluated, the hydrographs have been offset accordingly, and the ordinates have been summed up to produce the total outflow graph from the entire block. (In Fig. 14, curves I, II, III, and IV are the weighted hydrographs of flow into inlets from areas bearing the same numbers.)

This graph, and the companion graphs in Figs. 5, 6, and 7, can be accepted as representing, rather accurately, the manner in which the flood flow from this block will occur at the concentrating manhole *A*. The flow so represented may be accepted as that which would result from the conditions assumed. Among these conditions, the character of the infiltration capacity curve and the amount of retention are important. As has been previously discussed, such graphs, if they are to be used quantitatively in sewer design, should be

based on infiltration capacities and on retention values that have been developed out of actual flow gagings of such city lots. Undoubtedly the infiltration capacity for the turf on such lots and the characteristic retention on such turf will be found to be higher than those determined from the analysis of sprinkling plats. In this connection, the writers have made a quick preliminary review of the results of the gagings from station *B* (city block 4841 in St. Louis).[10] The development of this block as to character of surface cover and as to slopes is rather similar to the assumed city block outlined in Fig. 8(*a*). The principal difference is that the actual St. Louis block is nearly twice as long as that shown in Fig. 8(*a*). An analysis of the actual losses that occurred from the 70% pervious area in city block 4841 for a number of rains shows the results presented in Table 4. In this form, the values of losses have no specific

TABLE 4.—City Block 4841, St. Louis, Mo.; 70% Pervious

Date	Duration of rain, in min	Precipitation, in inches	Infiltration plus retention, in inches	Peak rate of runoff from area tributary to alley, in inches per hr	Date	Duration of rain, in min	Precipitation, in inches	Infiltration plus retention, in inches	Peak rate of runoff from area tributary to alley, in inches per hr
8– 2–15	60+	1.29	1.22	1.45	10–27–18	15	0.63	0.42	1.00
5–28–16	30	0.64	0.24	1.50	4–19–20	60+	1.30	0.68	1.95
6– 2–16	20	0.91	0.55	1.55	8–27–21	40	1.77	1.27	1.60
8–11–16	20	0.77	0.60	1.55	8– 8–23	50	3.51	3.00	2.45
8–12–16	60+	1.14	0.65	1.35	6–23–24	50	1.36	0.92	1.50
9– 7–16	10	0.78	0.54	1.65	8–24–24	15	1.13	0.89	1.80

significance. However, they can be broken down by analysis, between retention and total infiltration. The amount of infiltration, by comparison with the precipitation diagram, can be checked against infiltration opportunity. In this way, not only can mean infiltration capacity be determined, but by analogy to sprinkling plot results, an approximation can be made of the trend of infiltration capacity curve.

Of course, the peak rate of block flow such as the 1.68 in. per hr of Fig. 5(*a*) for manhole *A* is not the critical rate for which the sewer would be designed at the outlet of this block. The importance of this hydrograph relates to the design of the sewer at some point where the so-called "critical time" is approximately 60 min. At that point on the sewer, if all the blocks in the drainage basin had the same typical development as shown herein, the hydrograph of discharge may be determined by offsetting the hydrographs from the various blocks, adding the ordinates, and making some further modification in the shape of the hydrograph that would result from the translation of the flood wave under this condition. This process has been applied to each of the hydrographs, the routing being traced through a typical rectangular sewer system.

The outflow graph at manhole *A*, representing the hydrograph of outflow from the entire city block, has been calculated for each of the three duration periods and each of the four rain patterns, and the full hydrographs are shown in Figs. 5, 6, and 7. In these figures there has also been indicated a value of the net rainfall or supply graph for the block as a whole. This is a weighting

of the supply graphs for the pervious and impervious area with respect to the proportionate amount of these two types in the block.

For ease in inspection there has also been noted on each of the graphs the character of the hydrograph as it would appear after routing to the flow time point on the sewer system equivalent to the duration represented by the particular rain. By inspection, therefore, it is possible to note the following:

Comparison between the precipitation diagram and the weighted supply line shows the effect of the infiltration and the permanent retention that was introduced into the computations. A comparison between the supply line and the hydrograph at manhole *A* (Fig. 14) shows the combined effect of surface detention during overland flow and of gutter storage. The comparison between the hydrograph at manhole *A* and the hydrograph at the design point shows the equalizing effect resulting from the non-synchronous concentration of the flow from the various city blocks at this point. These last hydrographs, as to their peak values, are representative of the quantities such as have heretofore been commonly used in sewer design under the rational method. For those who are accustomed to thinking in terms of the coefficient of runoff, these peak rates may be compared with the mean precipitation intensity for the original duration period. These particular peak rates, of course, result from the particular network routing system used. This rectangular system with equally spaced branches, and with the flow time in the branch equal to the flow time in the main, is only one of a number of patterns that might have been applied. Higher peak rates would have resulted if the network system had been chosen of the sector type with the branches converging toward the time design point.

In order to give a general perspective view of the results of the calculations here presented, Table 5 has been prepared. All of the values given in this

TABLE 5.—Summary of Values for Assumed City Block, Fig. 8(*a*) (46% Impervious, No Impervious Surfaces Draining Over Turf)

No.	Unit Rates	20-Min Duration Precipitation Pattern				40-Min Duration Precipitation Pattern				60-Min Duration Precipitation Pattern			
		Uniform	Adv.[b]	Int.[b]	Del.[b]	Uniform	Adv.[b]	Int.[b]	Del.[b]	Uniform	Adv.[b]	Int.[b]	Del.[b]
	In Inches Per Hour:												
	Precipitation Rate—												
1	Mean	3.5	3.5	3.5	3.5	2.5	2.5	2.5	2.5	2.0	2.0	2.0	2.0
2	Maximum	3.5	6.9	6.9	6.9	2.5	5.0	5.0	5.0	2.0	4.0	4.0	4.0
3	Maximum supply rate	3.0	5.64	5.80	6.15	2.20	4.01	4.25	4.50	1.80	3.22	3.52	3.75
	Peak Flow Rate at:												
4	Manhole *A*	2.15	2.91	3.09	4.00	1.95	2.65	2.76	3.65	1.68	2.27	2.68	3.60
5	20-min point[a]	1.63	1.98	2.24	2.26	1.70	2.48	2.49	2.58	1.58	2.17	2.43	2.88
6	40-min point[a]	1.27	1.37	1.55	1.56	1.47	1.96	1.98	1.95	1.44	1.92	2.03	2.28
7	60-min point[a]	0.99	1.00	1.17	1.17	1.25	1.55	1.57	1.57	1.30	1.60	1.66	1.84
	In Inches of Depth:												
8	Mass precipitation	1.17	1.17	1.17	1.17	1.67	1.67	1.67	1.67	2.0	2.0	2.0	2.0
9	Mass infiltration	0.33	0.31	0.26	0.24	0.45	0.46	0.39	0.38	0.52	0.56	0.42	0.41
10	Mass supply	0.84	0.86	0.91	0.93	1.22	1.21	1.28	1.29	1.48	1.44	1.58	1.59

[a] Rectangular drainage network (see text). [b] Advanced, intermediate, and delayed, respectively.

table are expressed in inches per hour, or the approximate equivalent cubic feet per second per acre, except the mass values of supply Q and infiltration F, which are in inches in depth distributed over the entire tributary area. It should be noted again that, quantitatively, the values are responsive directly to the rainfall patterns and to the infiltration capacity values used, and as to the latter, result in runoff rates greater than would be expected with the frequency involved. The values given for peak flow at the various time-design points reflect the particular network sewer pattern previously described and would be materially different for a different type of collecting system. However, they all have been derived on the same basic assumption and through the same application of hydraulics, and therefore are definitely comparable.

Summary and Conclusions

Conventional Urban Areas.—The conception of a uniform rainfall rate for a specific duration period is an unsatisfactory basis with which to compare runoff rates. In the opinion of the writers a fully rational basis of design for urban storm drainage involves the following steps:

(*a*) A study of rainfall intensity distribution and of the sequence of intensities within certain arbitrary duration periods, such as those chosen in this paper.

(*b*) For a particular area, some further research is needed with respect to infiltration capacity and overland flow for the type of turf normally utilized for city lawns.

(*c*) With the necessary basic data, it is entirely practicable and not an unreasonably involved or expensive procedure to develop, through hydraulic processes, the type of hydrographs shown in this paper.

(*d*) The rates so determined, on the basis of the conditions for which they are derived, should be subjected to a further frequency investigation, and from these studies a series of design curves might be prepared that would give the peak rate of runoff expected from each of the typical blocks, for specific slope conditions, and at any particular frequency.

(*e*) No attempt should be made to reduce these basic design values to generally applicable flow rates at any "time point," but in the design of any particular sewer system they would form the basis of flow routing through the particular drainage network.

(*f*) The procedure in steps (*a*) to (*e*) is subject to organization and tabulation in a manner similar to that under which the rational method has been commonly applied. It removes, entirely, the completely irrational coefficient of runoff; it involves a further study and revision of the criteria of critical time; it implies that the critical runoff rates at the sewer inlets will be routed through the sewer network in such a manner that the maximum rate of flow which can occur at any design point will have been determined; its application initially will require the extensive preparation of essential basic data, and extensive studies as to the physical characteristics of typical city blocks. Its application in general, after basic hydrographs of determined frequency have been prepared, will be little more difficult than the methods now used.

PART III.—CONCLUSION

The demonstration presented herein for small conventional areas shows, in principle and outline, the manner in which hydraulic methods may be applied to the organization of basic data and the derivation of secondary data essential to the computation of flood flow rates. The writers believe that basic hydrologic data are now becoming available, of a quality and in an extent that justify the more detailed type of hydrologic analysis set out, and that hereafter the basis of design of important hydraulic structures may be developed in a more logical and dependable manner than has heretofore been considered feasible. That the computations involved will be tedious in some cases is recognized; but they need be no more extensive, nor complex, than the technical computations commonly underlying the design of an important bridge.

The procedure outlined in detail for small areas of conventional type has been applied with some modifications, but upon the same basic principles, to larger drainage basins and to natural channels.

When applied to larger areas, all matters may obviously be on a somewhat different scale, and some phases of the computation are properly of a more approximate character. However, even as to such large areas:

(*a*) Precipitation patterns can be prepared for each important sub-basin, preferably in units not exceeding 1,000 sq miles;

(*b*) Infiltration capacity values reasonably well related to actual soil and cover conditions can be accumulated separately for each sub-basin, and can be matched with the pertinent precipitation pattern in a logical manner;

(*c*) From such a combination, the rate of production of surface runoff from each sub-basin will be developed in the order and manner in which it would actually occur;

(*d*) The diagram of surface runoff can then be translated into the hydrograph of stream flow at any particular point in each sub-basin through the unit graph or a proper evaluation of surface detention and channel storage. The hydrographs for the sub-basins can then be synchronized and combined for the main stream. This application unquestionably requires more detailed physical data as to stream channels and valley storage areas, and involves a material increase in the cost of the design phase of the engineering of hydraulic structures; but it permits a marked reduction in the factors of ignorance that need to be applied thereto.

This procedure is restricted to the determination of flood flow from surface runoff. For many basins, ground water or base flow during the rise of the hydrograph will have a small value and may be neglected. For some conditions it may be large and may add materially to flood flow. Any detailed discussion of this question is beyond the scope of this paper.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

OPERATION EXPERIENCES, TYGART RESERVOIR

BY ROBERT M. MORRIS,[1] ESQ., AND THOMAS L. REILLY,[2] ESQ.

SYNOPSIS

Some of the problems involved in the operation of a large flood-control and water-supply project are reported in this paper. The writers demonstrate how effectively the Tygart Dam can perform the functions for which it was planned. For example, the effect of the reservoir in reducing flood crests at downstream points is demonstrated. The accuracy of these estimates is dependent on the method of flood routing and the thoroughness with which it is consummated. The elements involved in routing these floods are described only briefly because a complex problem of this type would require a separate paper for complete presentation.

INTRODUCTION

At the junction of the Monongahela and Allegheny rivers, in Pennsylvania, lies Pittsburgh, the steel city, favored by nature with bounteous coal supplies and three broad rivers for its transportation. Nature can be exacting for its favors, however. Records indicate that almost annually since the eighteenth century flood waters rolling down from the western slopes of the Appalachian Mountains have inundated some parts of the expanding city. At the time of the 1907 flood, when Pittsburgh had grown to be the steel center of the world, the valleys of the Allegheny and Monongahela rivers were lined with industrial developments. The low-lying "Triangle" at the junction of the two rivers was the crowded business center of this great industrial district. The flood of 1907, reaching then unprecedented heights, took a terrible toll in lives and property damage and awakened the citizenry to the realization that, since their city had been built practically in the flood plain of the rivers, some means must be found to check the annual flood menace.

The Pittsburgh Flood Commission, formed after the 1907 flood, made a detailed study of methods of flood control and published a complete report.[3]

NOTE.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 15, 1941.**

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[2] Ass't. Engr., U. S. Engr. Dept., Pittsburgh, Pa.

[3] Report of Flood Commission of Pittsburgh, Penna., April 16, 1912.

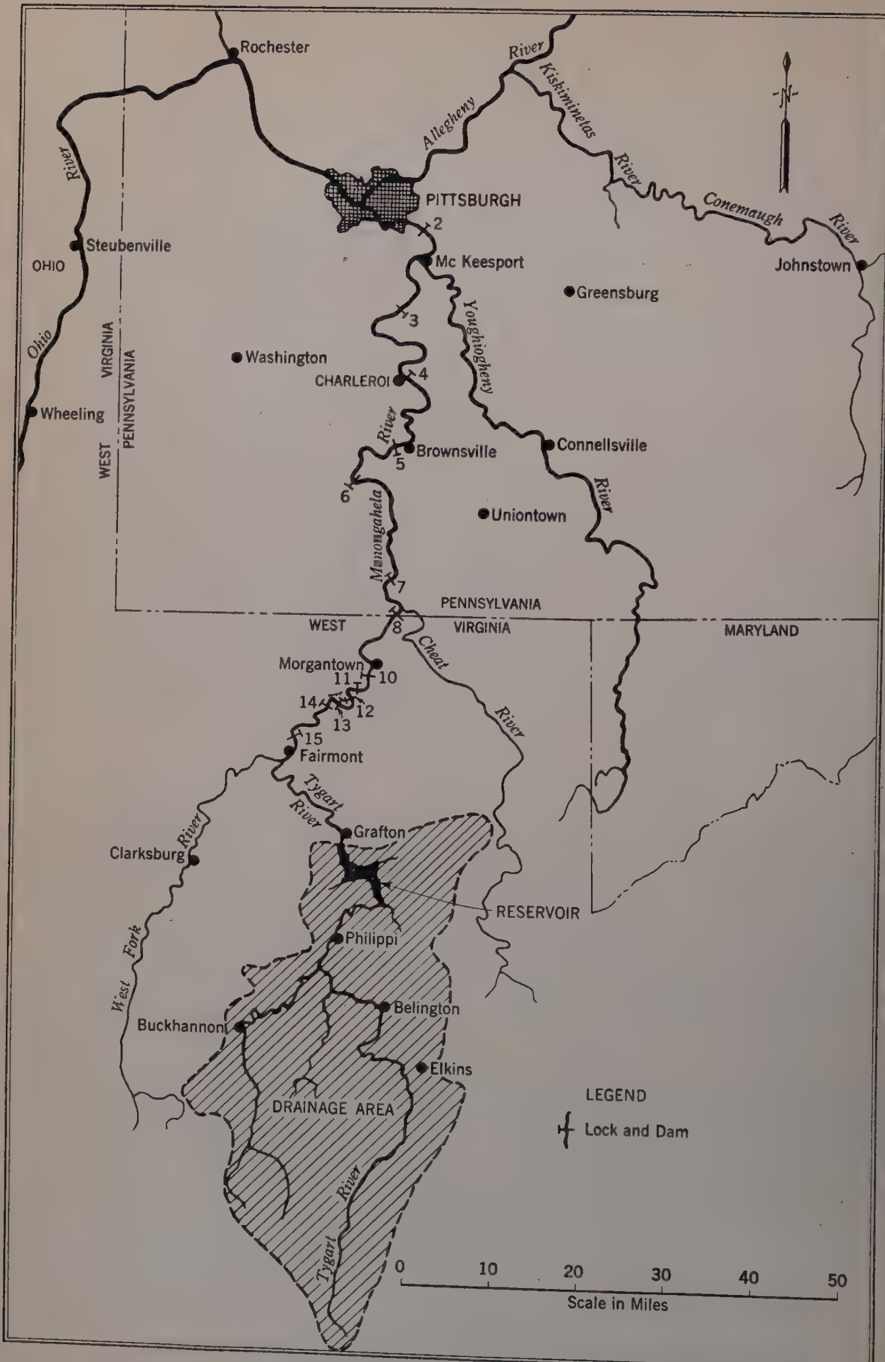

FIG. 1.—LOCATION MAP, TYGART DAM

Later surveys and studies were made by the United States Army Engineers. A complete plan for a ten-reservoir system for flood control in the Upper Ohio Basin was developed by the Army Engineers; and finally, in 1934, funds were appropriated by the federal government for the first unit in this system, a dam on the Tygart River, one of the main tributaries to the Monongahela. The latter river is one of the world's greatest industrial waterways, linking the bituminous coal fields of Pennsylvania and West Virginia with the steel mills and other industries of the Pittsburgh District. Since navigation on this stream was hampered periodically by low water in the dry summer and autumn months, this dam on the Tygart River was designed for the dual purpose of flood control and water supply.

The Tygart Dam and Reservoir, constructed and operated by the U. S. Army Engineers, have been in service since January, 1938. Following the great record-breaking flood of March, 1936, which caused an estimated monetary loss of $178,674,000 and heavy loss of life in the Ohio River basin above Wheeling, W. Va., money was appropriated by Congress for the extension of the reservoir system and four other units are now (1941) under construction. This paper purports to discuss some of the problems involved in the operation of a large flood-control and water-supply project and to demonstrate how effectively the Tygart Dam can perform the functions for which it was planned.

The dam is on the Tygart River in Taylor County, West Virginia. It lies just two miles by stream above the City of Grafton and 27 miles above Fair-

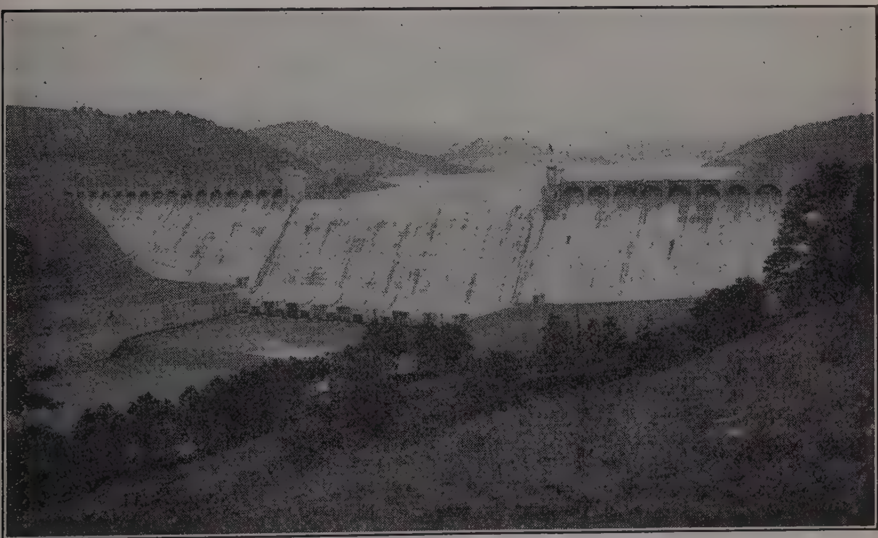

FIG. 2.—VIEW OF TYGART DAM

mont, W. Va., where the Tygart and the West Fork rivers join to form the Monongahela River. This latter river is canalized over its entire length of 128 miles, and from its mouth at Pittsburgh (where with the Allegheny River

it unites to form the Ohio River) to its source at Fairmont there are thirteen navigation locks and dams. The tributary drainage area of the Tygart Dam is 1,183 sq miles. This is 16.1% of the total Monongahela Basin (drainage area, 7,384 sq miles) and 6.2% of the combined drainage area of 19,117 sq miles of the Monongahela and Allegheny rivers at Pittsburgh.

This paper deals primarily with the operation of the reservoir rather than design or construction of the dam and only a brief description of the structure is presented herein for the purpose of clearness in explaining methods of operation.

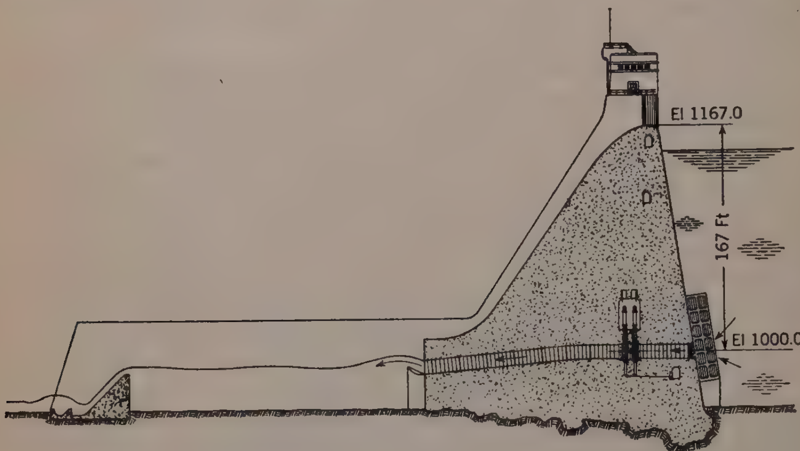

Fig. 3.—Spillway Section, Tygart Dam

The dam is of the concrete gravity type, 1,880 ft long, and rises about 230 ft above bedrock. The spillway section, at about the middle of the dam, is 489 ft long and is flanked by bulkhead sections 23 ft higher than the spillway crest. The outlet works consist of eight conduits 5 ft 8 in. by 10 ft, each controlled by two, vertical, hydraulically operated, steel slide gates and two, 48-in., internal, differential, needle valves for low-water regulations. A cushion pool is provided below the main dam for the dissipation of the kinetic energy of discharge through the outlet works and over the spillway. Fig. 1 shows the location of the dam; a view of the completed structure is shown in Fig. 2; and a cross-sectional drawing is given in Fig. 3. The cost of the completed project, including railroad relocation, was about $18,000,000.

General Features of Operation

Reservoir Capacity.—The size of a flood-control reservoir is governed both by the economic feasibility and the hydrologic potentialities of the project. The dam should be built to a height necessary to produce a reservoir of sufficient capacity to provide a reasonable degree of control over floods similar to those of past record and over theoretical floods that may possibly occur in the future. Where storage for low-water control must also be considered, as in the case of the Tygart Reservoir, this feature becomes doubly important. The height of

the dam and attendant reservoir capacity are also limited by the point at which the cost of the completed project, converted to annual charges, approaches the monetary benefits that will result from its operation. Exhaustive hydrologic and economic studies made by the Army Engineers, prior to the construction of the dam, limited the capacity of the reservoir to only 290,000 acre-ft at the spillway crest, which is equivalent to 4.5 in. of runoff from the tributary drainage area of 1,183 sq miles. Between the spillway crest and the top of the bulkhead sections an additional quantity of 88,000 acre-ft as surcharge storage is available. Fig. 4 shows the capacity of the reservoir in terms of elevation.

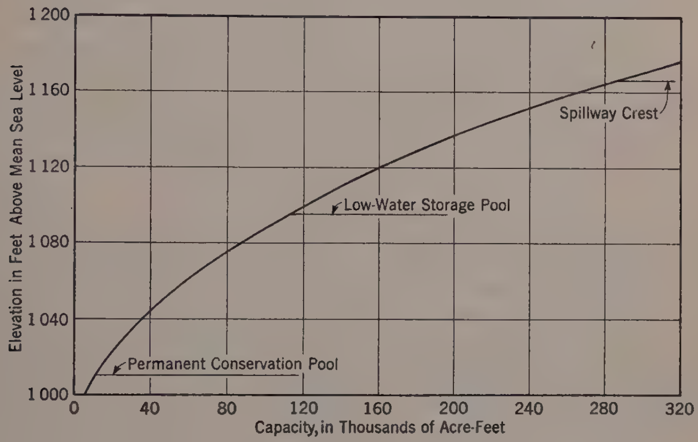

Fig. 4.—Curve of Reservoir Capacity

Seasonal Storage.—A schedule of dual-purpose operation is required for a flood-control reservoir of limited capacity if it must augment the seasonal low-water flow in the streams below the dam. The rate and quantity of low-water storage must be coordinated with flood-control operation to provide satisfactory results for both purposes. In allocating reservoir storage capacity to low-water regulation and flood control, during the months of low flow, a study was made to determine the water required to maintain satisfactory navigation conditions on the Monongahela River, under the most adverse conditions that might reasonably be expected. Using the historic 1930 drought as a criterion, it was determined that sometime it might be necessary to maintain a constant discharge of 340 cu ft per sec from the Tygart Reservoir for the $5\frac{1}{2}$-month period from July 1 to December 15. Translated into volume, this flow would be 675 acre-ft per day, or 113,400 acre-ft for the 168-day period. The record of the Tygart River during the 1930 drought reveals that, although on occasion the discharge was as low as 1 cu ft per sec, the total volume of discharge from July 1 to December 15 was approximately 13,000 acre-ft. Since the difference between the total volume of required discharge and the minimum expected inflow determines the volume of storage that must be in the reservoir prior to the low-water period the use of 1930 as a criterion would necessitate 100,000 acre-ft of storage by July 1.

In planning the operation of the reservoir, it was necessary to determine when, and at what rate, storage should be impounded in order to have the required storage by this time. It is obviously desirable to reserve the entire reservoir capacity for flood storage during the periods when floods are most prevalent. A study of Fig. 5 shows the distribution by months of floods at Pittsburgh, and indicates that flood frequencies increase from November to March and decrease from April through October. Therefore, maximum storage capacity is maintained in the period from November to March and, since the probabilities of damaging floods begin to decrease in April, at this time a gradual increase in storage may be started. An analysis of stream discharge during the months of April, May, and June in 1930 indicates that, by limiting the reservoir outflow to 100 cu ft per sec, the required 100,000 acre-ft of storage could have been obtained between April 10 and June 15. No ill effects would be caused by this schedule as a flow of 100 cu ft per sec is sufficient to maintain satisfactory sanitation conditions for the City of Grafton and, during this period of the year, the natural flow of the Monongahela River is normally much greater than the minimum required to maintain navigation.

Based upon the computed rate at which storage could have been obtained in 1930, a curve has been developed (Fig. 6) for a guide in impounding storage. Starting on April 1 the reservoir outflow should be adjusted (always maintaining a minimum of 100 cu ft per sec) so that the reservoir pool will rise at a rate that will keep it on or above the elevation shown on the guide line. Thus, even if conditions more severe than the low-water year of 1930 are encountered, 100,000 acre-ft of storage can be impounded before July 1. Referring again to Fig. 6, a depletion curve has been developed showing the rate at which the reservoir

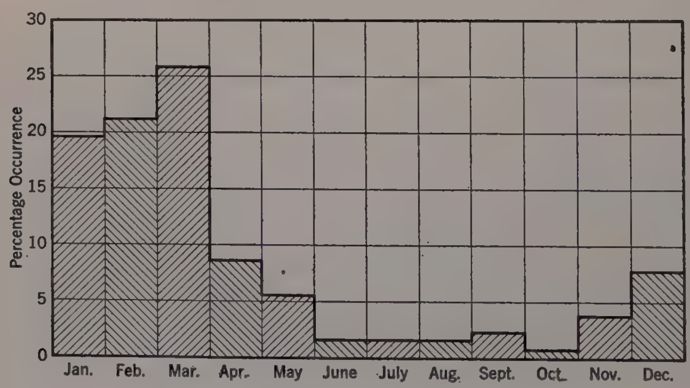

Fig. 5.—Monthly Distribution of Past Floods at Pittsburgh, Pa.

pool would have receded in 1930, under a constant outflow of 340 cu ft per sec, assuming the total inflow for the period to have been equally distributed. If the rate of drawdown throughout the low-water season keeps the pool elevation on or above the guide-line requirement, it is reasonably certain that the guaranteed minimum discharge of 340 cu ft per sec can be supplied continuously from July 1 to December 15.

General Features of Flood-Control Operation.—As previously mentioned, the gross capacity of the Tygart Reservoir is equivalent to 4.5 in. of runoff from the tributary drainage area. However, when the low-water storage of 100,000 acre-ft is in the reservoir, the remaining flood-control capacity is equivalent to only 2.9 in. of runoff. Obviously a reservoir of this capacity cannot completely contain the entire runoff from a great storm of the type that occurs frequently

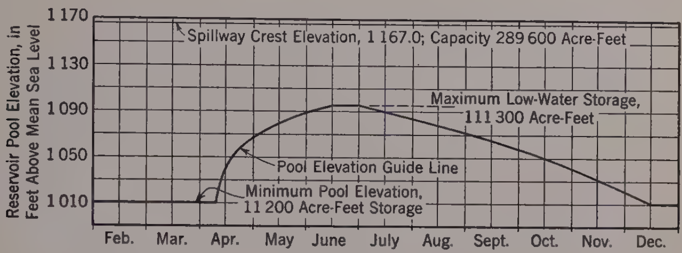

FIG. 6.—ANNUAL STORAGE SCHEDULE

over similar drainage basins in Eastern United States. This limited storage capacity, therefore, presents a problem of operation since the outlet works cannot be closed arbitrarily at the beginning of a flood and opened when the flood is past. A proper balance must be maintained between outflow and inflow so that the maximum benefits will be obtained from the available storage capacity. Flood storage in the reservoir should not begin until it will aid in preventing or reducing damage at some downstream point. As the flood passes and forecasts of stages indicate that an increased outflow from the reservoir will not arrive at the critical downstream points until after the rivers at those points have receded below damage stage, the rate of storage in the reservoir may be reduced by a gradual increase in reservoir outflow.

The true functions of a reservoir of this type during a general flood are to delay the rise of its tributary flow only until the streams to which it is affluent are falling below damage stage, and to keep the magnitude of their ultimate

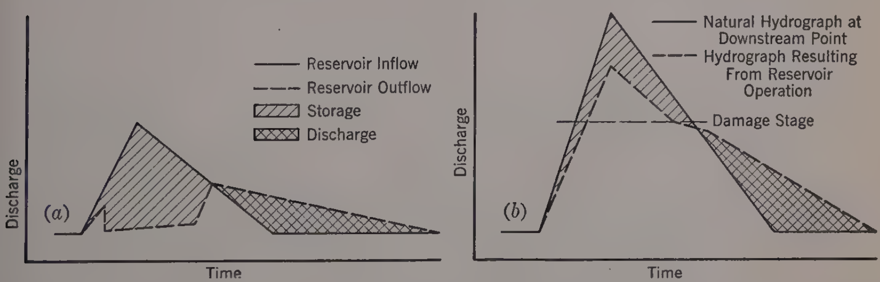

FIG. 7.—SCHEMATIC DIAGRAM OF RESERVOIR OPERATION

rise below this critical point. This principle is shown diagrammatically in Fig. 7, which illustrates the necessity to store, not the entire flood flow, but only that part represented by the single cross-hatched area of the hydrograph. The double cross-hatched area represents the discharge of storage; the two areas,

representing reservoir storage and draft, would, on an actual flood, be equal. The ideal result obtainable from this type of operation is shown diagrammatically in Fig. 7(*b*), which represents the desired effect at some distant downstream flood-damage point over which complete control cannot be procured. This condition obtains on the Monongahela River below Lock 7. Above this point the Tygart Reservoir can exercise almost continuous control over Monongahela River stages but beyond Lock 7 (which is immediately below the mouth of the Cheat River) the Monongahela Valley is still subject to floods from the uncontrolled drainage area of this large tributary stream.

Since many of the past floods of record in the Upper Ohio River Basin are of the double-peak and triple-peak variety wherein one wave rises immediately on the recession of another, the importance of ending flood storage and beginning discharge at the earliest possible moment is obvious.

Operation Schedules.—To obtain the aforementioned type of results, an effort has been made to determine fixed rules of operation and to establish criteria for the time and rate of flood storage and the time and rate of discharge of storage. The first attempt at establishing a fixed operation schedule was based upon stages at Pittsburgh, since it is in this district that the great monetary flood losses occur. From a study of past floods, a certain stage combined with a definite anticipated rate of rise on the Pittsburgh "Point" gage located at the confluence of the Monongahela and Allegheny rivers was set as the starting limit for storage in the Tygart Reservoir, and likewise a certain stage and rate of fall at Pittsburgh indicated the time for release of storage. Because of the complexities of the drainage basin above Pittsburgh it was soon evident that an operation schedule based entirely on stage conditions at this point would be subject to wide variation in results. The floods that occur at Pittsburgh are not always caused by simultaneous flood flows of the Allegheny and Monongahela rivers as either stream is sufficiently large, individually, to cause or contribute principally to flood stages in the Upper Ohio River. Therefore, although it is possible to set criteria at Pittsburgh that would have given excellent results from the Tygart Reservoir on some of the past great floods of record, recent experience has indicated that possible benefits from the reservoir would be sacrificed if the operation schedule were based on the Pittsburgh stage alone.

One very important factor, which could not be overlooked in establishing rules of operation, was the influence of navigation on the Monongahela River. The tonnage on this stream exceeds that on most of the inland waterways of the world, and the commercial traffic is vital to the industries of western Pennsylvania. In most instances, stages at which navigation is suspended on the Monongahela River, due to flooding of the locks, are below the stages at which actual flood damage begins in the cities and towns of the valley. The loss sustained by suspension of navigation is quite as real as physical flood damage in urban districts, and therefore the river stages affecting navigation must be taken into consideration in the regulation of the reservoir.

Because of its comparatively short dam section, Lock 5 at Brownsville, Pa., is usually the first lock on the Monongahela River to be inundated by a rising river and the last lock to return to normal service. This point, therefore,

forms a control index for the entire river. If the Tygart Reservoir is operated to delay the inundation of this lock as long as possible or prevent it altogether, and if, during reservoir discharge, the outflow is regulated to keep the stage at Lock 5 below the top of the lock walls, then the maximum benefits should normally be obtained throughout the length of the Monongahela River Valley. In the current operation studies Lock 5 has been used to a large extent as the point on which operating criteria must be based, and in a later paragraph the results obtained at Lock 5 on several actual floods will be demonstrated. The experience gained thus far strongly indicates that predetermined rules will serve only as a partial guide for successful reservoir operation and maximum efficiency can only be accomplished from judicious use of accurate forecasted stages at the critical points. The following example will tend to illustrate this point.

The average flood-wave translation time from Tygart Dam to Lock 5 is approximately 14 hr. When the rivers are rising the Tygart Reservoir, to be most effective, must begin storage at least 14 hr before the predicted time for Lock 5 to be overtopped by an uncontrolled flood wave. Storage must continue at the maximum rate in the reservoir until it is predicted that 14 hr later the stage at Lock 5 will have receded below the top of the walls. The rate at which the reservoir outflow can be increased must be equal to, or somewhat less than, the predicted rate of recession at Lock 5. If all these conditions are forecast accurately the reservoir can be operated so that the loss of lock service will be reduced to a minimum, and the maximum reduction of the flood crest will also be obtained at Pittsburgh and other critical points throughout the length of the Monongahela River.

One possible objection to the use of Lock 5 for operation control is that positive storage in the reservoir may be used for the benefit of the Monongahela River when the river at Pittsburgh is below flood stage. If a great general flood should occur in the Tygart Basin immediately following this period, sufficient in magnitude to overtop the spillway of the dam, the efficiency of the reservoir might be reduced, as part of the storage capacity would be occupied at the beginning of the flood. This possibility can be greatly lessened by accurate meteorologic forecasts and intelligent use of hydrologic data.

Flood Prediction.—In the preceding section the importance of predicted river stages below a flood-control dam has been emphasized. It is evident, of course, that the forecast of inflow into the reservoir itself is of great importance, particularly where the storage capacity is limited and where there is a possibility that the reservoir will be completely filled by a large flood or succession of smaller floods. In this section some of the factors affecting flood prediction will be discussed briefly, and the sources of river and rainfall data will be enumerated.

The first factor to be considered in making river-stage forecasts is the condition of the streams at the beginning of the storm period. This is very important as a flood of some definite magnitude may be comparatively small if isolated. If it has occurred closely after another flood, however, which has utilized all the natural storage in the stream valleys, it may result in extremely

harmful stages. The floods of January, 1937, in the Ohio Valley were of this type.

Another factor affecting flood flows is the percentage of runoff to be expected from a given rain. This is largely a function of the absorption capacity of the soil and condition of vegetation. A constant check on the rainfall-runoff relation must be maintained as it varies through the year. When they are unaffected by surface runoff, river stages represent outflow of ground water, or subsurface flow, giving an indication of soil conditions and the rate of infiltration to be expected with rainfall. Studies are being conducted along these lines to correlate these factors more accurately for use in determining runoff.

The most important factors in flood prediction, of course, are the rate and distribution of rainfall, combined when necessary with the water content of fallen snow. A network of rain gages and river-gaging stations is maintained over the Upper Ohio Drainage Basin by the U. S. Weather Bureau, U. S. Army Engineers, and U. S. Geological Survey. Daily reports from these stations are telegraphed to the Pittsburgh Office of the Weather Bureau at 7:00 a.m. and during periods of heavy rainfall at 6-hr intervals. The Weather Bureau, in conjunction with the Federal-State Forecasting Service, makes river-stage predictions for principal points in the Pittsburgh District of the U. S. Engineer Department.

In the tributary area of the Tygart Reservoir a network of ten rain gages and six river gages is maintained. From the reports of these stations, forecasts are made of reservoir inflow using runoff distribution percentages developed from a unit hydrograph for this basin. Fairly good results have been obtained and investigations are being made to define a small drainage basin above the dam to be used as an index of the runoff to be expected from the entire basin.

Accurate prediction of rainfall over an area before the rain has actually fallen represents the ultimate hope of those engaged in flood-protection work. Rapid strides in the science of air-mass analysis indicate that such predictions from data obtained by the Weather Bureau are a distinct possibility.

Flood Routing.—In simple terms, flood routing is an analytical redevelopment and transition of a flood wave—its inception from surface runoff in the headwater areas of one basin, its subsequent movement downstream augmented by inflow from successive tributary streams and diminished by the flattening of the wave as it progresses downstream, and the effect of natural valley storage. If the natural inflow from one of the tributary streams is withheld by a flood-control dam, the net effect at some downstream point will be a function of the valley storage between the dam and the point at which the reduction is computed. This amount of valley storage will itself be a function of the rate of discharge from the uncontrolled parts of the drainage basin.

The Army Engineers of the Pittsburgh District have been conducting extensive investigations into various analytical methods of flood routing and have adopted the so-called "coefficient method," first used in the Muskingum Watershed Conservancy Project studies, as the mathematical type most applicable to their region. In this method of routing, the stream is divided into

routing reaches. The ends of these reaches are marked by stream-flow stations, so selected that the time of flow translation between the reach limits is as nearly equal as possible and corresponds somewhat to the interval of time used in routing computations. The flood wave is then routed analytically through each reach. The inflow at the head of the reach is added to the computed tributary inflow throughout the reach; and the inflows are modified for time, and by valley and wave slope storage. The outflow at the reach termination is thus determined.[4] The outflow so determined at the end of the reach is then checked against the actual flood hydrograph. A representative group of floods in the Upper Ohio Basin have been routed by this method and very close agreements between the reproduced flood hydrographs and the actual flood hydrographs have been obtained. The flood routing coefficients determined by this method of routing have been used in computing the reductions effected by the Tygart Reservoir at Lock 5 and at Pittsburgh.

Results from Flood-Control Operation

Floods Encountered.—It is a curious fact that, since the beginning of construction of the Tygart Dam, four floods have occurred that exceeded in crest magnitude all but one of the previous floods of a thirty-year period of record at this site. In descending order of peak discharge the first five floods of record on the Tygart River are those of July, 1912, October, 1937, April, 1939, February, 1939, and March, 1936. Since the dam was not fully completed until January, 1938, it was not possible to control all of the four recent large floods. Construction was not far enough advanced to have any appreciable effect on the flood of March, 1936; some control was exercised over the flood of October, 1937; and, during the floods of February and April, 1939, the dam was in full operation. One other flood occurred in December, 1937, which, although of minor proportions on the Tygart River, occurred coincidentally with flood stage at Pittsburgh.

October, 1937.—This flood was the second greatest of record on the Tygart River, reaching a peak reservoir inflow of about 52,000 cu ft per sec. The installation of the slide-gate machinery in the dam was not yet completed and one of the slide-gate conduits was closed by a bulkhead at this time. The upper reaches of the reservoir had not been completely cleared and the available storage capacity, therefore, was limited. During this flood the dam was operated as a detention reservoir. All seven available slide gates and both needle valves were open from the beginning of the rise until the reservoir was again empty. The inflow into the reservoir so far exceeded the capacity of the outlet works at a low storage head that some 104,000 acre-ft of storage were impounded in the reservoir. The rainfall causing this flood on the Tygart River was general over the entire Monongahela Basin with the result that navigation locks on the Monongahela River were flooded out and flood stage at Pittsburgh was exceeded. Fig. 8(*a*) shows the hydrographs of the Tygart Reservoir inflow and outflow, and Figs. 8(*b*) and 8(*c*) demonstrate the effect of the reservoir on the flood hydrographs at Lock 5 and at Pittsburgh.

[4] "Muskingum River, Ohio (308), Vol. IV," report of U. S. Engr. Office, Zanesville, Ohio, December 1, 1934.

December, 1937.—This flood was the first on which the Tygart Dam could actually be operated. The flood on the Tygart River itself was not particularly severe, the peak reservoir inflow being 22,000 cu ft per sec, but the flood stage at Pittsburgh was exceeded and the reservoir effected some reductions at this point. Fig. 9(*a*) shows the reservoir inflow and outflow, and Figs. 9(*b*) and 9(*c*) show the effect of the reservoir on the hydrographs at Lock 5 and at Pittsburgh.

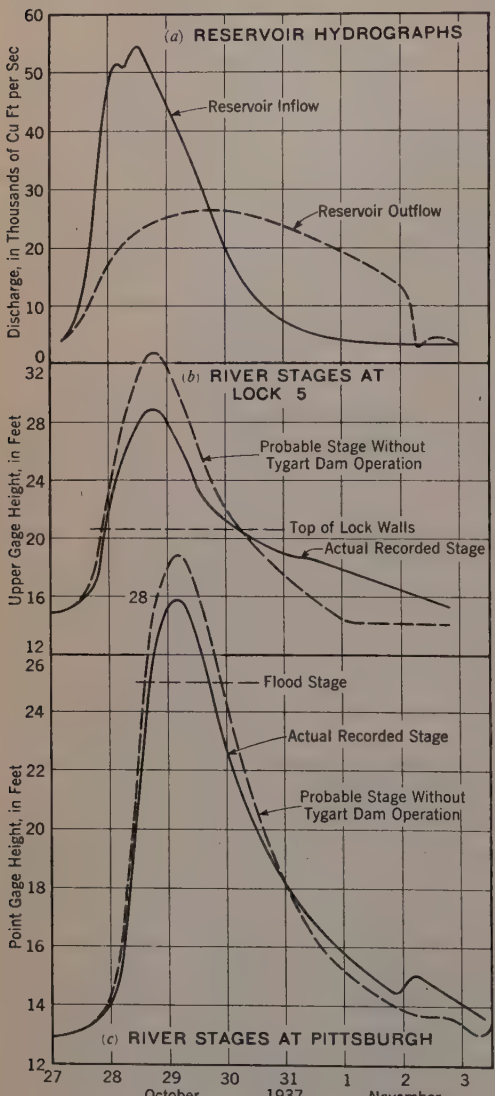

FIG. 8.—FLOOD OF OCTOBER, 1937

February, 1939.—This was a double-peak flood, the crests occurring on January 31 and February 4. The peak reservoir inflows were 28,000 cu ft per sec and 43,500 cu ft per sec, respectively, the latter being the fourth greatest of record in the Tygart River. Precipitation causing these floods was general over the Monongahela Basin and this river would have experienced a flood of major proportions had it not been for the Tygart Reservoir. Coming almost entirely from the Monongahela River and reaching flood stage at Pittsburgh, this flood furnished an example of the effectiveness of the Tygart Reservoir. The reservoir inflow and outflow are shown in Fig. 10(*a*), and the reductions of the flood hydrographs at Lock 5 and at Pittsburgh are shown in Figs. 10(*b*) and 10(*c*).

A double-peak flood of the type occurring on January 31 and February 4 furnishes a severe test of operation technique. Some of the storage impounded during the first peak was discharged as rapidly as possible when meteorological forecasts indicated the approach of additional rain. Referring to Fig. 10(*b*), it can be seen that the rate of discharge was so calculated that the Monongahela River was held stationary about 2 ft below the lock walls at Lock 5, thus allowing navigation to pro-

ceed on the entire river. Near the beginning of the second flood wave, storage was again begun in an effort to reduce this crest at downstream points. The total volume of reservoir inflow from January 30 to February 5, inclusive, was about 264,000 acre-ft. The maximum storage in the reservoir at any one time was only 128,700 acre-ft, so that it is evident that, by this system of operation, a possible third and greater peak could have been effectively reduced.

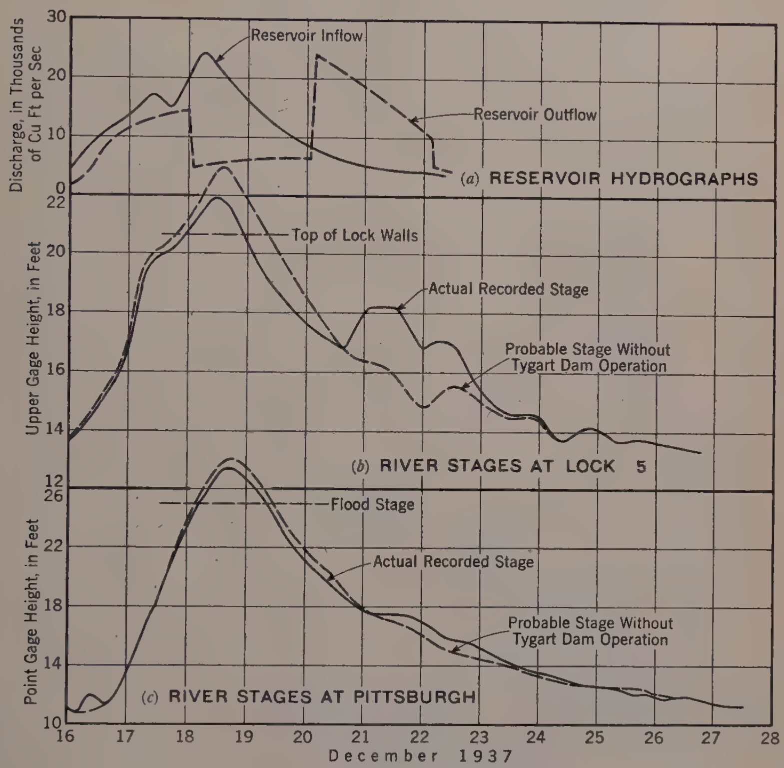

FIG. 9.—FLOOD OF DECEMBER, 1937

April, 1939.—At the beginning of April, 1939, storage was started in the reservoir for the purpose of impounding 100,000 acre-ft for use during the low-water season. The natural discharge of the Tygart River during the first two weeks of the month was such that about 90,000 acre-ft had already been impounded by April 14. With this volume of storage already in the reservoir another major flood occurred on the Tygart and Monongahela rivers. A peak reservoir inflow of 44,000 cu ft per sec on April 16 places this flood third in order of crest magnitude in the history of the Tygart River. A large flood of this type, occurring with the low-water storage already in the reservoir, furnished a thorough test of the capability of the Tygart project to perform its dual function of flood control and water supply. The results, as shown in Fig. 11, were satisfactory. During this flood the reservoir pool reached the

highest level yet attained at El. 1,144.75, representing a gross storage of 220,975 acre-ft. Although the net storage during the flood period was about 131,000 acre-ft there was still about 68,000 acre-ft of storage capacity available.

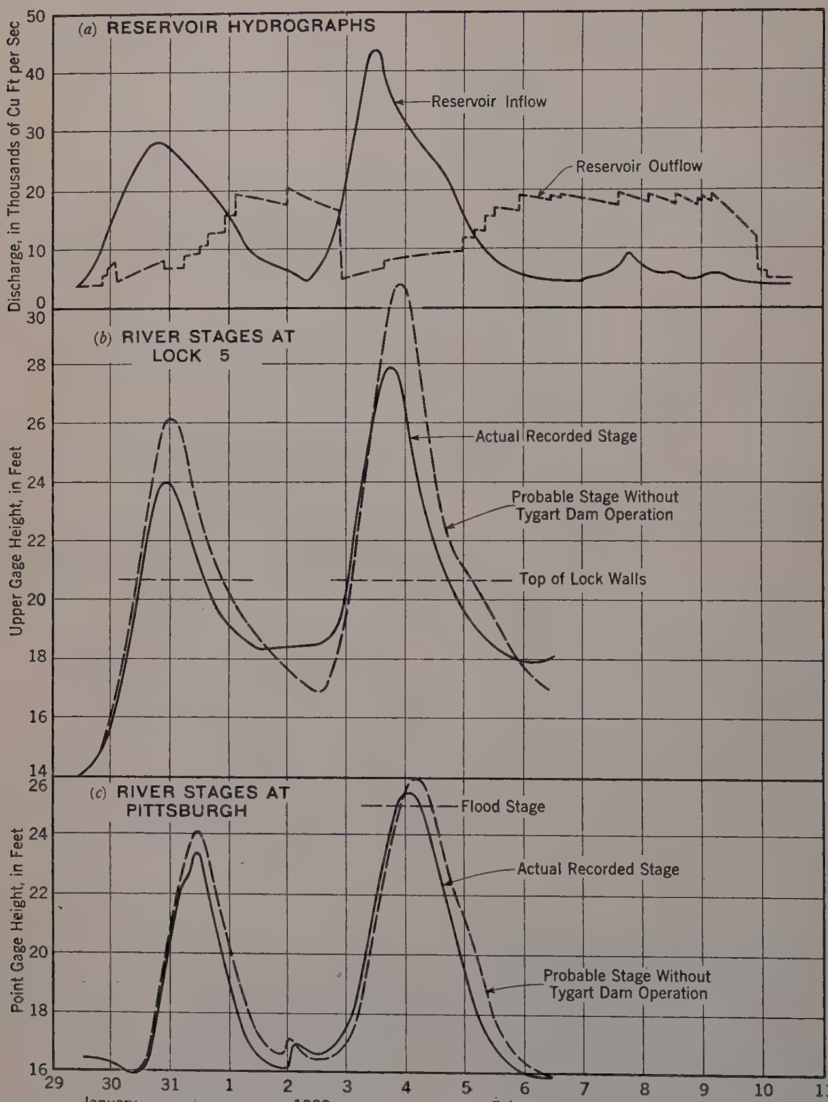

FIG. 10.—FLOOD OF JANUARY AND FEBRUARY, 1939

Minor Rises on Monongahela River.—There are frequent occasions throughout the year when a sharp rise in the rivers will threaten the suspension of Monongahela River navigation without approaching actual flood damage stages. On these occasions the Tygart Reservoir is utilized to equalize, as far

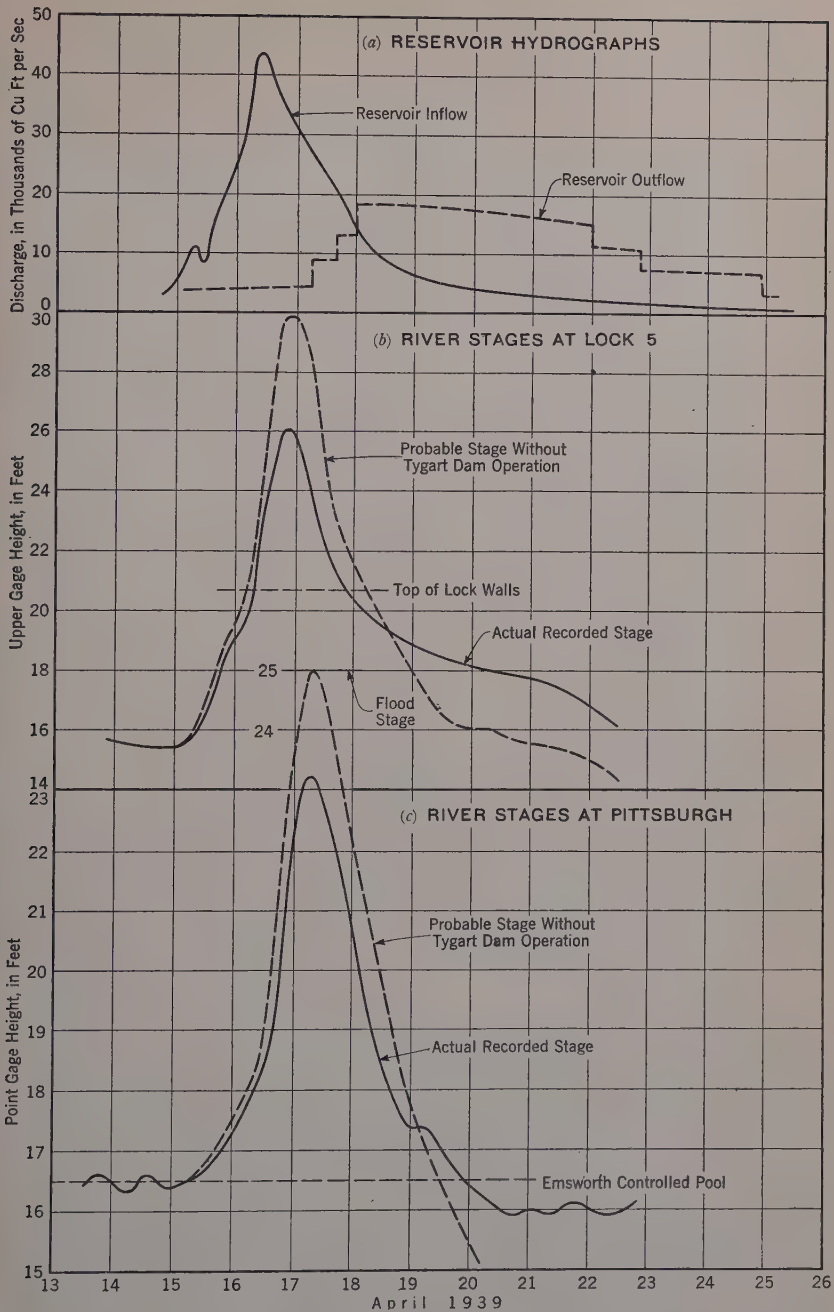

FIG. 11.—FLOOD OF APRIL, 1939

as possible, the flow of the Monongahela River without storing an unduly large volume of water. Experience has indicated that the detention reservoir method of operation, maintaining a fixed number of gate openings, is effective on these minor rises. A typical example of this type is given in Fig. 12(*a*), showing operation during a rise in March, 1940. Fig. 12(*b*) shows that the Monongahela River would have overtopped the walls at Lock 5 had it not been for the storage in the reservoir. Obviously, a somewhat greater reduction in the stage at Lock 5 could have been obtained by further reducing the reservoir outflow. However, on such frequent rises of this low-stage type, it is not believed advisable to impound too much storage in the reservoir lest its effectiveness be impaired should a major flood follow a series of minor rises.

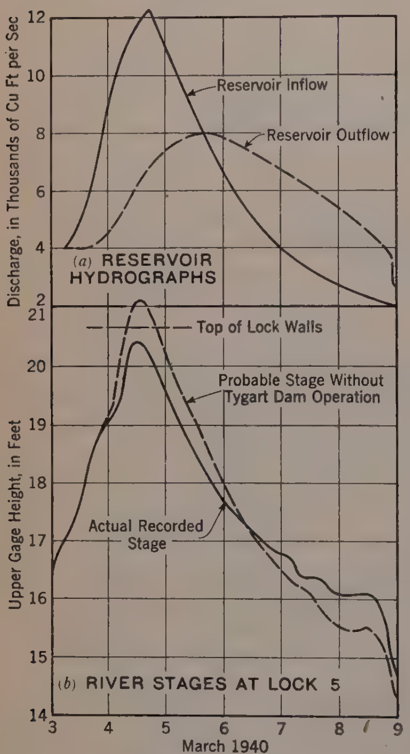

FIG. 12.—FLOOD OF MARCH, 1940

Review of Flood-Control Operation.—It is apparent that on each of the floods encountered thus far in the operation of the Tygart Dam a somewhat different operation procedure was followed. Studies are continuously in progress to determine the method of operation that will result in obtaining the maximum monetary benefits and will be equally satisfactory to flood control and navigation interests. The operation during these first two years, therefore, should be considered experimental. During the flood of December, 1937, an effort was made to correlate the time of reservoir storage with the river stage at Pittsburgh. As a consequence, the maximum possible rate of storage was delayed until nearly the time of the peak reservoir inflow. The rate of discharge, also based on the Pittsburgh gage, produced somewhat undesirable conditions on the Monongahela River. In the flood of January–February, 1939, it was found that an attempt to maintain a fixed passing flow during the storage period, and an increasing discharge rate after the flood had passed, resulted in excessive manipulation of the gates and valves and an extremely irregular hydrograph immediately below the dam. This flood also emphasized the fact that, to be most effective, storage must be started in the reservoir at the beginning of the rise in inflow without waiting for a predetermined stage at some downstream point.

Somewhat better results might have been obtained had storage been started about six hours earlier on the second peak. During the flood of April, 1939, the operation was predicated largely upon forecasted stages at Lock 5. The passing flow during the storage period was maintained at the capacity of one

slide gate, thus increasing the outflow as the available storage capacity was decreased. The rate of discharge after the flood peak had passed permitted the Monongahela River to maintain a steady recession at the navigation locks and was satisfactory to navigation interests. The operation during this April flood is believed to be the most successful yet attained and will be used as a guide on future floods of a similar nature.

Monetary Benefits from Flood Control.—In planning the system of flood-control reservoirs a very comprehensive study was made of flood damages in the Upper Ohio River Basin to determine the benefits that would result from reductions of flood peaks. The details of this damage study are quite extensive and only the results are presented herein. The Monongahela Valley has been divided into three damage districts as follows: Pittsburgh District, from the junction of the Ohio and Beaver rivers to Lock 2, Monongahela River; McKeesport District extending from Lock 2, Monongahela River to Lock 4, Monongahela River; Upper Monongahela District from Lock 4 to the head of the river. Damage curves for these three reaches of the river, as developed from the comprehensive study of flood damages, are shown in Fig. 13. The reductions of the crests effected by the Tygart Reservoir on the floods shown in Figs. 8 to 12 were applied to the damage curves to determine the flood-control benefits attributable to the Tygart project. The results in detail are presented in Table 1, the total sum being $2,818,000.

TABLE 1.—ECONOMIC BENEFITS, OPERATION OF TYGART DAM

Flood	Lock and Dam 5			Lock and Dam 3			Pittsburgh, Pa.			Total benefits
	Actual gage height[a]	Reduction[a]	Benefits[b]	Actual gage height[a]	Reduction[a]	Benefits[b]	Actual gage height[a]	Reduction[a]	Benefits[b]	
Oct. 29, 1937	28.9	3.8	180	28.9	3.7	980	27.8	1.6	520	1,680
Dec. 18 and 19, 1937	21.9	1.1	5	20.9	0.8	0	27.4	0.5	100	105
Jan. 31, 1939	24.0	2.2	23	22.9	1.9	160	23.3	0.8	0	183
Feb. 4, 1939	27.9	3.0	70	27.5	2.3	240	25.4	0.6	70	380
Apr. 17, 1939	26.1	3.8	85	25.3	3.0	385	23.2	1.8	0	470
Total			363			1,765			690	2,818

[a] Feet. [b] Thousands of dollars.

Low-Water Regulation; Navigation

One of the primary purposes of the construction of the Tygart Dam was to provide a storage reservoir to supply an adequate flow for navigation during the low-water season. Prior to the construction of the Tygart Dam, there were many occasions when the natural flow in the Monongahela River was not sufficient to maintain full pools behind the navigation dams. When the volume of water required for lockages, plus the normal leakage past the dams, exceeded the natural flow of the river, the pools would recede below the dam crests and the navigable depths in the pools would be reduced, thus necessitating lighter loading of barges and an increased number of towboat trips to haul the same tonnage. The historic example is the drought of 1930 at which

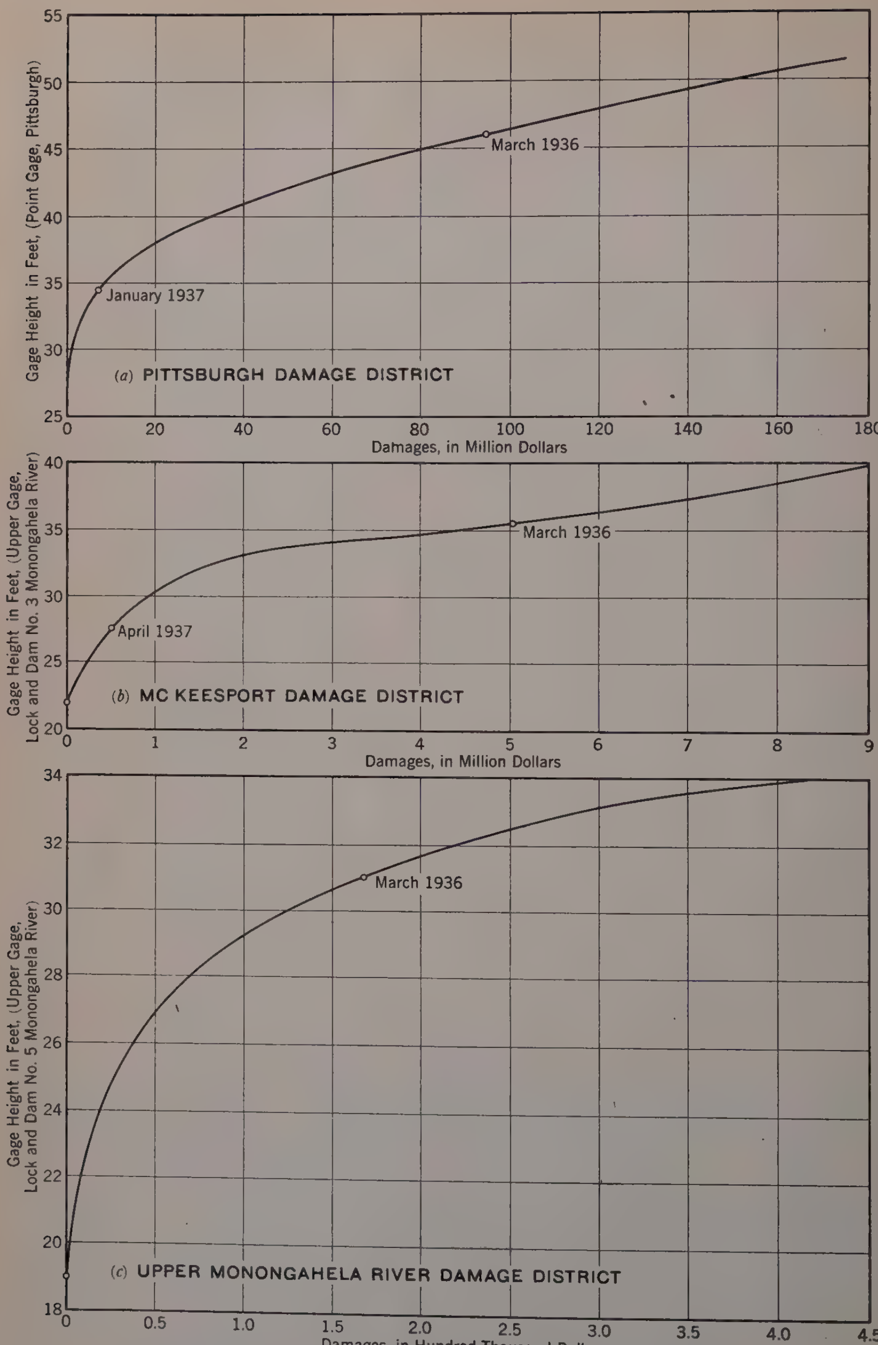

FIG. 13.—APPROXIMATE FLOOD DAMAGE (1936 DOLLAR STANDARD)

time navigation was completely suspended above the pool of Lock and Dam 7, and was maintained at a restricted rate below Lock 7 only by virtue of water obtained from a hydroelectric development on the Cheat River. The action of the Tygart Reservoir in alleviating this condition during the two years 1939 and 1940 has been effectively demonstrated, especially during the late summer and early fall of 1939 when the most severe low-water conditions since 1930 were encountered. Fig. 14 shows the daily reservoir pool elevations for 1938 and 1939, demonstrating the time and rate of storage and discharge.

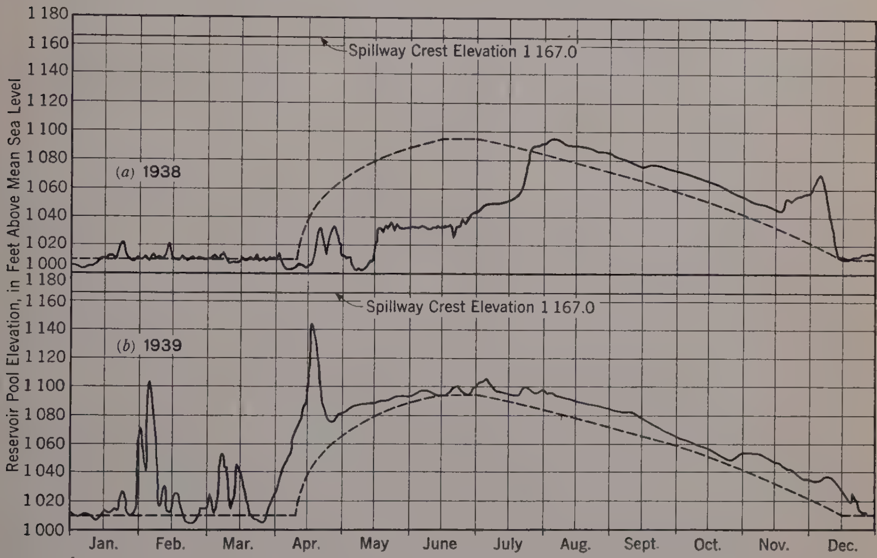

FIG. 14.—DAILY POOL ELEVATIONS

In 1938 it was not possible to begin storage in the reservoir in April (as called for on the operation schedule) because of complications in land acquisition in the reservoir area. Storage was finally begun late in June, and by July 20 about 40,000 acre-ft had been impounded by limiting the outflow to 100 cu ft per sec. Fortunately the natural flow of the Monongahela River was sufficient to permit this procedure. On July 20 and 21 a storm occurred over the Tygart Basin and, by storing almost the entire runoff, the full storage pool of 100,000 acre-ft was obtained by August 1.

Release of this storage was begun on August 6 when the Monongahela River began to recede to low stages, and proceeded continually to November 18. The data in Table 2, Col. 2, demonstrate the increase in the low-water flow provided by the reservoir storage during this period.

The low-water season of 1938 was not particularly severe except for about two weeks in late August and early September. However, the effect of the release of storage from the Tygart Reservoir produced conditions so far superior to those experienced in normal years that this feature of the reservoir operation resulted in widespread public approval. A head of 0.5 ft was maintained on the crests of the navigation dams on the Monongahela River from Dam 15

to Dam 7, inclusive, throughout the low-water period. Below Dam 7 the head on the dam crests was less than 0.5 ft due in part to the greater number of lockages in the lower reaches of the river. In late August and early September the pools behind Dams 5 and 6 receded to the dam crests and the additional

TABLE 2.—Low-Water Regulation

Description (1)	1938[a] (2)	1939[b] (3)
Total volume of storage released, in acre-ft	69,800	60,950
Average rate of release, in cu ft per sec	334	433
Average discharge, Monongahela River at Lock 15, in cu ft per sec	570	624
Percentage supplied from Tygart storage reservoir	58.7	69.5
Average discharge, Monongahela River at Lock 5, in cu ft per sec	1,404	1,135
Percentage supplied from Tygart Storage reservoir	23.8	38.2

[a] August 6 to November 18. [b] August 15 to October 25.

water being provided by the Tygart Reservoir was just sufficient to keep these pools full. It was quite apparent that without this water supply the normal depth in these pools could not have been maintained.

In April, 1939, storage was begun in the reservoir in accordance with the operation schedule. The flood that occurred on April 15 and 16, described previously herein, necessitated storage in the reservoir far above the low-water storage pool. Following this flood the reservoir was drawn down from the crest of El. 1,144.75 to El. 1,077 on April 27. Storage was then impounded at a gradual rate to attain the full low-water storage pool at El. 1,094 late in May. Natural river discharge during June and July was such that no discharge of the Tygart storage was necessary. Beginning on August 15 and extending to October 25 severe low-water conditions were experienced in the Monongahela Basin, and the release of storage from the Tygart Reservoir was responsible for maintaining unrestricted navigation on the Monongahela River. The data in Table 2, Col. 3, illustrate the effectiveness of the reservoir on the Monongahela River discharge.

As a result of this regulation by the Tygart Reservoir, conditions on the Monongahela River were maintained much the same as during the previous year. A 0.5-ft head was held on the dams from Lock 15 to Lock 7, whereas the pools at Locks 6 and 5 were barely full. The marked deficiency of rainfall and the acute low-water conditions on other streams of the Pittsburgh District made the effect of the Tygart storage on the Monongahela River quite apparent to the general public and many favorable comments were expressed by those interested in river transportation.

Evaluation of the monetary benefits attributable to the Tygart Reservoir during this period were made through recourse to the records of the 1930 drought. In that year the low water began about July 1 and by July 20 the pool at Lock 5 had receded 4 ft below the crest of the dam, and navigation was seriously impaired. Several slight rises in discharge increased the pool depth about 2 ft, but in mid-September a deficiency of slightly more than 4 ft existed and navigation authorities declared that, should conditions become any worse, it would be impossible to navigate towboats on the Monongahela River. Sub-

sequently, arrangements were made to obtain the storage impounded in the hydroelectric development on the Cheat River for low-water control. This additional flow was just sufficient to maintain navigation in the lower part of the Monongahela River, but in the upper reaches of the Monongahela River, above Lock 7, the pools with the exception of Lock 10 were completely empty and navigation had ceased entirely.

A comparison of the stream-flow records of 1930 and 1939 reveals, of course, that the 1930 drought was more prolonged and severe than that of 1939. However, the conditions experienced from mid-August until late October in 1939 are comparable with the early period of the 1930 drought.

Since the increase in flow (amounting to a total of 60,950 acre-ft) provided by the Tygart Reservoir was barely sufficient to keep the low Monongahela navigation pools full, it is certainly evident that under natural conditions these pools would have been well below the dam crests.

In 1930 information was obtained from the larger companies engaged in Monongahela River traffic to the effect that complete suspension of navigation would result in increased direct costs to these companies of about \$1,230,000 per month. It was also estimated at that time that restricted navigation with the pools below the dam crests resulted in increased costs of approximately \$139,000 per month. The average monthly Monongahela River tonnage for July, August, and September of 1930 was 2,130,930 tons. In August, September, and October of 1939 the average monthly tonnage was 2,160,900 tons.

The navigation pools of the river would have unquestionably fallen below the critical dam crests for at least two months in 1939 and, since the volume of traffic was comparable to that of 1930, a direct saving of at least \$140,000 per month, or \$280,000 for this low-water period, was effected. A determination of the exact amount which the navigation pools would have receded under natural conditions was considered inadvisable, as the quantity of water lost by leakage can only be approximated. It is problematical, therefore, whether or not navigation could have been maintained even constantly in a restricted manner throughout the dry period without additional water supply. The cost figures obtained in 1930 pertain only to the additional cost of providing coal by means other than river shipment to the industries normally served by water transportation. The far-reaching indirect effects of disrupted transportation and delayed shipments upon the plants immediately concerned and the general effect upon other related industries cannot be readily reduced to a money basis. Therefore, the estimated saving of \$280,000, representing only direct additional costs of coal shipments and based on the assumption that navigation could have continued in a restricted manner without the Tygart Reservoir, is believed to be extremely conservative.

Low-Water Regulation; Water Supply and Sanitation

General.—The seasonal storage in the Tygart Reservoir and the regulation during the low-water periods is based primarily on the demands of Monongahela River navigation. However, since this river is extensively used as a source of domestic and industrial water supply and at the same time as a disposal channel for sanitary and industrial wastes, any increase in the seasonal low-water flow

constitutes a major benefit to industry and to the general public in the Monongahela Valley. Deplorable conditions existed in the Monongahela River during the 1930 drought, constituting a serious menace to the public health. The improved condition of the river water since the Tygart Dam has been placed in operation has been quite apparent at the water works and industrial plants on the river.

Quality of Tygart River Water.—In the drainage area tributary to the Tygart Reservoir there is a sewered population of from 15,000 to 20,000 persons discharging domestic wastes directly into the river. Industrial development above the dam is sparse, there being but two plants of any size using the river for industrial disposal. During periods of low-water flow the West Virginia State Department of Health has recognized major sewage nuisance conditions in the vicinities of Philippi, Buckhannon, Belington, and Elkins and sewage treatment plants are contemplated for the latter two towns. There has been rather extensive coal-mining activity in the area above the reservoir, and the drainage from abandoned mines imposed an acid load upon the streams. Activities of the federal government in the mine-sealing field have resulted in the sealing of 124 out of 187 abandoned mines above the Tygart Reservoir.

TABLE 3.—Increase in River Discharge

Period ending	No. of days	Tygart River[a]		Monongahela River[c]	
		Average discharge[b]	Percentage from storage	Average discharge[b]	Percentage from storage
Aug. 31[d]	26	637	52	2,181	15
Sept. 15	15	485	89	1,186	36
Sept. 30	12	460	50	1,325	17
Oct. 31	31	447	86	866	45
Nov. 18	18	546	58	1,444	22
Average	(102)[e]	520	67	1,404	25
June 15[f]	9	785	56	3,053	15
June 27	4	2,430	51	8,288	15
July 19	13	1,714	48	5,312	15
July 29	6	2,355	28	6,987	9
Aug. 31	31	830	45	1,985	19
Sept. 30	30	558	87	1,054	46
Oct. 26	26	464	93	1,126	38
Nov. 30	27	651	57	1,562	24
Average	(146)[e]	858	57	2,303	21
Average[g]	(248)	719	60	1,933	22

[a] At the dam. [b] In cubic feet per second. [c] At Charleroi, Pa. [d] Beginning August 6, 1938. [e] Total. [f] Beginning June 7, 1939. [g] Average, both periods.

The storage period of the Tygart Dam begins early in April (see Fig. 6). Ordinarily the major part of the 100,000 acre-ft of storage will be impounded during the spring freshets of April and May. During these periods of high river stages the water is turbid but relatively pure, and this vast supply in the reservoir serves as an effective dilution basin for the contaminated low-water flow in the dry summer months. The City of Grafton, immediately below the dam, obtains its raw water supply directly from the reservoir and considers this source far superior to the run-of-river supply previously used. The West Virginia Department of Health has classified the water in the reservoir as "safe."

Chemical analyses of the water discharged from the reservoir during the past summers have been made and the average values are as follows: Alkalinity, 5 ppm; soap hardness, 29 ppm; turbidity, 7 ppm; free carbon dioxide (CO_2), 3 ppm; potential of hydrogen (pH), 6.1; temperature, 68° F; and indi-

cated number of B. coli (24 hr presumptive), 1 per 10 million. Turbulent discharge through the needle valves insures a dam-site flow that is high in dissolved oxygen, thus aiding in the assimilation and purification of domestic and other wastes that have a high oxygen demand.

Increase in Flow of Monongahela River.—Disposal by dilution is an accepted method of handling almost all of the common wastes incident to human and industrial activities. The success of the method is dependent upon the quantity and quality of diluting water available. The quality of the Tygart Reservoir water has been discussed, and Table 3 gives the increase in low-water flow supplied to the river during various critical periods of 1938 and 1939. Under the heading: "Low-Water Regulation; Navigation," average values were given for the increase in the low-water flow for a continuous period in 1938 and 1939. The values contained in Table 3 are for intermittent critical periods and are based at different points.

Chemical Changes in Monongahela River.—An estimate has been made of the probable effect of the Tygart Reservoir on the chemical characteristics of the Monongahela River during the low-water seasons of 1938 and 1939. Chemical analyses of the reservoir water and of the river as recorded by the

TABLE 4.—TYGART DAM OPERATION EFFECT ON ACIDITY AND HARDNESS IN MONONGAHELA RIVER

(Reductions in Parts per Million; Averages Are Weighted for Time)

Date, 1938 and 1939	Charleroi, Pa.		McKeesport, Pa. (Above Youghiogheny River)		McKeesport, Pa. (Below Youghiogheny River)		Pittsburgh, Pa.	
	Hardness	Acidity	Hardness	Acidity	Hardness	Acidity	Hardness	Acidity
Aug. 6 to 31	21	4	25	6	19	5	19	2
Sept. 1 to 15	60	11	73	13	49	12	65	7
Sept. 19 to 30	45	8	46	9	27	6	31	4
Oct. 1 to 31	78	13	131	31	83	19	97	8
Nov. 1 to 18	44	4	61	12	46	10	53	4
Average (1938)	51	8	73	16	49	11	57	5
June 7 to 15	22	4	28	4	16	2	18	1
June 24 to 27	23	2	18	2	13	1	9	0
July 7 to 19	13	1	11	1	9	1	10	0
July 24 to 29	8	1	4	1	5	1	6	0
Aug. 1 to 31	30	3	25	4	25	4	28	2
Sept 1 to 30	129	14	121	14	105	11	111	5
Oct. 1 to 26	133	16	144	15	114	11	112	7
Nov. 4 to 30	62	8	28	4	30	4	32	1
Average (1939)	72	8	64	8	55	6	57	3
Average (1938 and 1939)	63	8	68	11	52	8	57	4

various water works and industrial plants along its course were obtained continuously throughout these periods. The approximate natural discharge of the river, as it would have been without the additional discharge from Tygart Reservoir, was determined. Stream-flow records were investigated to find years prior to the construction of Tygart Dam when the Monongahela River discharge compared closely with the natural discharge during 1938 and 1939.

From the past records of water works and industrial plants the chemical characteristics of the water during these years of similar flow were obtained. After a thorough study of all the factors involved a computation was made of the chemical effect of the Tygart Reservoir water and the results are shown in Table 4.

Monetary Benefits.—The total quantity of water consumed for various purposes throughout the length of the Monongahela River was determined by a field investigation of the records of the municipal water works and industrial plants. These data are shown in Table 5*A*. At the same time time the method

TABLE 5*A*.—WATER CONSUMPTION, MONONGAHELA RIVER, IN MGD

TABLE 5*B*.—MONETARY BENEFITS (ADD 000)

River reach (see Fig. 1)	Year	Domestic	Industrial		Domestic		Industrial		Total annual values
			Cooling water	Boiler feed	Neutralization	Softening	Cooling water treatment	Boiler water treatment	
Fairmont, W. Va., to Charleroi, Pa.	1938	10	95	2	0.4	8.5	3.9	2.6	15.4
	1939	10	175	3	0.6	17.1	10.2	7.9	35.8
Charleroi, Pa., to McKeesport, Pa.	1938	4	120	5	0.2	6.3	7.3	7.9	21.7
	1939	4	220	9	0.2	9.9	12.8	22.3	45.2
McKeesport, Pa., to Pittsburgh, Pa.	1938	18	415	12	0.7	24.3	16.9	16.2	58.1
	1939	18	750	21	0.6	36.8	24.6	42.9	104.9
Totals: 1938	....	....	....	....	1.3	39.1	28.1	26.7	95.2
1939	....	....	....	....	1.4	63.8	47.6	73.1	185.9
1938 and 1939	....	....	....	....	2.7	102.9	75.7	99.8	281.1

and cost of water treatment were ascertained to be as follows:

Neutralization (lime at $12.00 per ton), in cents per million gal, per ppm of acidity neutralized.......................... 5

Softening (lime at $12.00 per ton and soda ash at $1.25 per 100 lb), in cents per million gal, per ppm of hardness removed...... 25

A combined adjustment of water consumption with the reduction in acidity and hardness effected by the Tygart Reservoir and the unit costs of water treatment (in thousands of dollars) produce total monetary benefits as shown in Table 5*B*.

Review of Water-Supply Benefits.—The monetary benefits in Table 5*B* include only the estimated savings in actual water treatment. No attempt has been made to assign monetary values to various factors, such as the increased oxygen-carrying capacity that aids in the assimilation of domestic sewage and other organic and inorganic wastes. The possible effect on the Ohio River below Pittsburgh has also been omitted. The estimates of monetary benefits, therefore, are believed to be conservative.

Although the operation of the Tygart Dam for low-water supply is only one step forward in the desired purification of the rivers of the Upper Ohio Basin, the presence of this reservoir does insure the population of the Monongahela Valley against repetition of the conditions of 1930. This feature of the

project is an important by-product of the primary functions of navigation and flood control.

Conclusion

The total monetary benefits attributable to the Tygart Dam for the first two years of operation are as follows: Flood control, $2,818,000; navigation, $280,000; and water supply, $281,100—a total of $3,379,100. Monetary benefits have been determined only where direct savings to the public can be shown. The direct benefit for the maintenance of navigation is most misleading because, as explained previously, many intangible factors, including delay and disruption of industrial production, are too complicated to be evaluated fully in a paper of this nature. The effect of the Tygart Reservoir in maintaining unrestricted navigation on the Monongahela River throughout the summer and fall of 1939 is probably the greatest single benefit attributable to this project.

The moderate reductions effected on flood crests at Pittsburgh by this one reservoir, controlling only about 6% of the total drainage area of the combined Allegheny and Monongahela rivers, serve as an indication of results that may be obtained from the proposed ten-reservoir system. It is also worthy of mention that, although the net reduction in the flood peak is the most important factor in evaluating flood damages, there is also the factor of time that the river remains above damage stage. On some of the floods discussed herein the degree of synchronization of the peak discharges of the Tygart River with those at Pittsburgh was such that only a small reduction in crest stage was effected, but it was accompanied by a much greater reduction during the recession of the flood, with a corresponding reduction in the time that the river remained above flood stage. This time element is particularly important at the locks of the Monongahela River, and the reduction in the time that these locks would have been out of service is one of the important intangible benefits to which no monetary values have been ascribed.

These first two years of operation most probably do not represent average annual conditions since three of the five largest floods of record on the Tygart Basin occurred during this time and the Monongahela Basin experienced one of the worst droughts in its history. On the other hand, only small floods with low damages have occurred at Pittsburgh during this period. A small reduction on a larger flood would result in greater flood-control benefits than have thus far been obtained on any individual flood.

In conclusion, it is believed that the results achieved thus far by the Tygart Dam, even in the experimental stage, amply justify the funds expended for its construction and operation and are the most conclusive proof that can be offered for the extension of the flood-control system of the Upper Ohio River Basin.

Acknowledgments

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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PAPERS

CONSUMPTIVE USE OF WATER FOR AGRICULTURE

By Robert L. Lowry, Jr.,[1] M. Am. Soc. C. E., and Arthur F. Johnson,[2] Assoc. M. Am. Soc. C. E.

Synopsis

Transpiration and evaporation, together accounting for practically all consumptive use of water, have been shown by experimental investigations to be influenced by climatic factors, of which temperature gives one of the better correlations. Consumptive use in a number of adequately watered irrigated valleys and humid watersheds, representing a wide range in climate, latitude, elevation, and type of crops, is shown in this paper to bear a straight-line relation, within narrow limits, to accumulated daily maximum temperatures above 32° F during the growing season. Factors responsible for deviations from average consumptive use are discussed. The relation of consumptive use to growing-season temperatures offers to the engineer a ready means of estimating probable consumptive use on projects under investigation as an initial step in determining the irrigation requirement at the farm or at the point of diversion. Short descriptions of each area studied, with summaries of annual data, are given in the Appendix.

Introduction

The problems of consumptive use of water have been studied intermittently by engineers of the Bureau of Reclamation since the early years of the organization. The relation of consumptive use to temperatures was first studied in the Bureau in 1920. Other approaches have been made from time to time. Many engineers have collaborated in the studies, and this paper, therefore, is the result of accumulated effort of the organization.

The study was made for the purpose of obtaining a working basis for estimating the probable annual consumptive use of water on projects under

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **August 15, 1941.**

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investigation for conditions of full development with an adequate water supply. With consumptive use known, estimates of irrigation requirement may be made by taking into account the effective precipitation and expected losses incident to the diversion and application of the water consistent with the characteristics of the individual project studied. Efforts were directed toward finding a dependable correlation between consumptive use and some factor that influences consumptive use, or some related factor for which records are readily available in most areas, such correlation to be used in estimating the consumptive use on the areas under investigation. Growing-season temperatures were found to be more satisfactory than other factors studied, and this paper discusses their use for this purpose.

The determination of consumptive use for agriculture, in its full sense, involves allowance for consumptive use by nonagricultural areas also. Where nonagricultural use is limited to precipitation, it need not be considered, but, where it is supplied in part from present or potential irrigation supplies, it cannot be neglected. On many projects, control or elimination of nonproductive water uses may determine the feasibility of additional agricultural development. In this study an attempt has been made to evaluate nonagricultural losses contributing to valley consumptive use in order to arrive at the true value of consumptive use for agriculture.

Definition

In a basic sense, consumptive use[3] is "* * * the quantity of water, in acre-feet per cropped acre per year, absorbed by the crop and transpired or used directly in the building of plant tissue, together with that evaporated from the crop-producing land." The Committee's definition of valley consumptive use includes also the consumptive use on the uncropped area and nonrecoverable deep-soil percolation. Valley consumptive use is represented by the difference between annual inflow to the valley (consisting of surface and subsurface movement of water into the valley and of precipitation) and the total outflow therefrom in the same period (consisting of surface and subsurface movement of water out of the valley), with appropriate correction for changes in surface and subsurface storage. For the purpose of this paper, the Committee's definition of valley consumptive use has been modified by the omission of the term "nonrecoverable deep percolation loss" and the substitution of the term "equivalent valley area" for cropped or irrigated area. "Nonrecoverable deep percolation loss" is a form of outflow independent of the factors influencing transpiration and evaporation, the forms of use primarily considered herein. An effort has been made to limit consideration to valleys where deep percolation losses are believed to be negligible. The equivalent valley area in each case comprises the entire area or valley (except for areas consuming no stream flow), reduced to an area consuming water at a rate equivalent to that by the cropped area. Every area considered contains open water or waterlogged areas from which the loss of water is materially higher than on cropped lands, and the abnormal use on such areas counterbalances the incomplete supply on

[3] "Consumptive Use of Water in Irrigation," Progress Report of the Duty of Water Committee of the Irrigation Div., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 1349–1377.

cropped areas. There are often also extensive areas with partial supplies feeding on ground waters or surface wastes from the irrigated areas.

Earlier Investigations

Many studies have been made for the purpose of determining consumptive use of water. These earlier studies usually have been undertaken for special purposes. They reflect many different conceptions of the meaning of consumptive use, and therefore are not generally comparable. Their results must be interpreted in terms of their purposes.

Tank Experiments.—Some of the first experiments on consumptive use were aimed at measuring plant transpiration in relation to growth, and many of these were designed to eliminate surface evaporation. The laboratory experiments of T. A. Kiesselbach,[4] L. J. Briggs and H. L. Shantz,[5] and many others, with tanks and potometers, have demonstrated that, with other factors constant, production of dry matter for any one crop varies in direct proportion to the quantity of water transpired, over a wide range, but that plant efficiency is affected by soil fertility, frequency of cutting, and many other factors. The efficiencies of various plant species were also found to vary widely under similar conditions.

Other experimenters have used tank data as a basis for estimating consumptive use. J. C. Stevens,[6] M. Am. Soc. C. E., used the Briggs and Shantz data for various crops in estimating consumptive use by applying the relation between transpiration and dry matter produced to crop yields. W. C. Hammatt[7] and E. B. Debler,[8] Members, Am. Soc. C. E., and others, have made more direct application by endeavoring to simulate natural conditions. The integration method used by H. F. Blaney,[9, 10] M. Am. Soc. C. E., involves the determination of the consumptive use of various types of crops, native vegetation, and soil surfaces by experiments using undisturbed samples in tanks set in the native habitat of the plants. The rates of use so determined are weighted by the area devoted to that type of use within the study area or valley to obtain the total consumption and weighted average use.

Because of the difficulty of simulating natural conditions and eliminating individual differences, tank experiments were declared by the Duty of Water Committee[3] to be of questionable value. The improved equipment and methods now available remove many of the uncertainties characterizing early tank experiments.

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[4] "Transpiration as a Factor in Crop Production," by T. A. Kiesselbach, *Research Bulletin No. 6*, Nebraska Agri. Experiment Station, 1916.

[5] "The Water Requirements of Plants," by L. J. Briggs and H. L. Shantz, *Bulletins Nos. 284 and 285*, Bureau of Plant Industry, 1913; "Relative Water Requirements of Plants," by the same authors, *Journal of Agricultural Research*, Vol. 3, 1914, pp. 1–64; and "The Water Requirements of Plants at Akron, Colorado," by H. L. Shantz and L. N. Piemeisel, *loc. cit.*, Vol. 34, 1927, pp. 1093–1190.

[6] "The Duty of Water in the Pacific Northwest," by J. C. Stevens, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 2094.

[7] "Determination of the Duty of Water by Analytical Experiment," by W. C. Hammatt, *loc. cit.*, p. 200.

[8] "Final Report on Middle Rio Grande Investigations," by E. B. Debler, Bureau of Reclamation, May, 1932 (unpublished); see also discussion by Ivan E. Houk, M. Am. Soc. C. E., of "Experiments to Determine Rate of Evaporation from Saturated Soils and River-Bed Sands," by Ralph L. Parshall, Assoc. M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), pp. 982–995.

[9] "Rainfall Penetration and Consumptive Use of Water in Santa Ana River Valley and Coastal Plain," by H. F. Blaney, and Colin A. Taylor and A. A. Young, Assoc. Members, Am. Soc. C. E., *Bulletin No. 33*, California Div. of Water Resources, 1930.

[10] "Regional Planning—Part VI, The Rio Grande Joint Investigation in the Upper Rio Grande Basin in Colorado, New Mexico, and Texas, 1936–1937, Pt. III, Water Utilization," by H. F. Blaney and others, National Resources Committee, 1938, pp. 295–427.

Plot and Farm Experiments.—A large number of plot experiments are constantly in progress at the many federal and state agricultural experiment stations. Although many of these studies are directed toward comparison of plant varieties as to yield, resistance to heat, cold, plant diseases, etc., there have also been many experiments (such as those conducted by J. A. Widtsoe[11, 12] and F. S. Harris[13] in Utah; W. L. Powers[14] in Oregon; M. R. Lewis,[15] M. Am. Soc. C. E., and D. Bark[16] in Idaho; R. G. Hemphill[17] in Colorado; and W. H. Snelson[18] in Alberta, Canada) that have been directed toward use of water. These experiments have shown definite relations between production and application of water under farm conditions and have contributed much to irrigation practice. Generally, they have not attempted to measure deep percolation losses and surface wastes, and so have fallen short of determining consumptive use.

Project Studies.—In some cases, studies of project areas, such as those by W. G. Steward and D. J. Paul[19] and L. Crandall,[20] M. Am. Soc. C. E., in Idaho, have met the requirements for determination of valley consumptive use in method. In general, such studies do not account for uses of water outside the irrigated or cropped area, and the entire use is charged against such area. The increasing necessity for conservation of water resources requires that use of water by nonproductive areas be considered in relation to irrigation efficiency, and in the future most studies must be made valley wide.

Valley Studies.—Many studies of valley consumptive use have been made using inflow-outflow methods and the data submitted as testimony in water-right adjudications and interstate suits. Among these may be mentioned the following: Sevier Valley, Utah[21]; Cache la Poudre Valley, Colorado[17]; Cache la Poudre and South Platte valleys, Colorado, and Little Laramie Valley, Wyoming[22, 23]; Arkansas Valley, Colorado[24]; San Luis Valley, Colorado[25];

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[11] "The Production of Dry Matter with Different Quantities of Irrigation Water," by J. A. Widtsoe, *Bulletin No. 116*, Utah Agri. Experiment Station, 1912.

[12] "The Yield of Crops with Different Quantities of Water," by J. A. Widtsoe, *Bulletin No. 117*, *loc. cit.*

[13] "The Duty of Water in Cache Valley, Utah," by F. S. Harris, *Bulletin No. 173*, *loc. cit.*, 1920.

[14] "Irrigation and Soil Moisture Investigations in Western Oregon," by W. L. Powers, *Bulletin No. 140*, Oregon Agri. Experiment Station, 1914.

[15] "Experiments on the Proper Time and Amount of Irrigation, Twin Falls Experiment Station, 1914, 1915, and 1916," by M. R. Lewis (U. S. Dept. of Agriculture cooperating with Twin Falls County Commissioners, Twin Falls Canal Co., and Twin Falls Commercial Club, 1919).

[16] "Experiments on the Economical Use of Irrigation Water in Idaho," by D. Bark, *Bulletin No. 339*, U. S. Dept. of Agriculture, April 21, 1916.

[17] "Irrigation in Northern Colorado," by R. G. Hemphill, *Bulletin No. 1026*, U. S. Dept. of Agriculture, 1922.

[18] "Irrigation Practice and Water Requirements for Crops in Alberta," by W. H. Snelson, Irrigation Series, *Bulletin No. 6*, Dept. of the Interior, Canada, 1922.

[19] "Report on Drainage Investigations of Pioneer and Nampa-Meridian Districts in Boise Valley for the Years 1916 and 1917," by W. G. Steward and D. J. Paul, Bureau of Reclamation, 1919 (unpublished).

[20] "Report of Use of Water on Twin Falls North Side Project, 1918," by L. Crandall (unpublished).

[21] "Hearing of the Colorado River Commission, Salt Lake City, Utah," by C. J. Ulrich, March 27 and 28, 1922.

[22] Various investigations, mostly unpublished, by R. I. Meeker, M. Am. Soc. C. E. Studies in the South Platte Valley were based on data published in Colorado Agricultural Experiment Station *Bulletin No. 279*, 1922.

[23] "Return Flow Water from Irrigation Developments," by R. I. Meeker, *Engineering News-Record*, Vol. 89, July 20, 1922, pp. 105–109.

[24] "Arkansas River Basin Investigations—Consumptive Use of Arkansas River Waters, Pueblo to Holly," by C. L. Patterson, Colorado Water Conservation Board, 1940 (in course of preparation); see also Amsley Reports, 1922–1925, state engineer of Colorado (unpublished).

[25] "San Luis Valley Field Investigations, Consumptive Use Determinations, Evaporation Experiments, Drainage Measurements, 1930, 1931, 1932," by R. J. Tipton and Francis C. Hart, Assoc. Members, Am. Soc. C. E., Colorado State Engr's Office (unpublished).

Truckee River, Nevada[26]; Owens Valley, California[27]; San Gabriel Valley, California[28]; Columbia River and its tributaries, Washington-Oregon[29]; and Rio Grande in Colorado, New Mexico, and Texas.[10] The results from some of these studies have been used in this paper. Others were either incomplete or involve valleys with an inadequate water supply and so have been excluded.

The methods and results of these studies are not discussed herein as many of the reports are readily available. Several of the studies are discussed in Society publications and reviewed in the Committee Report.[3]

Influence of Meteorological Factors

Although small-scale experiments using tanks, potometers, etc., for determining consumptive use are of questionable value in some cases, they have been of great value in pointing out the importance of transpiration and evaporation as the chief processes in consumptive use and the influence on these phenomena of meteorological factors. Considerable research has been directed toward the determination of both transpiration and evaporation.

The potometer experiments on transpiration of plants of various kinds conducted by Messrs. Briggs and Shantz[30; 31] are among the most extensive of their kind. They indicate a very close correlation between transpiration and meteorological phenomena, including evaporation from water surfaces, air temperatures, solar radiation, and wet-bulb depression readings. Experiments on sorghum, wheat, oats, rye, alfalfa, and pigweed, which cover a wide range of rates of water use, all show similar changes in hourly rate of use throughout the daily cycle.[32] Experiments on the rate of transpiration of tules at the Prado Station in California[33] show similar hourly cycles for transpiration, evaporation, and temperature, of air and of water.

The latter experiments showed that 93.5% of the total daily transpiration occurred between 9 a.m. and 9 p.m. Observations of ground-water levels in an alfalfa field in the Escalante Valley, Utah,[34] showed a distinct lowering of the water table during daylight hours and a recharge at night. Similar results were obtained[35] for native vegetation in the Middle Rio Grande Valley. Ex-

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[26] "Irrigation Studies on the Truckee River, Nevada," by S. T. Harding, M. Am. Soc. C. E.—data from testimony in Truckee River Adjudication, 1918.

[27] "The Determination of Safe Yield of Underground Reservoirs of the Closed-Basin Type," by Charles H. Lee, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVIII (1915), p. 148; also, *loc. cit.*, Vol. 94 (1930), pp. 1382–1397.

[28] Discussion by H. Conkling, M. Am. Soc. C. E., of "Consumptive Use of Water in Irrigation," Progress Report of the Duty of Water Committee of the Irrigation Div., *Transactions*, Am. Soc. C. E., Vol. 94 (1930), p. 1378.

[29] "Water Powers of the Cascade Range—Pt. III—Yakima River Basin," by G. L. Parker, M. Am. Soc. C. E., and F. B. Store, *Water Supply Paper No. 369*, U. S. Geological Survey, 1916.

[30] "Hourly Transpiration Rate on Clear Days as Determined by Cyclic Environmental Factors," by L. J. Briggs and H. L. Shantz, *Journal of Agricultural Research*, Vol. 5, January 3, 1916, pp. 583–650.

[31] "Daily Transpiration During the Normal Growth Period and Its Correlation with the Weather," by L. J. Briggs and H. L. Shantz, *loc. cit.*, Vol. 7, October 23, 1916, pp. 155–212.

[32] "Hourly Transpiration Rate on Clear Days as Determined by Cyclic Environmental Factors," by L. J. Briggs and H. L. Shantz, *Journal of Agricultural Research*, Vol. 5, January 3, 1916, p. 617.

[33] "Water Losses Under Natural Conditions—Pt. I—South Coastal Basin Investigations," by H. F. Blaney, *Bulletin No. 44*, California Div. of Water Resources, 1933.

[34] "A Method of Estimating Ground Water Supplies Based on Discharge by Plants and Evaporation from Soil," by W. N. White, *Water Supply Paper No. 659–A*, U. S. Geological Survey, 1932, pp. 87 and 88.

[35] "Regional Planning—Part VI, The Rio Grande Joint Investigation in the Upper Rio Grande Basin in Colorado, New Mexico, and Texas, 1936–1937, Pt. II—Ground Water Resources," by C. V. Theis, National Resources Committee, 1938, pp. 272–277.

periments on evaporation[36] at Fort Collins, Colo., showed that two thirds of the daily evaporation occurs during daylight hours. Obviously, solar radiation or some secondary phenomenon connected with sunlight is an important factor in both transpiration and evaporation. In fact, the Escalante Valley experiments showed a marked decrease in transpiration on cloudy days from that on clear days.

The Briggs and Shantz experiments over long-time periods show the variation in rates from day to day as affected by changes in solar radiation and temperatures. A comparison of daily changes in solar radiation and temperatures indicates that the former is a function of the altitude of the sun and thus varies with the seasons, whereas the latter (temperature), which is dependent upon absorption of heat by air and earth, and the circulation of the air, lags behind the seasons. Although solar radiation gives one of the best correlations with transpiration and evaporation, growing-season temperatures more nearly parallel the cycle of plant growth. Fig. 1 shows clearly the relation of consumptive

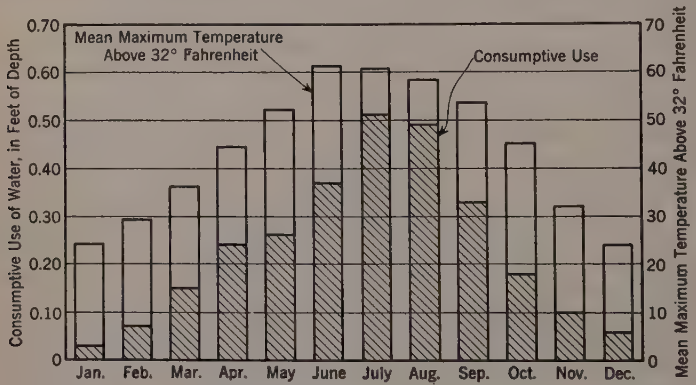

FIG. 1.—COMPARISON OF MONTHLY DISTRIBUTIONS OF CONSUMPTIVE USE AND MEAN TEMPERATURES ABOVE 32° F

use to the monthly mean of maximum daily temperatures above 32° F for the Mesilla Valley in New Mexico, one of the few areas for which monthly values of consumptive use are available. Maximum daily temperatures are used as being more representative of daylight hours when most of the transpiration occurs.

Consumptive use (see Fig. 1) is relatively lower with respect to temperature in winter than in summer and has a different distribution in the spring than in the fall. These variations are produced by differences in plant water requirement as related to plant development. In winter, perennial plants are practically dormant and annual plants are entirely so. As spring advances, annual crops are seeded and all plants increase in size. As their leaf area enlarges, their water requirements increase, and reach a maximum soon after maximum temperatures pass the peak of the season. In the fall, water requirements drop rapidly as the function of the plant is transferred from growth to processes

[36] "Evaporation from Free Water Surfaces," by C. Rohwer, Assoc. M. Am. Soc. C. E., *Technical Bulletin No. 271*, U. S. Dept. of Agriculture, December, 1931, pp. 23–26.

of ripening and reproductive development; and growth practically ceases with harvest or with the onset of sub-growing temperatures. The pronounced lag in consumptive use noted in May is due in part to the cutting of alfalfa and the harvest of winter crops and in part to the preparation of seedbeds for planting cotton and other summer crops. In the Mesilla Valley 91% of annual consumptive use and 85% of annual evaporation occur during the eight-month growing season, March to October.

Hedke Method of Estimating Consumptive Use.—The method of estimating consumptive use derived by C. R. Hedke,[37] M. Am. Soc. C. E., is based on the principle that each crop requires a definite quantity of heat above its germinating temperature to bring it to maturity, and that moisture and plant food are required in proportion to the heat utilized. Basing his study on the years 1916 and 1917 and the estimated normal year, in the Cache la Poudre Valley, Mr. Hedke accumulated mean daily temperatures, in degrees Fahrenheit above the germinating temperature, for each crop produced in the valley over its period of growth; and he weighted each crop total by the area devoted to that crop. The weighted average consumptive use of heat per cropped acre was then plotted against consumptive use of water in acre-feet per cropped acre to obtain a correlation curve. In determining consumptive use of water, 50% of the annual precipitation was added to the measured stream depletion. The ultimate object of the method was apparently a guide for estimating stream depletion in other valleys, and it was assumed that effective moisture not obtained from precipitation must be supplied from stream flow. Mr. Hedke assumed that precipitation, falling on the soil surface, evaporates twice as rapidly as normal soil moisture drawn from depth. He did not allow for variations in ground-water storage in computing consumptive use, and he charged the total valley use to the cropped area without regard to relative uses by non-cropped land.

Method of Study

The present study, like that of Mr. Hedke, is predicated on correlating consumptive use of water to effective heat during the growing season; but in details it departs widely from the Hedke method. Of necessity the study has been confined to a rather limited number of areas for which essential data are available. These data have been utilized in accordance with a method of procedure found by trial to give the most satisfactory results.

Measurement of Use of Water.—On each area used in the study, adequate records or estimates were obtained of surface inflow to, and outflow from, the area; of changes in surface and ground-water storage, if appreciable; and of precipitation. The study was confined to areas believed to have negligible or compensating underground inflow and outflow. Where ground-water storage was considered to be fairly stable from year to year, no correction for changes in such storage was deemed necessary, as any error due to this factor is practically eliminated from the mean consumptive use over a period of several years. Precipitation was computed by weighting the recorded quantity at

[37] "Consumptive Use of Water by Crops," by C. R. Hedke, New Mexico State Engrs. Office, July, 1924 (unpublished).

available stations by the area nearest each station. The full year, rather than the growing season, was used as a basis of study in order to eliminate the effect of seasonal changes in water table and soil moisture, which are affected by the surplus or deficiency in precipitation compared with consumptive use. For the sake of uniformity, the calendar year was used for all areas, although it is perhaps less satisfactory on some areas than some other operating year for securing accurate measurements because of unfavorable distribution of precipitation, snow storage, high stream discharge, or ground-water changes. As these factors vary with the locality, no one period would be satisfactory for all areas.

Correction for Years of Short Water Supply.—In most of the areas studied, shortages of water supply occurred in one or more years of the period of record. Omitting the records of the dry years in arriving at mean values for each area introduces errors in those cases where no records of ground-water storage are available because it is usually withdrawn in dry years, through consumptive use and natural drainage into streams, and is replaced in wet years. When dry years are omitted, ground-water recharge enters into the mean as consumptive use. It was believed preferable, therefore, to include all years of record in the mean and to add a correction factor to the water supplies in low years sufficient to make them adequate. From an inspection of the records of inflow and consumptive use in dry years, it appears that consumptive use deviates from the mean much less, relatively, than does total inflow. In the humid regions the lessened rainfall is more fully used because the soil capacity of the root zone is overtaxed less frequently with reduced escape of water to the underlying water table. Runoff is also relatively less. In the irrigated areas, the same results are achieved by reduced irrigation applications and reduction in surface waste. Withdrawals from ground water both for plant life and through natural drainage are important in such years. In succeeding wet years there is a recharge of depleted ground water.

Equivalent Area.—Instead of referring use to cropped area or to gross valley area as a basis for computing consumptive use (as has been done frequently), an estimate was made of the area, within the limits defined by points of measured inflow and outflow, consuming water at a rate equivalent to that on the cropped area or land representative of prevailing conditions in the valley. In irrigated valleys, high areas consuming only precipitation and contributing a negligible quantity of runoff were omitted. Corrections were made in the remaining area, as believed advisable, to compensate for unbalanced areas in swamp land consuming water at a higher rate than cultivated land and in natural vegetation adjacent to irrigated areas consuming less water than cultivated land. Although these corrections are a matter of judgment, the net effect is small, since the cropped land in most agricultural valleys amounts to 85% or more of the gross area.

Correction for Variations in Irrigated Area.—In irrigated valleys the acreage actually irrigated and cropped may vary from year to year. After full development has been reached, fluctuations in area are normally so small that they may be neglected in computing consumptive use; but, if economic conditions cause major changes, the variations in area should be considered. Since

1933 the program of the Agricultural Adjustment Administration (AAA) in many localities has effected reductions in the area devoted to certain staple crops and has freed the land for other uses. In some cases land has been removed from cultivation and has received little or no regular irrigation. Where the cropped area in a given valley has been reduced during the period of study, consumptive use values in the affected years have been computed on an equivalent area, resulting from a deduction of one half the area temporarily out of cultivation that year, on the theory that a limited weed growth and soil evaporation would still occur. This is equivalent to fixing the consumptive use on the fallow land at one half that on cropped land.

Effective Heat.—As used in this paper, the term "effective heat" is the accumulation, in day-degrees, of maximum daily temperatures above 32° F during the growing season. This measure of heat was decided upon after comparative studies had been made using mean daily, as well as maximum, temperatures with bases both above and below 32°. The choice of the maximum temperature is in line with experiments showing that transpiration and evaporation take place almost entirely during the daylight hours, when temperatures are normally higher than at night. Although evaporation of moisture from soil and other surfaces continues at a low rate during the winter, transpiration is almost entirely confined to the growing season. During the winter season, maximum temperatures are frequently considerably above the freezing point even when minimum temperatures prevent plant growth. Because of this break in the relation of consumptive use to heat, a much better correlation is obtained between annual consumptive use and accumulated growing-season heat units than with total annual heat units.

Length of Growing Season.—The growing season is frequently considered the time between killing frosts. For annual crops the growing season is necessarily shorter than the frost-free period, in order to permit planting in the spring after danger of frosts is past and to permit maturing and possibly harvesting before the first serious frost in the fall. For most perennial crops, growth starts as soon as the maximum temperature rises far above the freezing point for any extended period of days and continues in spite of later freezes throughout the season—and sometimes for many weeks after the first so-called killing frost. In the spring, and to a less extent in the fall, daily minimum temperatures may fluctuate 1° or 2° above and below 32° for several days before remaining above or below that point, even though the temperature for most of the day is quite warm and the hardier crops are able to grow unharmed by the few hours of subfreezing temperature. In fact, many hardy crops, and especially grasses, mature although temperatures drop below freezing at frequent intervals throughout the summer.

In order to define the limits of the growing season more definitely than the dates of the latest and earliest frosts, minimum temperature data were smoothed by obtaining twice-repeated five-day moving averages, equivalent to applying the expression, $\frac{a + 2b + 3c + 4d + 5e + 4f + 3g + 2h + i}{25}$, to the consecutive daily minimum temperatures (represented in the expression by letters) and offsetting the result to the middle day of the series to maintain proper

phase. The ends of the growing season are thus defined by the dates between which the smoothed temperatures remain above 32°.

Valley Areas Studied

The work of the Bureau of Reclamation is concerned primarily with the arid West where agricultural development is dependent upon irrigation. In this paper, available studies on consumptive use were utilized. Some of them were on Bureau of Reclamation projects, constructed or under investigation; others were studies by various agencies in connection with local and interstate litigation. Many other areas were examined but discarded for lack of adequate records.

As a check on the validity of the conclusions reached for irrigated areas, the study was extended to a number of valleys in the humid regions of the United States and to one mountain basin in Colorado, where an essentially adequate water supply is obtained from precipitation. In these cases the entire watersheds are included in the study areas, and consumptive use is computed by deducting outflow from precipitation. Consumptive use in these humid valleys was found to vary with effective heat of the growing season in exactly the same manner as in irrigated arid valleys.

Table 1 lists the various valley areas used, with comparative data on area, elevation, and number of years of study. Fig. 2 shows the average consumptive

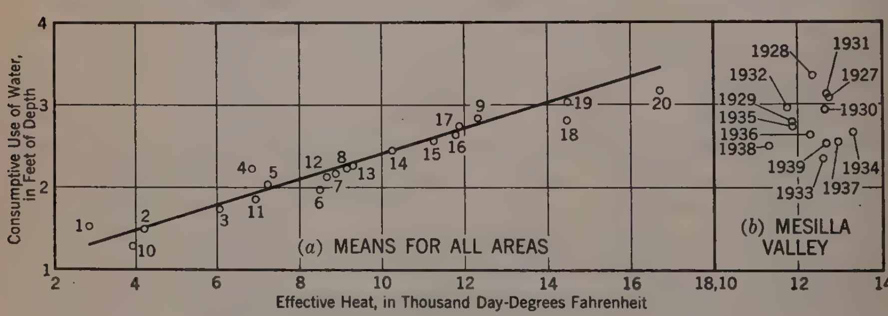

FIG. 2.—RELATION OF CONSUMPTIVE USE TO EFFECTIVE HEAT (NUMBERS IDENTIFIED IN TABLE 1)

use on each area for the period of study, plotted against the corresponding average effective heat. A brief description of each area, with annual data on consumptive use, effective heat, and source of basic information, is given in the Appendix.

The areas included in the study represent practically the entire range of growing conditions and types of agriculture found in the United States. Climatic conditions range from arid to humid and from frigid to subtropical, growing seasons range from a few days to the entire year, latitudes range from near the northern to the southern boundaries of the nation, and altitudes from near sea level to over 10,000 ft; crop diversification varies from the mountain meadow hay and alpine forests to cotton and citrus fruits. Notwithstanding the wide differences in crops, the relation between consumptive use and effective heat is maintained closely.

TABLE 1.—COMPARATIVE DATA ON VALLEY AREAS STUDIED

No.	Valley (see Appendix)	State	Approximate mean elevation, in ft	Equivalent area, in acres	Length of record, in years	Mean consumptive use, in ft	Mean effective heat, in day-degrees (F)
	(a) IRRIGATED VALLEYS						
1	New Fork	Wyoming	7,400	25,000	1	1.53	2,860
2	Michigan and Illinois	Colorado	8,300	43,000	3	1.50	4,210
3	Southwest Area, San Luis	Colorado	7,750	390,000	1	1.74	6,070
4	West Tule Lake	California	4,150	6,300	7	2.22	6,890
5	Garland Division of Shoshone Project	Wyoming	4,350	41,900	4	2.02	7,240
6	North Platte	Wyoming-Nebraska	4,100	462,000	9	1.98	8,560
7	Mason Creek and Boise	Idaho	2,500	13,570	2	2.17	8,940
8	Uncompahgre	Colorado	5,500	137,700	2	2.24	9,200
9	Mesilla	New Mexico-Texas	3,800	109,000	13	2.83	12,370
	(b) NON-IRRIGATED WATERSHEDS						
10	Wagon Wheel Gap "A"	Colorado	10,000	222	14	1.30	3,980
11	Black River	Wisconsin	1,200	494,000	13	1.85	6,950
12	Mad River	Ohio	1,100	307,000	13	2.15	8,760
13	Skunk River	Iowa	700	1,850,000	16	2.25	9,340
14	Sangamon River	Illinois	700	1,640,000	16	2.43	10,270
15	North Fork of White River	Missouri	1,200	755,000	14	2.58	11,260
16	Green River	Kentucky	600	5,000,000	6	2.62	11,810
17	Tallapoosa River	Alabama-Georgia	900	1,060,000	13	2.75	11,900
18	East Fork of Trinity River	Texas	600	531,000	14	2.82	14,460
19	Cypress Creek	Texas	300	545,000	13	3.02	14,500
20	San Jacinto River	Texas	200	1,159,000	9	3.19	16,710

ANNUAL VARIATIONS IN CONSUMPTIVE USE

The plotted points of Fig. 2(*a*) show only a small deviation from the curve. When individual years instead of means are plotted, a rather wide variation is noted. As an example, Fig. 2(*b*) shows the relation of consumptive use to effective heat for the Mesilla Valley, New Mexico, for the period of study 1927 to 1937. Measurements on this valley are probably more complete than on other areas studied, and yet deviations from the mean consumptive use amount

to as much as 18.7% and deviations from the mean effective heat to 6.4% for individual years.

Variations from the mean may arise from many causes, among which are: (1) Inaccurate records of discharge; (2) inaccurate estimates of ground-water storage changes; (3) inadequate precipitation and temperature records; (4) differences in occurrence and intensity of precipitation, and variability of temperatures, wind, hail, cloudiness, humidity, etc.; (5) inaccuracy in estimating the area equivalent in rate of use to the cropped area; and (6) changes in crop distribution, damage by plant diseases, insects, animals, and weeds.

In many valleys where the stream flow is quite large as compared with stream depletion within the valley, small percentages of error in stream measurement may be reflected by large discrepancies in estimates of consumptive use.

Corrections for surface storage can be made accurately where reservoir-capacity data and storage records are available but, because of the heterogeneous nature of most valley fills, it is difficult to evaluate ground-water storage changes. Water-level measurements from only a few wells may be misleading. Application of a single yield coefficient for converting differences in depth into storage changes gives a rough estimate at best and may not reflect actual conditions, especially where changes in water-table levels are not uniform throughout the valley.

Inadequate coverage by precipitation stations may result in wide discrepancies where rainfall distribution tends to "spottiness." Even with adequate coverage, variations still will prevail where there are marked differences in seasonal distribution. For the same annual precipitation, soil-surface evaporation will be much greater in a year when there are numerous light showers than when there are only a few heavy rains which penetrate deeply and also produce considerable surface runoff. The distribution between summer and winter also affects consumptive use, as evaporation is higher in summer than in winter.

Temperatures do not usually vary greatly over a valley area, and for that reason inadequate coverage by recording stations is not likely to be an important source of error in estimating effective heat. Differences in temperature from year to year will normally cause corresponding differences in consumptive use; but changes in consumptive use may vary widely from the relation exhibited by valley means (see Fig. 2(*a*)) because of changes in temperature distribution and plant efficiency. Abnormally low temperatures may retard plant development, or unusually high temperatures may cause plants to become dormant. The latter effect is especially noticeable in the case of the southern study areas where consumptive use is actually reduced in the hotter years, even when the water supply is adequate. Consumptive use may vary widely in years of equal accumulated temperatures because of deviations from the normal seasonal distribution, as transpiration is influenced by the area of leaf surface and plant functional requirement (both related to stage of maturity), as well as by temperature. The effect of increased wind movement is normally to increase evaporation; and the effect of increased humidity and cloudiness is to reduce it. Hail may damage crops and thus reduce, materially, their rates of transpiration.

The annual consumptive use varies with type of management and land use, which are influenced by economic factors. These factors may change crop distribution and methods of irrigation. Thus, a change from grain to alfalfa will increase transpiration. Consumptive use by native vegetation on non-cropped areas may be expected to remain more or less constant from year to year where the water supply is adequate, but surface evaporation will increase in wet years because of the increase in area of water surface and moist soil.

In irrigated valleys there is usually a lavish application of irrigation water when the supply is abundant; and there is a consequently larger amount of waste and return flow available for nonproductive consumption, as well as an increased soil evaporation on the cropped lands. For these reasons the equivalent valley area is seldom a constant; actually, it increases in wet years and decreases in dry years. Plant diseases and pests reduce consumptive use by inhibiting plant growth. The spread of noxious weeds may reduce the area irrigated if crops cannot be grown in infested areas. Indirect results of infestation may appear in widespread changes in crops.

Most of the factors mentioned fluctuate from year to year. Consumptive-use studies continued over a series of years may be expected to eliminate most of the inaccuracies in measurements and smooth out short-time trends due to changes in crop distribution and other factors.

Application

An essential feature of the water-supply investigation of an irrigation project, whether it is an undeveloped tract needing a full supply or a developed project needing supplemental water, is the determination of its water requirement. Annual consumptive use is a convenient approach in this determination. From the relation developed in this paper it is possible to estimate consumptive use from temperature data, which are usually available. The crop irrigation requirement may then be estimated by deducting from consumptive use that part of the annual precipitation that it is estimated will be consumed. The farm delivery requirement then may be estimated by adding to the crop irrigation requirement an allowance for deep percolation and surface waste incident to application of water to the land, consistent with anticipated irrigation practice under local conditions. The diversion requirement then may be estimated by providing for canal and lateral operation waste and losses in transit from stream or reservoir to the farm. In case the project is to be devoted chiefly to a single crop, differing materially in its water requirements from the average requirements common to the temperature conditions of the project, the consumptive use estimated from the relation curve may be corrected as required. Although the determination of the project-diversion requirement leaves much to judgment after the proper consumptive use has been adopted, a mass of experimental data on the other factors is available to guide the investigating engineer in his determinations.

Acknowledgment

Data for this paper were secured from various sources, and where possible the source has been stated. Data on land use in the humid areas were obtained

from U. S. Census reports, or from personal knowledge, and climatic and runoff data used in connection with area studies, not specifically acknowledged, were obtained from publications of the Weather Bureau and Geological Survey, respectively. Appreciation is expressed for the advice and assistance of R. I. Meeker, M. Am. Soc. C. E., and R. J. Tipton, Assoc. M. Am. Soc. C. E., in the conduct of this study.

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APPENDIX

Description of Areas Studied

New Fork Valley, Wyoming.—The valley is in the upper Green River Basin above Pinedale, Wyo., at an elevation of about 7,400 ft. It has a gross area of 25,400 acres, of which 16,650 acres in meadow hay and pasture are irrigated. A part of the remainder is in sagebrush and a part in willows and seeped alkali land. The equivalent area is estimated at 25,000 acres. The climate is severe in winter. Summer temperatures reach 90°, but frosts frequently occur at night in every month. Precipitation averages 10.2 in., of which 4.3 in. fall in the months of May to August. Irrigation by wild flooding is practiced, and the area always has adequate water. Consumptive-use studies were made by the Bureau of Reclamation in 1939 (Table 2, No. (1)) and include inflow and outflow measurements and precipitation. Ground-water storage, as indicated by test wells, did not show any change for the year, and ground-water movement is believed to be small. Because of the prevalence of summer frosts, the effective growing season is doubtless longer for the type of crop grown than that computed for the study.

Michigan and Illinois Rivers, North Park, Colorado.—The study area includes the narrow valley of the Michigan River between the Haworth School and Cowdrey and the tributary valley of the Illinois River below Rand, in North Park, near Walden, Colo. Grass and hay, covering 35,250 acres, are the only crops. The remainder of the area is in willows and water surface. The equivalent area is estimated at 43,000 acres. The average elevation is about 8,300 ft. The climate is rigorous. Temperatures range from − 49° to 109° with a May-to-September average of 53°. The frost-free period is about 60 days. Annual precipitation averages 10 in., about one half falling during the growing season. The meadows are irrigated by flooding until hay is cut,

and the supply obtained from the Michigan and Illinois rivers is adequate. Data (Table 2, No. (2)) were supplied by the Colorado Water Conservation Board.[38] Winter runoff not covered by records was estimated. Ground-water storage as shown by well measurements was essentially the same each year with no movement into, or out of, the basin.

TABLE 2.—CONSUMPTIVE USE SUMMARY, IRRIGABLE VALLEYS ((1) TO (4)) IN WYOMING AND COLORADO[a]

Year	(1) NEW FORK VALLEY		(2) MICHIGAN AND ILLINOIS		(3) SAN LUIS VALLEY		(4) WEST TULE LAKE	
	U	H	U	H	U	H	U	H
1933	....	....	....	....	....	....	1.47	5,520
1934	....	....	....	....	....	....	2.66	6,850
1935	....	....	....	....	....	....	2.31	6,440
1936	....	....	....	....	1.74	6,070	1.82	6,900
1937	....	....	1.70	4,630	....	....	2.19	6,890
1938	....	....	1.27	4,550	....	....	2.24	7,200
1939	1.53	2,860	1.53	3,570	....	....	2.86	7,520
Mean			1.50	4,250			2.22	6,890

[a] U = consumptive use, in ft; and H = effective heat, in degree-days (F).

Southwest Area, San Luis Valley, Colorado.—The area includes the valley lands tributary to the Rio Grande on the west, amounting to 400,000 acres. The topography is comparatively flat—about 7,750 ft in elevation. About 56% of the area is farmed, principally to hay, grain, and potatoes. The equivalent area is estimated at 390,000 acres. The climate is moderately warm, dry, and windy during the short summers, and cold in winter. Precipitation averages about 7 or 8 in. per yr, about 65% of it in the growing season. Water for irrigation is supplied from the Rio Grande and its tributaries. Subirrigation is practiced widely. Consumptive use has been determined by a number of agencies from inflow-outflow measurements, but the present study (Table 2, No. (3)) includes only the 1936 determination by the Bureau of Agricultural Engineering as a part of the Rio Grande Joint Investigation.[10]

West Tule Lake, Klamath Project, California.—The area includes land under the J-1 lateral of the Tule Lake Division of the Klamath Federal Reclamation Project lying within the dikes west of Lost River. The mean elevation is about 4,150 ft. The effective area is estimated at 6,300 acres, of which 5,700 acres have been irrigated in recent years. The soil is a sandy lake-bed deposit and drains well. Alfalfa, cereals, and potatoes are the principal crops. Temperatures vary from sub-zero to 100°. The growing season is about 140 days. Precipitation averages about 8.5 in., most of which occurs in the fall and winter. Irrigation is necessary for agriculture, and an adequate supply is obtained from the Lost and Klamath rivers. Inflow is measured in the supply lateral and outflow at drainage pumps. Ground-water storage is maintained nearly constant by well-regulated drains; and, because of the small differential between

[38] "Jackson County Investigations—Special Studies on Consumptive Use," by C. L. Patterson, Colorado Water Conservation Board, 1940 (in course of preparation).

Lost River and drains, little underground flow is believed probable. Data (Table 2, No. (4)) were furnished by the Klamath Project office and from the report by J. R. Iakisch,[39] Assoc. M. Am. Soc. C. E.

Garland Division, Shoshone Project, Wyoming.—The area consists of 48,400 acres of fairly level bench land, 4,200 to 4,600 ft in elevation, lying north of the Shoshone River near Powell, Wyo. Although only 30,000 acres were irrigated at the time of the study, 41,900 acres are irrigable and estimated to consume water equivalent to the rate on cropped land. Soils vary from sand to clay. About 50% of the cropped area was in alfalfa, the remainder being mainly grain, sugar beets, and potatoes. Inflow is supplied by the Garland Canal and some storm runoff; outflow is measured in the Frannie Canal, Bitter Creek, and river drains. Complete records of inflow and outflow are not available in any one year, but the missing data (storm inflow, and drains to river and Bitter Creek) have been estimated from measurements available in other years. Deep percolation losses are believed negligible. The water supply was average in two years, abundant in one, and deficient in one. Data in Table 3, No. (5), were supplied by the Bureau of Reclamation project office and from reports by H. H. Johnson.[40]

TABLE 3.—CONSUMPTIVE USE SUMMARY, VALLEYS (5), (7), AND (10), IDAHO, WYOMING, AND COLORADO[a]

Year	(5) Garland Division			(7) Mason Creek		(10) Wagon Wheel Gap[c]		
	U_p	U_r	H	U	H	U_p	U_r	H
1912	....	....	....	....	....	0.86	1.05[b]	3,990
1913	....	....	....	....	....	1.30	1.30	3,920
1914	....	....	....	....	....	1.27	1.27	4,200
1915	....	....	....	....	....	1.35	1.35	3,600
1916	....	....	....	2.13	8,760	1.48	1.48	3,710
1917	....	....	....	2.20	9,130	0.75	0.91[b]	3,930
1918	....	....	....	....	....	1.65	1.65	4,410
1919	....	....	....	....	....	1.43	1.43	4,060
1920	....	....	....	....	....	1.14	1.14	3,910
1921	....	....	....	....	....	0.97	1.13[b]	4,060
1922	....	....	....	....	....	1.54	1.54	4,520
1923	2.06	2.06	7,480	....	....	1.43	1.43	3,960
1924	1.64	1.84[b]	6,960	....	....	0.75	1.02[b]	3,860
1925	2.16	2.16	7,120	....	....	1.43	1.43	3,540
1926	2.02	2.02	7,390	....	....	....	....	....
Mean	1.97	2.02	7,240	2.17	8,940	1.24	1.30	3,980

[a] U = consumptive use, in ft; U_p = computed consumptive use; U_r = corrected consumptive use and H = effective heat, in degree-days (F). [b] Dry year corrected by a quantity considered adequate to insure normal consumptive use. [c] Non-irrigable watersheds; Nos. (5) and (7) are irrigable.

North Platte Valley, Wyoming, and Nebraska.—The study area includes the valley between Whalen, Wyo., and Lisco, Nebr. This valley has an average elevation of 4,100 ft, a length of 120 miles, and a width of 7 miles. It includes an equivalent area estimated at 462,000 acres, which includes the North Platte Federal Reclamation Project and private lands. The bottom soils are black loam, the terrace soils sandy loam. There is a considerable area of river bed,

[39] "Report on Pumping from Tule Lake and Wild Life Refuge Development," by J. R. Iakisch and others, Bureau of Reclamation Project Investigations Report No. 5, April, 1938.

[40] "Consumptive Use of Water, Garland Division of Shoshone Project," by H. H. Johnson, Reports for years 1923–1926, Bureau of Reclamation (unpublished).

reservoir surface, and seeped bottom land. Part of the area once seeped has been drained. The underlying Brule clay precludes deep percolation, and as the channel is cut into this material there is little underflow at the river gaging stations. The area is in general crops with alfalfa, sugar beets, and cereals prevailing. Temperatures vary from sub-zero to more than 100°. The growing season is about 165 days. Precipitation averages about 17 in., of which 75% falls in the growing season. Additional water is supplied by irrigation from the North Platte River and from reservoir storage. The supply has

TABLE 4.—CONSUMPTIVE USE SUMMARY, VALLEYS (6), (8), (9), AND (16)[a]

Year	(6) NORTH PLATTE			(8) UNCOMPAHGRE		(9) MESILLA			(16) GREEN RIVER[d]		
	U_p	U_r	H	U	H	U_p	U_r	H	U_p	U_r	H
1927	....	....	....	....	....	3.09	3.14[c]	12,750	....	....	....
1928	....	....	....	....	....	3.36	3.36	12,370	....	....	....
1929	....	....	....	....	....	2.81	2.81	11,840	....	....	....
1930	....	....	....	....	....	2.96	2.96	12,630	....	....	....
1931	1.58	1.71[b]	8,230	....	....	3.13	3.13	12,670	2.77	2.77	11,740
1932	2.31	2.31	7,740	....	....	2.98	2.98	11,750	2.69	2.69	11,630
1933	1.93	1.93	8,930	....	....	2.35	2.43[c]	12,570	2.57	2.57	11,680
1934	0.95	1.82[b]	8,380	....	....	2.67	2.79[c]	13,230	2.41	2.41	11,430
1935	1.69	1.69	7,720	....	....	2.63	2.82[c]	12,240	2.60	2.60	11,740
1936	2.10	2.10	9,380	....	....	2.73	2.76[c]	11,860	2.21	2.66[b]	12,660
1937	2.30	2.30	8,020	....	....	2.54	2.54	12,940	....	....	....
1938	2.09	2.09	9,310	2.20	9,200	2.50	2.54[c]	11,310	....	....	....
1939	1.83	1.83	9,370	2.27	9,190	2.53	2.56[c]	12,630	....	....	....
Mean	1.86	1.98	8,560	2.24	9,200	2.79	2.83	12,370	2.54	2.62	11,810

[a] U = consumptive use, in ft; U_p = computed consumptive use; U_r = corrected consumptive use; and H = effective heat, in degree-days (F). [b] Dry year corrected by a quantity considered adequate to insure normal consumptive use. [c] Years of reduced acreage, U_r based on equivalent area less one half of reduction in area. [d] Non-irrigable watershed; Nos. (6), (8), and (9) are irrigable.

been adequate except in years such as 1931 and 1934. Records of inflow and outflow (Table 4, No. (6)) were corrected for changes in surface storage, but no correction for ground storage was considered necessary. Records were obtained from the North Platte Project Office.

Mason Creek Area—Boise Project, Idaho.—The area includes 13,570 acres of the Pioneer Irrigation District in the Boise Project of the Bureau within the Mason Creek drainage unit near the towns of Caldwell and Nampa. The elevation is about 2,500 ft and the topography fairly flat. It is all irrigated except for right of ways and farmsteads. At the time of the study it was more than 50% in alfalfa and clover hay, and the remainder was largely grain. An adequate water supply is furnished through the Phyllis Canal from Boise River and Arrowrock Reservoir. Precipitation amounts to about 10 in. per yr, little of it occurring in the growing season. Outflow is measured in the various drains and by observing the ground-water changes in 55 wells. There is apparently little difference in ground-water inflow and outflow as the area is surrounded with uniformly well-drained irrigated land (see Table 3, No. (7)).[19]

Uncompahgre Valley, Colorado.—The valley is at the foot of the San Juan Mountains in the Gunnison Basin and includes Montrose and Delta, Colo., at an average elevation of about 5,500 ft. Most of the area is in the Uncompahgre Federal Reclamation Project. It has a gross area of 169,000 acres, of

which 86,500 acres are irrigated and cropped to alfalfa, cereals, sugar beets, and onions. The soil is heavy, much of it has been seeped, and a part is affected by alkali. From a field study of the non-cropped land, the area equivalent in rate of use of water to the cropped land is estimated at 137,700 acres. Temperatures range from −25° to 102° with an average during the growing season of about 67°. The growing season is about 145 days. The average annual precipitation is 9 in., of which about one half falls in the growing season. Abundant water for irrigation is obtained from the Uncompahgre and Gunnison rivers and the Taylor Park Reservoir. Ground-water inflow and outflow are believed to be small. Corrections were made for ground-water storage change effected by drainage operations. Data were secured in a cooperative study by the Bureau of Reclamation and Colorado Water Conservation Board (see Table 4, No. (8)).

Mesilla Valley, New Mexico-Texas.—The Mesilla Valley is one of the many narrow valleys of the Rio Grande. It extends from Fort Seldon, New Mexico, almost to El Paso, Tex., and comprises the Leasburg (N. Mex.) and Mesilla divisions of the Rio Grande Federal Reclamation Project. The valley bottom is flat and varies in elevation from 3,700 to 3,900 ft. Of the gross area of 109,000 acres, about 77,000 acres are cropped, principally to cotton and alfalfa, 13,000 acres are in native vegetation, and the remainder in miscellaneous uses. Studies by the U. S. Bureau of Agricultural Engineering[10] in 1936 indicate the consumptive use by the non-cropped land to be essentially at the same rate as by the cropped land except in the case of areas temporarily out of cultivation, which vary from year to year. An equivalent area of 109,000 acres has been used except for the years noted in Table 4, No. (9). Temperatures range from −8° to 106° with an annual mean of about 60°. Annual precipitation is about 8 in., two thirds falling in the 240-day growing season. Irrigation water is supplied from the Rio Grande with storage in Elephant Butte and Caballo reservoirs. The data in Table 4, No. (9), were supplied by the Rio Grande Project Office, and for the years 1927 to 1936 were included, in uncorrected form, in the study by Mr. Debler.[41] The effect of annual differences in the factors affecting consumptive use of water is well illustrated by the annual variations in consumptive use in the Mesilla Valley.

At Agricultural College, N. Mex., precipitation in 1931 was the highest of the years of study and consumptive use of water second highest. In 1935 precipitation was second highest in total, but approximately one half of the total fell in one day and consumptive use was below the average of the period. In 1928, the year of greatest consumptive use, precipitation occurred in many light rains. The year of least rainfall, 1933, was also the year of least consumptive use.

The effective heat in 1933 was approximately the same as in 1931 but consumptive use was 0.78 ft less. Lower temperatures in April, May, and June may account for a part of the lower use in 1933, with rainfall and other factors accounting for the remainder.

Land use is one of the most important factors causing variation in consumptive use. The cotton acreage in the Mesilla Valley declined between

[41] "Valley Consumptive Use," by E. B. Debler, *Transactions*, Am. Geophysical Union, Pt. II, National Research Council, 1937, Washington, D. C., pp. 532–536.

1930 and 1935 and has increased since that time. Compensating changes were made in general crops or in the fallow acreage. The alfalfa acreage has remained practically constant for many years. The effect of differences in land use is most apparent in 1933, 1934, and 1935 (Table 4, No. (9)), when the government cotton-reduction program retired large areas not used for other crops. The difference of 0.73 ft in consumptive use for the years 1928 and 1935, years of nearly equal effective heat, is largely due to a reduction of 13,882 acres in irrigated area. It is possible, too, that water-surface evaporation and consumption by native vegetation growing along the Rio Grande was much less in 1935 than in 1928 as the runoff of the river at Leasburg, head of the valley, was only 69% as great as in 1928. The reduction in consumptive use in 1933 as compared with 1931 is explained in part by the fact that 6,870 acres were plowed under in 1933 and no other use made of the land that year. Nonproductive uses of water in these two years were probably about the same.

The low consumptive use in 1937 cannot be explained by any one of the factors mentioned. Unusually favorable circumstances must have prevailed in that year to account for the high crop yields and low consumptive use of water.

The corrected use data in Table 4, No. (9), have been corrected to a full-area basis by reducing the equivalent area by 50% of the area temporarily out of cultivation. This is based on an estimated consumptive use, by the land that is temporarily uncropped, of 50% of the cropped land. In 1927 development was incomplete; in 1933, 6,870 acres of cotton were plowed under but reported in crop; in 1934, 9,220 acres of cotton land were fallow but not reported; and, in other years, the corrected full irrigation development was set at 76,900 acres, which is the mean for the period 1928 to 1934, and the year 1937.

Wagon Wheel Gap Area, Colorado.—The area consists of a 222-acre watershed tributary to Rio Grande at Wagon Wheel Gap, Colo. Elevations range from 9,400 to 11,300 ft. The area is forested with Douglas fir and other growth typical of the central Rocky Mountains, but includes no agricultural land. Its water supply is limited to precipitation, which varies little from the average of 21 in. per yr. Runoff, and hence consumptive use, varies from year to year because of differences in intensity and occurrence of precipitation. There are no known seepage losses from the area. The data in Table 3, No. (10), are for watershed "*A*," used in the experiments of the Forest Service and Weather Bureau described by C. G. Bates and A. J. Henry.[42]

Black River, Wisconsin.—The drainage basin above Neillsville, Wis., comprises 770 sq miles of gently rolling land, 62% of which is cropped, 30% pastured, and the remainder wooded. The average elevation is about 1,200 ft. Temperatures vary from $-40°$ to 101°, although summer temperatures are normally moderate. The growing season is 164 days. Annual precipitation averages only 31 in. but as a large percentage occurs in the growing season, and as winter precipitation is held in snow storage until spring, agriculture is successfully practiced without irrigation. During the period of runoff records,

[42] "Forest and Streamflow Experiment at Wagon Wheel Gap, Colorado," by C. G. Bates and A. J. Henry, *Monthly Weather Review*, Supplement No. 17 (1922) and Supplement No. 30 (1928).

water shortage was severe in only one year. Consumptive use (Table 5, No. (11)) was obtained by deducting runoff from precipitation on the entire watershed.

Mad River, Ohio.—The watershed above Springfield, Ohio, covers an area of 480 sq miles of flat to rolling land with an average elevation of 1,100 ft lying about half way between the Ohio River and Lake Erie. About 33% of the area is cropped, 38% is pastured, and the remainder is woodland.

TABLE 5.—CONSUMPTIVE USE, VALLEYS (11), (12), (13), (14), AND (17); NON-IRRIGABLE WATERSHEDS[a]

Year	(11) Black River			(12) Mad River			(13) Skunk River			(14) Sangamon River			(17) Tallapoosa River		
	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H
1921	....	....	....	....	....	....	2.53	2.53	9,560	2.80	2.80	10,900	....	....	....
1922	....	....	....	....	....	....	2.29	2.29	9,990	1.96	2.06[b]	11,360	....	....	....
1923	....	....	....	....	....	....	2.26	2.26	8,930	2.80	2.80	9,550	....	....	....
1924	1.77	1.77	6,590	1.75	1.75	8,370	2.05	2.05	9,130	2.29	2.29	9,380	2.72	2.72	12,150
1925	2.09	2.09	6,380	2.27	2.27	9,130	2.32	2.32	9,490	2.09	2.09	10,020	2.51	2.51	13,040
1926	1.93	1.93	7,020	2.65	2.65	8,520	2.10	2.10	8,750	2.53	2.53	9,740	3.10	3.10	11,590
1927	1.76	1.76	6,860	1.99	1.99	8,830	1.99	1.99	9,340	3.01	3.01	9,020	2.66	2.66	12,800
1928	1.60	1.60	6,400	1.77	1.77	8,470	2 55	2.55	8,800	2.42	2.42	9,150	3.04	3.04	11,010
1929	1.60	1.60	7,020	2.15	2.15	8,720	1.91	1.91	9,240	2.17	2.17	10,360	2.91	2.91	11,820
1930	1.94	1.94	7,090	1.50	1.79[b]	8,680	1.86	1.86	9,100	1.69	2.19[b]	10,100	2.38	2.38	11,230
1931	2.04	2.04	7,110	2.43	2.43	9,630	2.84	2.84	9,830	2.70	2.70	11,410	2.35	2.35	11,810
1932	1.70	1.70	6,870	2.29	2.29	8,760	2.43	2.43	8,910	2.52	2.52	10,540	3.59	3.59	11,500
1933	1.39	1.80[b]	7,770	1.71	1.83[b]	8,990	1.66	2.04[b]	9,530	2.11	2.11	10,850	1.90	2.25[b]	11,420
1934	2.27	2.27	7,970	1.61	2.28[b]	9,090	2.05	2.05	10,240	2.52	2.52	10,900	2.91	2.91	11,860
1935	1.94	1.94	6,060	2.77	2.77	7,510	2.71	2.71	8,770	2.44	2.44	10,540	2.90	2.90	12,910
1936	1.40	1.68[b]	7,220	1.92	1.92	9,110	1.86	2.12[b]	9,870	2.19	2.19	10,500	2.38	2.38	11,510
Mean	1.80	1.85	6,950	2.06	2.15	8,760	2.21	2.25	9,340	2.39	2.43	10,270	2.72	2.75	11,900

[a] U = Consumptive use, in ft; U_p = computed consumptive use; U_r = corrected consumptive use; and H = effective heat, in degree-days (F). [b] Dry year corrected by a quantity considered adequate to insure normal consumptive use. [c] Non-irrigable watersheds; Nos. (5) and (7) are irrigable.

Temperature extremes vary from −34° to 108°. The growing season is six months. Precipitation averages 38 in. per yr, of which about 21 in. occur in the growing season. This is adequate in all except a few short years for crop production. Consumptive use (Table 5, No. (12)) was obtained by deducting runoff from precipitation on the watershed.

Skunk River, Iowa.—The drainage area above Coppock, Iowa, is about 145 miles long and 20 miles wide and contains 2,890 sq miles of gently sloping prairie land, lying immediately east of Des Moines. The average elevation is about 700 ft. Crop land occupies 68% of the area, pasture 26%, and woodland 6%. Temperatures vary between extremes of −30° and 110°, and relative humidity is high. The growing season is about 205 days. Precipitation averages 32 in. per yr but has dropped as low as 18 in. Nearly two thirds of the annual precipitation falls in the spring and summer months, and the supply is ordinarily enough to mature crops. In a few recent years the supply has been inadequate. Consumptive use (Table 5, No. (13)) was obtained by deducting runoff from the precipitation falling on the entire watershed.

Sangamon River, Illinois.—The area comprises 2,560 sq miles of Sangamon River watershed above Springfield, Ill. It has an average elevation of 700 ft. The topography is gently rolling; 78% of the watershed is cropped, 18% is

pastured, and the remainder is wooded. Temperatures vary from −24° to 107°, and the relative humidity is high. The growing season is about 220 days. Precipitation averages 36 in. per yr and is favorably distributed for crop use. A full water supply is obtained in nearly every year. Consumptive use (Table 5, No. (14)) was obtained by deducting runoff from precipitation over the entire watershed for the period 1921 to 1936.

North Fork of White River, Missouri.—The watershed above Tecumseh, Mo., comprises 1,180 sq miles of rough foothill country, with an average elevation of 1,200 ft, lying along the southern edge of the Ozark Plateau. There is a larger percentage of woodland than in any other area studied except the small Wagon Wheel Gap area. Only about 25% of the area is cropped and 28% pastured. Extremes of temperature vary from −29° to 106°. The growing season is about eight months. Precipitation, which furnishes the sole water supply, averages 42 in., of which 26 in. occurs during the spring and summer. The amount of precipitation fluctuates considerably but is seldom short. Consumptive use (Table 6, No. (15)) was computed for years of recorded runoff by deducting runoff from precipitation on the entire watershed.

TABLE 6.—CONSUMPTIVE USE, VALLEYS (15), (18), (19), AND (20); NON-IRRIGABLE WATERSHEDS[a]

Year	(15) White River			(18) Trinity River			(19) Cypress Creek			(20) San Jacinto River		
	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H	U_p	U_r	H
1924	2.78	2.78	10,630	2.00	2.65[b]	14,330	....	....	....	....	....	....
1925	2.63	2.63	11,180	1.87	2.72[b]	16,070	2.76	3.20[b]	15,470	....	....	....
1926	2.43	2.43	10,880	3.60	3.60	12,480	3.95	3.95	14,950	....	....	....
1927	3.33	3.33	11,520	3.18	3.18	14,640	3.00	3.00	15,650	....	....	....
1928	2.22	2.22	10,440	2.66	2.66	14,120	2.93	2.93	14,270	....	....	....
1929	2.45	2.45	11,150	2.46	2.46	13,990	2.21	2.21	14,170	3.47	3.47	15,920
1930	2.46	2.46	10,950	2.36	2.86[b]	14,890	2.92	2.92	13,360	2.91	2.91	16,340
1931	2.54	2.54	12,020	2.33	2.90[b]	14,800	3.04	3.04	13,430	3.03	3.24[b]	19,480
1932	2.54	2.54	11,490	2.81	2.81	13,420	2.57	2.74[b]	13,280	2.90	2.90	13,630
1933	2.43	2.43	11,700	2.52	2.85[b]	16,450	2.70	2.90[b]	14,980	2.51	3.15[b]	16,840
1934	2.23	2.23	12,350	2.23	2.79[b]	14,870	2.32	2.80[b]	15,290	3.20	3.20	17,880
1935	3.04	3.04	11,210	3.04	3.04	14,240	3.12	3.12	14,110	3.56	3.56	16,110
1936	2.04	2.63[b]	11,160	1.94	2.70[b]	15,240	2.10	2.90[b]	14,700	3.02	3.02	16,410
1937	2.35	2.35	10,970	2.33	2.33	12,880	3.12	3.12	14,850	2.93	3.30[b]	17,820
Mean	2.53	2.58	11,260	2.52	2.82	14,460	2.83	3.02	14,500	3.06	3.19	16,710

[a] U = Consumptive use, in ft; U_p = computed consumptive use; U_r = corrected consumptive use; and H = effective heat, in degree-days (F). [b] Dry year corrected by a quantity considered adequate to insure normal consumptive use.

Green River, Kentucky.—The watershed above Livermore, Ky., has an area of 7,800 sq miles, covering a large part of Kentucky and part of Tennessee, and is the largest area studied. The topography is rolling, with an average elevation of 600 ft. About 40% of the area is cropped, about the same percentage is pastured, and the remainder is wooded. Temperatures vary widely between winter and summer. The growing season is about eight months. Annual precipitation averages about 45 in., of which more than one half occurs in the growing season. This is normally sufficient for full crop requirements, although occasional shortages occur. Consumptive use (Table 4, No. (16))

was obtained by deducting runoff at Livermore from precipitation over the entire watershed in the period of runoff record, 1931 to 1936.

Tallapoosa River, Alabama-Georgia.—The Tallapoosa River above Wadley, Ala., drains an area of 1,660 sq miles ranging from an elevation of 1,200 ft at the southern end of the Blue Ridge Mountains in Georgia to 200 ft on the coastal plain of the Gulf of Mexico. The area is only 42% cropped, with 15% in pasture, the remainder being woodland. Temperatures have an extreme range of −8° to 103° but are normally moderate due to the proximity of the Gulf. The area is occasionally subjected to severe storms. The average annual precipitation is about 52 in., a considerable part of which occurs in winter. Spring and summer rainfall is usually adequate for crop requirements. Consumptive use (Table 5, No. (17)) was obtained by deducting runoff from precipitation on the entire watershed.

East Fork of Trinity River, Texas.—This area of 830 sq miles is upstream from the stream gage near Rockwall in northeastern Texas in the blackland prairie region, one of the richest agricultural sections of the state. The topography is relatively flat, ranging in elevation from 400 to 800 ft. Of the total area, 75% is in crops, 16% is in pasture, and 9% is woodland. The distance from the Gulf of Mexico is sufficient so that temperature variations are wide. The growing season is from April to October. Precipitation furnishes the entire water supply but in many years is inadequate. It averages 21.6 in. in the spring and summer months. Runoff is flashy and dependent on storm rainfall. Consumptive use (Table 6, No. (18)) was determined by deducting runoff from precipitation on the entire watershed.

Cypress Creek, Texas.—The drainage area, containing 848 sq miles upstream from the gaging station at Jefferson, is in eastern Texas. The elevation ranges from 200 to 400 ft and the topography is gently rolling. The area was once heavily timbered but is now (1941) about 50% cropped and 20% pasture. Soils vary from sandy to tight clay. A wide variety of crops, including much truck and fruit, is grown. Temperatures vary widely from below zero to 110°. The growing season is about 280 days. Precipitation, the sole source of water supply, averages 43.9 in. per yr, of which about 70% occurs in the growing season and, except in dry years, is adequate for crop production. Relative humidity is high. Consumptive use (Table 6, No. (19)) was determined by deducting runoff from precipitation on the entire watershed.

San Jacinto River, Texas.—The San Jacinto River Valley is a flat sandy area of 1,811 sq miles, varying from 50 to 400 ft in elevation, on the coastal plain of the Gulf of Mexico a few miles north of Houston, Tex. Of the total area, 29% is pine forest, 38% prairie, and 33% cultivated. The climate is mild and humid. The precipitation is adequate for crop requirements in most years, but is frequently so intense that high runoff and shortages of moisture occur. Consumptive use (Table 6, No. (20)) was estimated as the difference between precipitation and measured outflow. Ground-water movement out of the basin was considered negligible.

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PAPERS

TUNNEL CONSTRUCTION, SIXTH AVENUE SUBWAY, NEW YORK, N. Y.

By Jacob Feld,[1] M. Am. Soc. C. E.

Synopsis

As part of the contract to construct Section 10 of the Sixth Avenue Subway, between 40th and 47th streets, in New York City, one double-track, and two single-track, tunnels were driven for three blocks under a busy city street, under the foundations of an operating elevated railroad, very close to buildings varying from two to sixteen stories in height, through a variety of rock formations, and with scant cover. This paper describes how the work was planned and organized to provide efficient and continuous progress, how it was executed, and at what cost; and it describes the type and adaptability of the equipment used. The major factor in the execution of this work was the necessity for safety.

General Description of the Contract

The contract included the construction of a four-track subway for a distance of 1,800 ft in Sixth Avenue, which is 100 ft wide, and crossed seven intersecting streets. The southerly part of the work, including a station, was built in open cut. The northerly end of the contract, about 260 ft long, was also built in open cut. Between these two areas the four tracks were built in tunnel without disturbing the street surface. The work was done under a unit price contract, the tunnel work being included in the total contract items. As far as plant and equipment were concerned, the construction of the tunnels was merely part of the total contract, except as certain special equipment was found necessary for the tunnel operation (see Fig. 1). The contract plans and specifications were prepared by the Board of Transportation of the City of New York. The contract was awarded on February 6, 1936, and the tunnel work described herein was executed from June, 1937, to April, 1938.

General Plan and Specifications

The plan of the east half consisted of a double-track tunnel running in a straight line under the easterly half of Sixth Avenue, from a point 50 ft south

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by August 15, 1941.

[1] Cons. Engr., New York, N. Y.

of 46th Street for a distance of 720 ft. At approximately the middle of this length there is a ventilating and emergency exit shaft serving both of these tracks and running to the street surface. At two other points, in 44th Street and in 45th Street, the easterly tunnel is widened out to provide space for a

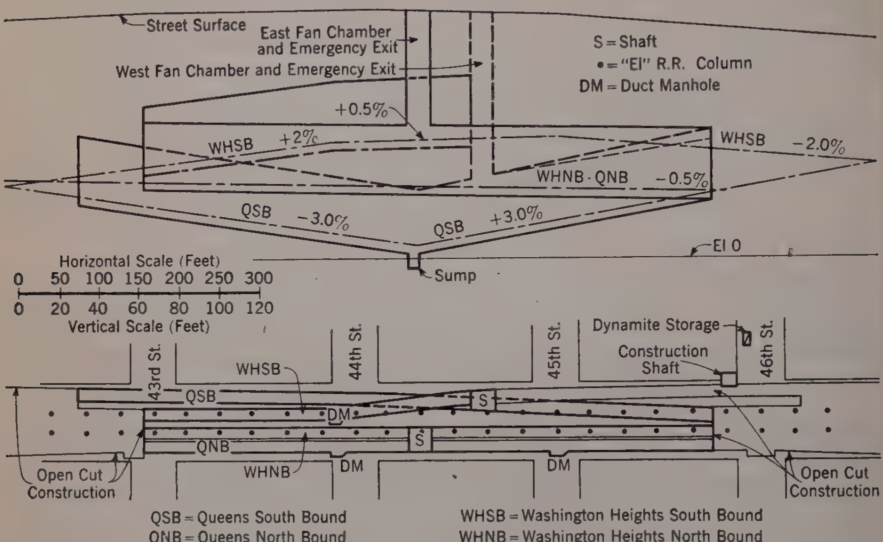


Fig. 1.—Plan and Profile of Tunnels

cable-duct manhole connected to the street surface with a cable-pull pipe 10 in. in diameter. Starting at the 46th Street end, the third track from the east is in a single-track tunnel that dips sharply downward to the west emergency exit shaft (see Fig. 1) and a sump to which the drainage of all the tunnels is connected, and then is carried upward at the same grade, making a total distance of 814 ft, emerging into an open-cut area south of 43d Street at the west side of the street. This tunnel is built on a curve to change the track position from the third to the fourth lane. South of 45th Street, at about its lowest point, this tunnel connects to a ventilation and emergency exit shaft carried to the street surface. From the southerly face of this shaft the fourth tunnel starts near the west building line of Sixth Avenue directly over the third tunnel and crosses from the fourth to the third lane, entering the open-cut area at 43d Street. The double-track tunnel required an excavation approximately 18 ft high and 32 ft wide. Each of the single-track tunnels required an excavation approximately 18 ft high and 18 ft wide.

Specifications provided that the methods used for tunneling should be suitable to the local conditions, and must not cause any injury to the foundations, walls, or other parts of adjacent buildings, structures, or surfaces, such methods to be subject to the approval of the engineers before work was begun. Such methods were to be changed from time to time as local conditions required, but at no time was the approval of the engineers to relieve the contractor of

his responsibility for the proper execution of the work. The contractor had the option to design and submit for approval any method of temporary timbering for the support of the rock before the concrete lining was placed. As local rock conditions made it advisable, the engineer had the right to order the tunnel lining placed before the excavation was completed.

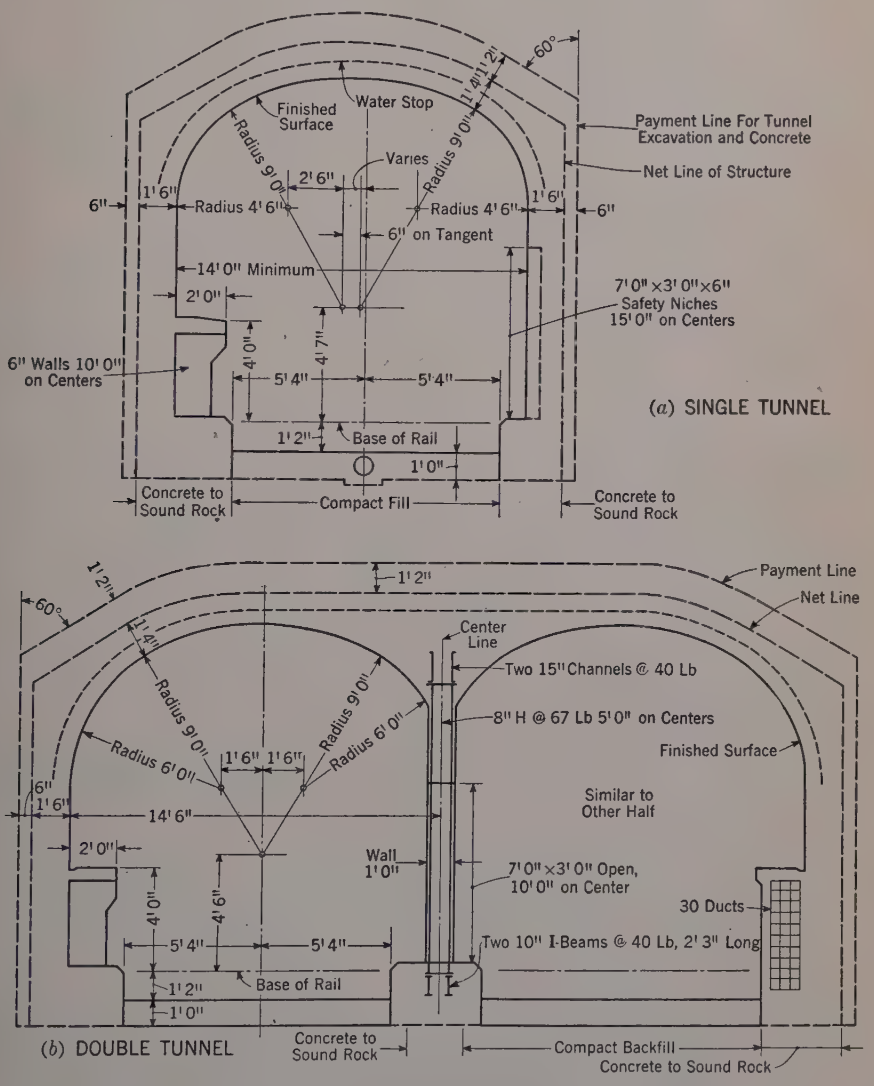


Fig. 2.—Typical Tunnel Sections

The single-track tunnels were to be lined with concrete as shown in Fig. 2, such lining to include no reinforcing or structural steel. The double-track tunnel was also lined. The two arches met in an umbrella section, in which a

series of bents was concreted, consisting of double channels supported on two columns. The center wall between the two tunnels in which these bents are concreted has a series of cross-connecting openings spaced 15 ft apart. In each of the single-track tunnels there was a series of man safety niches in one side, spaced 15 ft apart. On one wall of the double-track tunnel, and on one wall of one single-track tunnel, a line of clay cable ducts was to be built, consisting of thirty individual 3.5-in. ducts connecting the manholes. On the other wall of the double tunnel, and in the other single tunnel, a raised walkway was to be built at the level of the subway car platforms for emergency and patrol use. The invert of all tunnels is 12 in., with cast-iron drain pipes laid in certain locations, cross-connected between the two tunnels near the north end, all draining to the sump.

The contract provided that the surface of the concrete lining was to be smooth and of a type known as "finished surface." The contractor had the option to use steel forms so as to eliminate the need for rubbing the concrete. Grout holes were left in the concrete lining for filling any spaces left between the rock and the concrete. The installation of track, cables, and other operating equipment was not part of this contract.

Borings and Geology of Subsurface

Prior to beginning the work, the city had several sets of borings made in the area under which the tunnel was to go, and reports of these were furnished to the contractors. Four of the borings were driven almost 200 ft below the street surface, chiefly to check what leakage could be expected from the Catskill Aqueduct approximately that distance below Sixth Avenue. These borings were one in each block (260 ft apart) and showed a great variety of rock recoveries, the samples removed ranging from 1% to a maximum of 93% of the boring depth. In addition, there were seven other borings taken to rock surface with a 10-ft core into the rock, and the recoveries varied from 1.4 to 7.7 ft of that distance.

In addition to the borings several maps were also available showing the accumulation of previous information, none of which, however, indicated the complexity of rock formations actually encountered. For instance, a field report of June, 1937, described

> "* * * rock in both tunnels of great variety, from soft, disintegrated schist to an extremely hard igneous syenite with seams of feldspar, which in some cases is so disintegrated that it can be taken out by hand. Seams are generally almost vertical, slight tip to the West and running almost to South slightly to East."

In general, the basic rock was schist, but in the volume occupied by the tunnels there were a great number of intrusions in the form of bands running southward about 20° to the east and almost vertical, sometimes tipped 5° to 20° to the west. At the north end of the tunnels a band of very hard syenite with quartzite intrusions was found crossing the double tunnel. In the vicinity of 45th Street a very hard and fine-grained syenite crossed all four tunnels. To the south of this band was a layer of disintegrated syenite which was so

soft that it could be broken easily by hand, and, in the terms of the blasters, was "dead." South of this point there was more mica schist, which again changed to disintegrated syenite a few feet in thickness, in contact with several layers of chlorite and serpentine, interspaced with disintegrated syenite. These latter layers were encountered at the extreme south end of the double tunnel, affecting the westerly part only, and crossed the south end of the Washington Heights southbound single tunnel near its southerly end, and also crossed the Queens southbound tunnel about 300 ft from its southerly end. Keeping in mind that these layers were encountered practically on edge and running diagonally across the face of the headings, the complexity of the drilling and blasting operations can be pictured easily. Practically no water was encountered at any point.

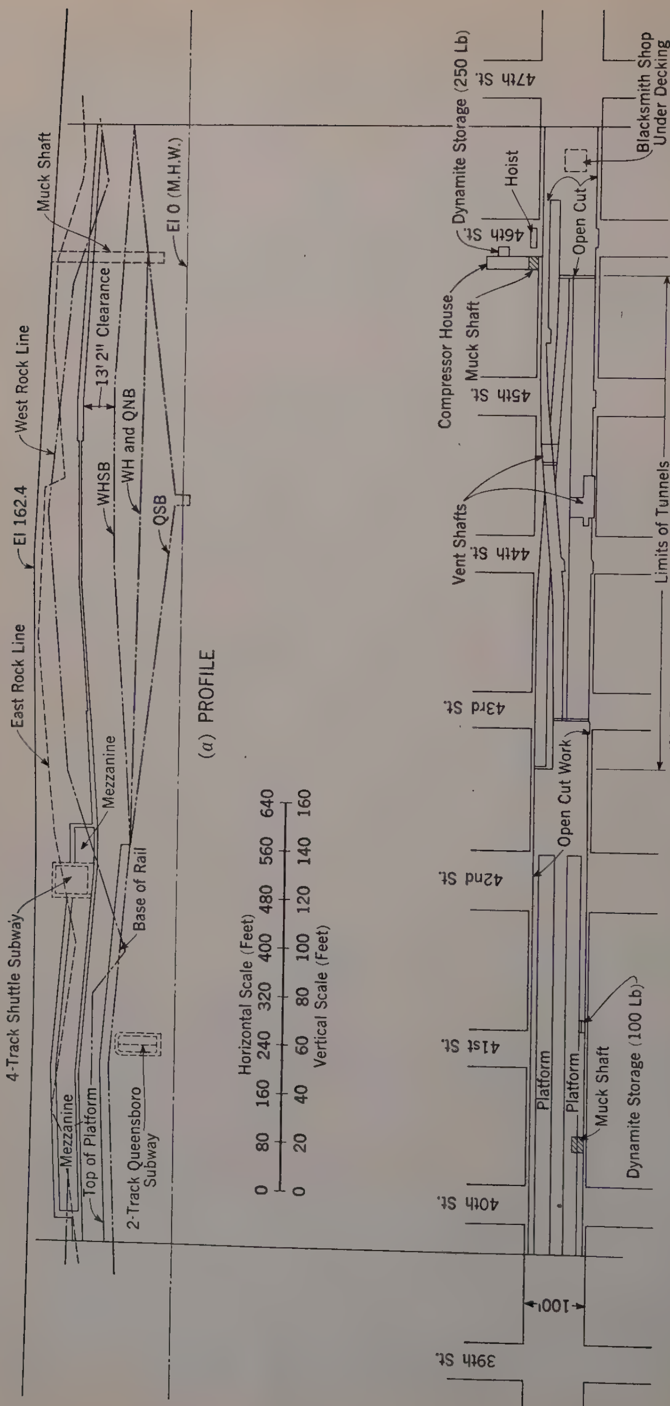
Surface and Subsurface Conditions

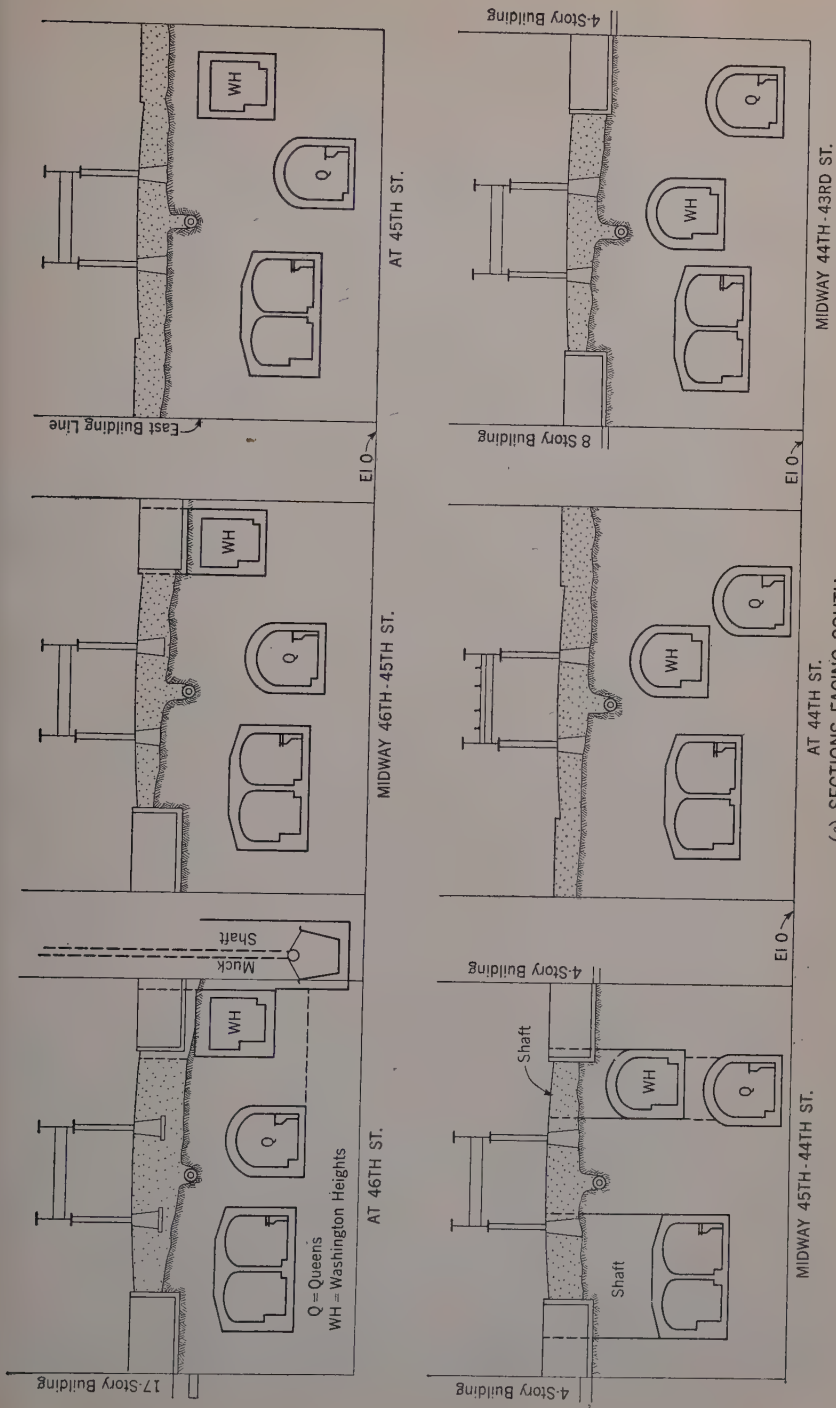
The street width is 100 ft, and buildings occupy every available piece of ground the full length of the tunnels. These buildings vary from altered four-storied brownstone residences with stores on the ground floor, built about 1880, to modern, fireproof, seventeen-story office buildings. In the middle of the street there was a two-track elevated railway, supported on wrought-iron columns spaced on 43-ft centers longitudinally, with two columns 23 ft apart in each bent. Train operation of this railway was continuous, with the average headway about 5 min each way. The columns rested on brick piers, some of which were carried to rock, and others were in the nature of spread footings several feet above rock elevation.

Prior to the beginning of any construction, a careful examination was made of each building on Sixth Avenue, and also the buildings within a distance of 100 ft east and west of Sixth Avenue. These examinations were made by representatives of the Board of Transportation and the contractor, and usually the owner of each building. Stenographic notes were taken during the examination, and the report was signed by all those present. Where unusual conditions were found, the contractor also took photographs for his records. At the completion of the work these buildings were again examined to determine whether any changes had occurred.

To avert breakage of windows as far as possible, a wire cable and timber trussing were placed over each window with either dimension greater than 6 ft. This included all of the show windows. These "spiders" were kept in repair during the entire operation, and not one of the show windows along the line of the tunnel was broken.

In the avenue itself, below the surface, there was an old brick sewer in the center about 14 ft down, built in 1890, or before, in a rock trench. At higher levels there were two cast-iron water mains, one 20 in. and one 12 in., with 12-in. lines crossing the avenue at each street. Two steel gas mains were constructed in the avenue under the subway contract, prior to any tunnel work, to replace a great variety of old cast-iron gas mains that were found. In some points as many as twelve old gas mains were thus replaced. In the easterly part of the roadway there was a large bank of telephone cables laid in clay ducts, containing approximately 25,000 telephone circuits. There were


4-Track Shuttle Subway
East Rock Line
El 162.4
West Rock Line
Muck Shaft
13' 2" Clearance
WHSB
WH and QNB
QSB
El 0 (M.H.W.)
Mezzanine
Base of Rail
Mezzanine
Top of Platform
2-Track Queensboro Subway
(a) PROFILE
Horizontal Scale (Feet)
0 80 160 240 320 400 480 560 640
0 20 40 60 80 100 120 140 160
Vertical Scale (Feet)
47th St.
46th St.
45th St.
44th St.
43rd St.
42nd St.
41st St.
40th St.
39th St.
Dynamite Storage (250 Lb)
Hoist
Compressor House
Muck Shaft
Vent Shafts
Open Cut
Blacksmith Shop Under Decking
Limits of Tunnels
Open Cut Work
Platform
Platform
Muck Shaft
Dynamite Storage (100 Lb)
100'
(b) PLAN



(c) SECTIONS FACING SOUTH

FIG. 3.—CONTRACT PLAN AND STATIONS

also two banks of clay ducts containing low-tension and high-tension power cables, one at each curb, with sub-branches and service lines. Similar subsurface equipment also existed in each of the cross streets.

Approximately 200 ft below the street surface there is a concrete-lined aqueduct tunnel of 15 ft inside diameter, bringing water under pressure into the city. The fear that subway construction would affect this tunnel delayed the construction of the Sixth Avenue Subway link until such time as the second aqueduct tunnel was completed in 1934. Such fear was unfounded, as proved by the record that no serious leakage was encountered at any point along the Sixth Avenue Subway construction.

Relation of Tunnels to Remainder of Contract

The open-cut work was the most important part of the contract, and the location of shafts for tunnel work was controlled by points fixed in the contract. Therefore, the plant layout in the tunnels, with the exception of special equipment used in the tunnel work, was included in the layout for the entire contract (see Fig. 3). As far as the tunnel excavation was concerned, all material was removed in cars on industrial track to a shaft at the southwest corner of 46th Street, which was located at the west building line about 60 ft north of the portals of three of the tunnels and about 200 ft north of the fourth portal. The subgrade elevation of the track adjacent to the shaft was 10 ft higher than the elevation of the subgrade of the tunnel portals. The disposal shaft was carried approximately 20 ft below the subgrade of the adjacent track, and a crosscut at the elevation of the lower tunnels was excavated to permit bringing cars to the shaft at tunnel grade. In the shaft a steel tower was constructed to accommodate a 6-cu-yd steel skip. By bridging the crosscut, cars could be dumped into the skip at two different levels 10 ft apart, the lower level taking care of the three tunnels, and the upper level taking care of most of the open-cut and the fourth tunnel. The skip was operated by a hoist located above 46th Street on a steel and timber framework, the drum thereby being spaced 50 ft from the guide sheave so as to permit a reasonable fleet angle of the hoist cable as it ran on the drum. The guide sheave was pinned for rotation about the vertical axis.

Adjacent to the shaft, there was a compressor house and transformer vault, from which compressed air and electricity were supplied for the entire contract. Equipment for the compressor house consisted of three, two-stage electrically driven air compressors, each with a rated capacity of 1,320 cu ft at 120-lb pressure. In the transformer room there were three sets of 100-kva units, providing current at 208 volts. The air receivers and coolers were directly below the compressor room, which was at truck elevation above street surface.

All water for cooling was taken from city mains and was metered. During tunnel operation, and also including supply for the open-cut work, power consumption per month ran from 130,000 to 190,000 kw-hr. Water consumption for compressors, for wet drilling, and for all other uses, averaged about 400,000 cu ft per month.

The cost of air production at the compressor plant, as determined during the last six months of 1937 (during which time the plant was operated on

three shifts with an average production of 1,800 cu ft per min), was $0.0865 per 1,000 cu ft. This cost includes the following, per shift: labor, $13.30; insurance, $2.66; power, $52.00; water, $8.50; and miscellaneous, $1.42—a total (without plant construction or equipment costs) of $77.88.

Dynamite for the entire contract was stored in two separate magazines, one at 41st Street and the other near the compressor plant. The 41st Street magazine had a maximum capacity of 100 lb and the 46th Street magazine of 250 lb. Practically all of the dynamite for the tunnels came from the 46th Street magazine, with the exception that some of the night shifts would draw on the other magazine for completing a round. Delivery of dynamite could be made only during daylight hours, and to some extent the blasting was controlled by the quantity that could be stored at the end of each day (approximately 4:30 p.m. during the winter months) for use until 8:00 a.m. the next day. At each magazine, a dynamite watchman was assigned on each shift, and his duties were to mark each stick of dynamite with the license number of the magazine, keep a record of the withdrawals requested by each blaster, and prepare the information required by the Fire Department of the City of New York for the daily check on the use and purchase of dynamite. In the tunnel work, all dynamite was 40% gelatin. Most of the blasting in the tunnels was by delayed-action blasting caps.

The blacksmith shop for the tunnel operation was in the open-cut already excavated to subgrade beneath the street decking, a short distance north of the tunnel portals. The blacksmith shop was connected to all of the tunnels by industrial track.

All drill steel for the tunnels was $1\frac{1}{4}$-in. hollow steel, purchased directly from the mill in full random lengths. Typical gage diameters for the bits (standard four edge) were $1\frac{7}{8}$ in. for drill steel 12 ft long, increasing $\frac{1}{8}$ in. for each 2-ft reduction in steel length. All steel was sharpened, shaped, and shanked at the blacksmith shop, using two complete sharpening units with oil-fired furnaces. Two of these units were operated on one shift, with only one unit on each of the other two shifts. This continuous use required relining of the furnaces twice during the entire contract. This excellent service was due mostly to the use of a special firebrick and latite mortar. During the last six months of 1937 the drill-sharpening cost per cubic yard of rock was as follows: labor, $0.313; insurance, $0.04; fuel, etc., $0.038; and water, $0.006—a total cost per cubic yard of $0.40 (plus equipment costs). The average steel required from the blacksmith was 3.64 pieces per cubic yard of rock, making an average total cost of 11 cents per drill sharpened. A blacksmith unit, consisting of three men, produced 350 pieces of drill steel, sharpened, per shift. The extra gang of men on one shift took care of preparing new steel and reshanking old steel as required.

Labor Requirements, Rates, and Organization in the Field

Under the terms of the contract, labor requirements, work assignment, and the minimum rates were fixed by the State Labor Law. Under such rules, with the exception of the foremen, no man could work more than five days in one week nor more than eight hours in one day, except in cases of emergency.

In addition, except in cases of emergency, no Sunday work was permitted by city ordinances.

The job was organized on a three-shift basis, with eight hours in each shift, including one-half hour for lunch, payment being made on a straight basis for eight hours each shift. The foremen, including the blasters, worked six shifts a week, and the other men were rotated so that no man worked more than five days in one week. The exception to this rule was the "graveyard" shift, which omitted the shift starting midnight Sunday and only worked five days a week. Tunneling operation was continuous, therefore, from 8:00 a.m. Monday morning until midnight Saturday. However, concrete was poured only during the day shift, except that a part of the day shift was called at 6:00 a.m. to prepare the forms, and in cases of necessity the 4:00-o'clock shift would complete the concreting at the expense of their regular work.

The tunnel operation was considered a part of the entire contract, and under the supervision of the superintendent and master mechanic. However, three junior civil engineers were assigned to tunnel work only, one for each shift, in addition to the office work and planning, which were done by the engineering staff for the entire contract.

Labor rates were fixed in the contract, and they held for the duration of the work under an agreement with the union organizations. Practically no labor difficulty was encountered during the entire tunnel operation. The labor rates were as follows:

Item No.	Classification	Rate	Item No.	Classification	Rate
1	Miners, drillers, and underpinners	$1.25	8	Operating enginemen	$1.65
2	Helpers for item 1	1.00	9	Conway operators (by agreement)	1.25
3	Laborers	0.80	10	Dinkey runners and maintenance enginemen	1.125
4	Blacksmiths	1.40	11	Blasters	1.65
5	Blacksmith's helpers	1.00			
6	Carpenters	1.40			
7	Electricians (raised to $1.70 by agreement)	1.40			

Actually, distinct, separate trades were used as drillers and drill helpers, timbermen who did the underpinning and shoring, and timbermen helpers. By special arrangement, mucking operators were paid $1.25, the operating engineman classification only applying to men on the hoists and on a large concrete pump. The operator of the small concrete pump was an apprentice engineman, as were also burners, welders, pipemen, and other mechanics, all at the rate of $1.125.

Each group of drillers and helpers was under the direct supervision of the blaster; the timbermen and helpers were under their own foremen. The foremen were responsible to two shift bosses, each covering twelve hours, who in turn were directly under the superintendent. All the mechanical men were under the master mechanic, who assigned the work and was responsible for the performance of the equipment as well as the operators.

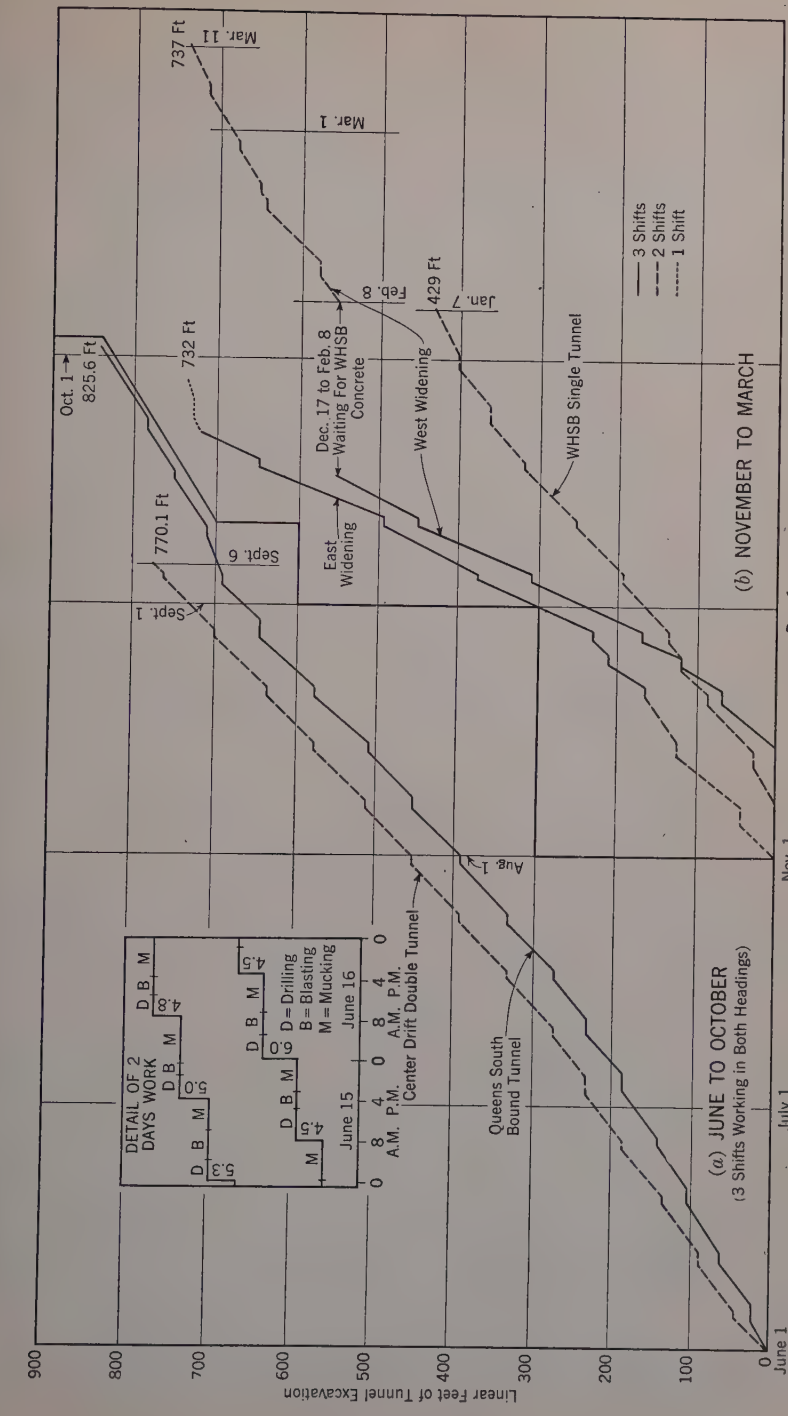


FIG. 4.—EXCAVATION PROGRESS

Sequence of Operations

The general sequence of operations, as planned and as actually performed, was to start excavation of the Queens southbound single tunnel and the center cut of the double tunnel at the same time at the north end, and proceed south. The rock was shored simultaneously with the excavation during the drilling periods, except that, where bad rock was encountered, drilling was postponed

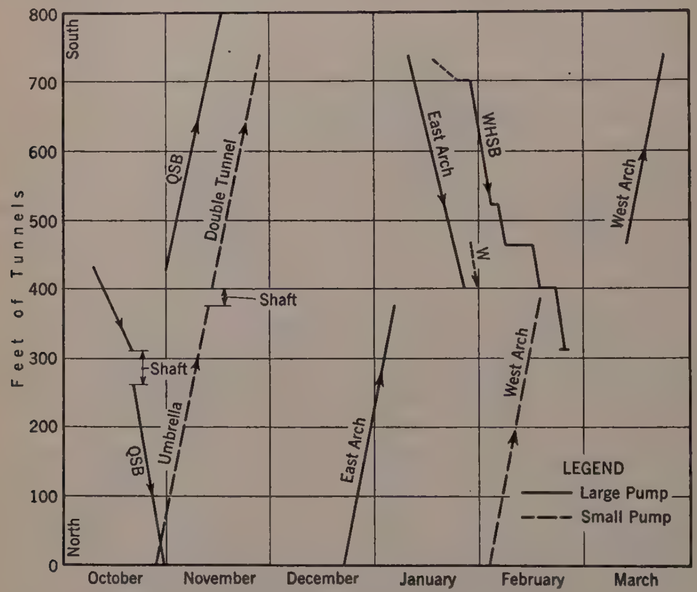


Fig. 5.—Progress, Pouring of Arch Concrete

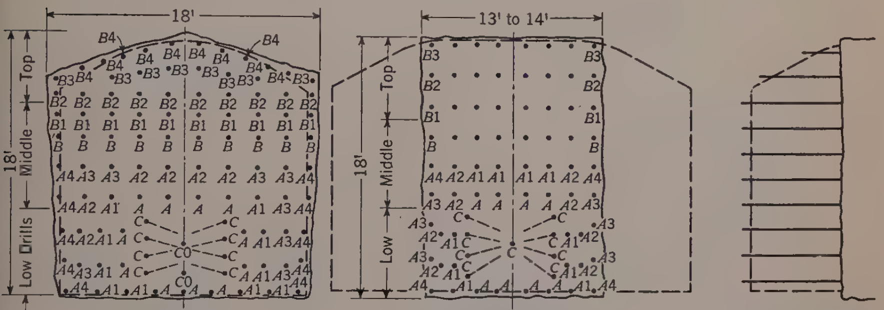
until the rock was shored to the face. For the single tunnel, the concrete operation was first to pour the two benches and follow with the complete arch, using the benches to support the form. Concreting the invert was postponed until the remainder of the tunnel was completed.

In the center cut of the double tunnel after completion of excavation, the next step was ring drilling for the widening on each side of the center cut; then followed the concreting of the center bench of the "umbrella," erecting the structural steel columns, and concreting the umbrella section to the roof so that the side supports of the roof bracing could be eliminated. The posts of the roof bracing were then used to line the two faces of the umbrella concrete while the ring-drilled rock on each side was shot out and removed. This operation, in turn, was followed by concreting each of the benches, completing the arch, grouting, and concreting the invert. The excavation progress and timing are illustrated in Fig. 4. The corresponding concrete progress is shown in Fig. 5.

Although the original plan was to place the cut shot at the bottom of each heading, it was found too difficult in the field to drill diagonal holes nearer

subgrade than about 15 in. The cut holes were raised about 18 in. above subgrade (see rounds C, Fig. 6). The typical hole locations were varied daily to suit rock and seam conditions in the heading, the normal loading schedule (Fig. 6) being as follows:

Descriptive	Round *C*	Round *A*	Round *B*
Cut shot	*C*	....	....
Blank	....	*A*	*B*
First delay	....	*A*1	*B*1
Second delay	....	*A*2	*B*2
Third delay	....	*A*3	*B*3
Fourth delay	....	*A*4	*B*4



SINGLE TUNNEL
90 TO 103 HOLES

CENTER DRIFT OF
DOUBLE TUNNEL
77 HOLES

SIDE ENLARGEMENT
10 HOLES, 2 FT APART

FIG. 6.—DRILLING DIAGRAMS

EQUIPMENT

To a large extent equipment purchased for the entire contract was used in the tunnels, except that the muckers were purchased chiefly for tunnel operation but were used partly in open-cut work. The drill jumbo and steel-erection jumbo were made especially for the tunnels; the concrete pumps were used chiefly in the tunnels; and the smaller unit was used in some open-cut work. The cars, track, dinkeys, pumps, and other small equipment were adapted for use both in the tunnels and in the open cut.

The drilling jumbo was fabricated at the site from drawings made to fit conditions of the job. The first design called for mounting six drills, and this was later changed for eight drills, seven of which were used at all times and one for a standby. In general, the idea was to have drills at three levels, dividing the 18 ft of total height into approximately three equal sections. Hinged side platforms at the intermediate and top levels permitted the use of the jumbo in the 18-ft single-track tunnel as well as in the 14-ft center cut of the double tunnels. To compensate for the lack of dead weight, the structure being made as light as possible for easy shifting, screw jacks were placed at the top and bottom of the vertical columns for wedging against the roof and the floor of the excavated tunnel. The mounting was on trucks from small muck cars used on a previous contract.

Tunnel drilling operation was started with drills with 3-in. pistons. As soon as it was found that drilling time was slowing up the expected schedule of a round per shift, these drills (which had been used on a previous contract) were replaced by drills with 3½-in. pistons. With these larger drills a round was drilled out in three hours. When the drill jumbo was changed from six to eight drills, the two additional drills installed were equipped with 4-in. pistons.

During the tunnel construction it was found necessary, due to changes in the New York City Industrial Code, to change all the drills for better dust and fogging control. This was done by installing round pistons to eliminate the air leakage into the water tube, to change the location of the exhaust port, and to increase the size of the water tube. With these changes, fogging of the smaller drills was entirely eliminated and greatly reduced in the larger drills.

All the mucking in the tunnels was with electrically driven machines. These machines were equipped with 50-hp, 208-volt motors; they were designed for an 18-ft cleanup width, and were equipped with a hinged boom and a chain-operated bucket at the end of the boom, with hard steel teeth and a rock-breaker knob. Four muckers of this type were purchased and used to some extent in the open-cut excavation before tunnel work was started. They were equipped with a 20-in. belt running in a 24-in. beam on the flat. In the first days of operation it was found that the flanges of the beam prevented proper contact between the mucked material and the belts. The upper flanges of the beam were cut away and rewelded to the web of the beam to form a sloping trough. This correction was made on the first two machines in the field and on the other two, which had not yet been shipped, in the shop. It was also noticed that considerable of the slip could be eliminated by a small stream of water running continuously down the sloping belt. Such slip was caused chiefly by the large quantity of mica in the mucked rock.

At no time were four of the mucker units actually in the tunnel, the fourth unit being kept as a reservoir for spare parts. In addition, quite a large stock of parts was kept in the storeroom. The use of the rock-breaker knob on the under side of the bucket was discontinued after a short trial because it was found much easier and more economical to drill and shoot a large boulder between mucking periods than to spend the time required in the mucking cycle to break up the few large pieces of rock that were too large for the mucker to handle through its throat.

Considerable study was given to the belts used in the mucking machines, not only because of the high cost of such belts, but also because of the time wasted when a belt broke and required replacement. The belts originally supplied by the manufacturer had been developed as the result of considerable research in the California tunnels. However, the rock conditions encountered on this contract were much more severe, especially because of the sharp edges and large quantity of mica. The average life of a belt was about 2,500 cu yd of rock removed.

All track was of standard 36-in. gage, composed of 40-lb industrial rail, with standard switch points and frogs. Dump cars were 5-cu-yd, water-level, steel body, side dump, with side top hinged. They were dumped into the skip

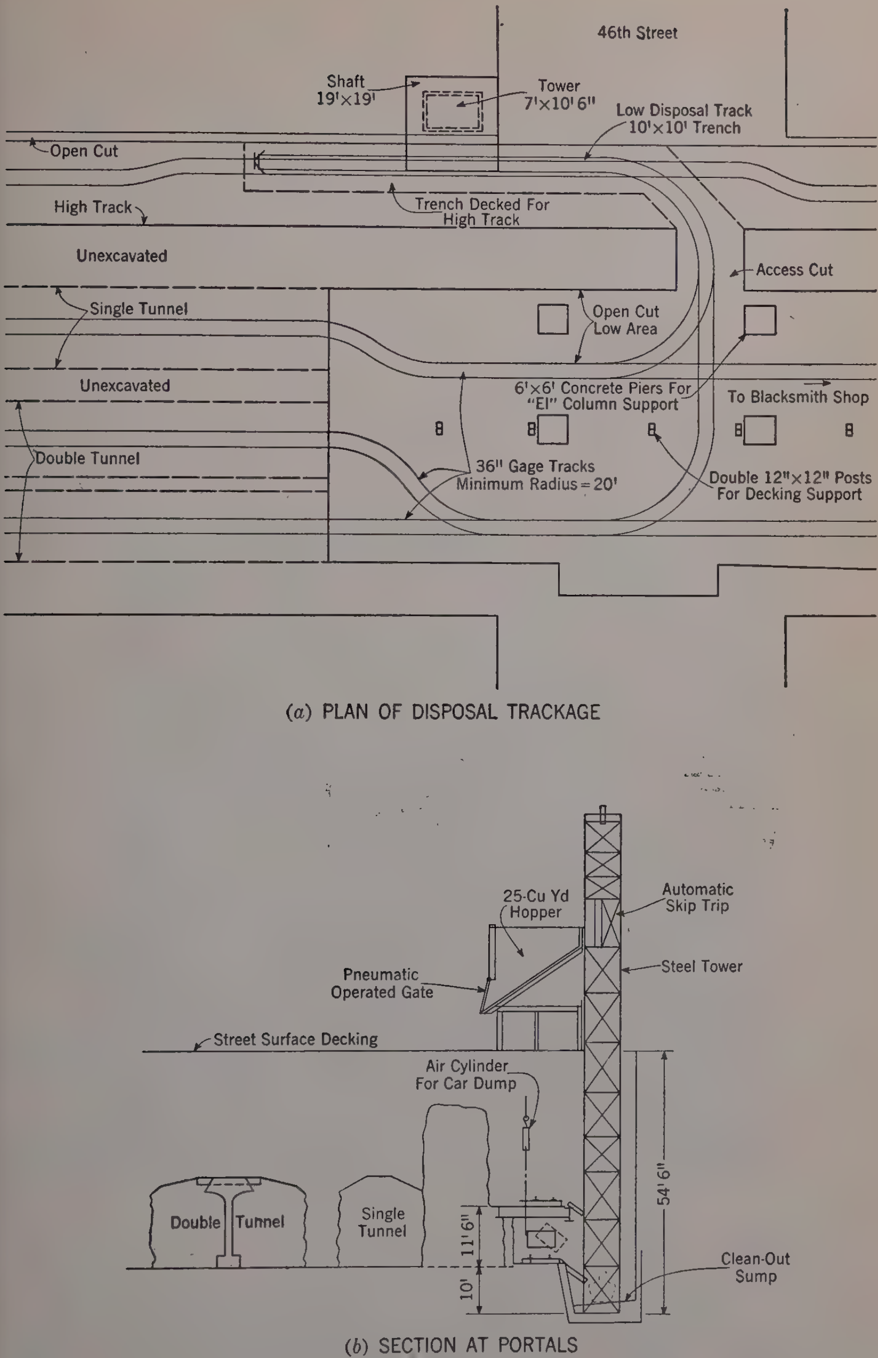


FIG. 7.—DISPOSAL TRACKAGE

at the 46th Street shaft by the use of a pneumatic cylinder 12 in. in diameter with sufficient stroke to unload the car surely by gravity. Two dump levels were provided directly over each other, and by operating the dumping with only one cylinder, to which the car at either level could be attached, the danger of some one attempting to dump two cars at one time was eliminated (see Fig. 7).

The dinkeys for pulling the cars, the muckers being self-propelling, were electric-battery locomotives of 6-ton rated capacity, equipped with 42-plate batteries of the high cap type. Each dinkey had two sets of batteries, and four dinkeys in all were available for the tunnel work. The batteries were charged with a 30-kw, 108-volt, 1,750-rpm, 278-ampere generator direct-connected to a 50-hp, three-phase, 60-cycle, 220-volt motor. The batteries were charged at the upper level beneath the decking near the 46th Street shaft. By means of an overhead trolley hoist and beam track, batteries were raised from the dinkeys and placed into charging position quite efficiently.

Due especially to the long wheelbase of the muckers, it was found necessary to use a minimum radius of 40 ft on the track and to maintain the gage of the track with standard railroad track-gage rods made over to the 36-in. gage. Wood ties were used, spaced about 30 in. on centers; usually 6-in. by 12-in. timbers were available from decking lumber, or 6-in. by 8-in. timbers were purchased specially.

Practically all the concrete was placed by two concrete pumps. A small, gasoline-driven unit, with a 6-in. pipe, and with a conical hopper and integral agitator, was used in pouring benches. This smaller pump was also used on the umbrella and a part of the arch of the double tunnel, when the larger pump was being used elsewhere. The larger pump was the same make as the smaller, with a 7-in. pipe. It was propelled by a gasoline motor but of a model antedating the time when integral agitators were installed. The smaller pump gave an actual production of from 20 to 25 cu yd per hr, compared with 30 to 35 cu yd per hr for the larger pump.

Some of the smaller items in the tunnel, such as catwalk and duct-protection incasement, were poured manually from a special car with a high box body and side chute. However, about 98% of the concrete volumes, including the inverts, were placed with the pumps.

Miscellaneous equipment also used included two blowers, each of 2,000-cu-ft-per-min capacity, interconnected with each other and with the ventilator pipe in each tunnel, so that a maximum capacity of 4,000 cu ft for either blowing or sucking could be provided in either tunnel. The vent pipe was composed of 8-in. thin-shell steel pipe with dresser couplings. Two short lengths of flexible tubing were provided, but after a trial it was found just as easy to remove one or two sections of the steel pipe from the heading as to use the flexible pipe.

For trimming the roof, a stoper drill, with automatic feed, for 1-in. hexagonal shankless steel, was used. All other drill steel was $1\frac{1}{4}$-in. round.

The drill-sharpening plant (which was situated just north of the tunnels beneath the decking and connected by rail with all the tracks in the tunnels) consisted of two complete units for making up and sharpening drilling steel,

as well as a small blacksmith shop. Each drill-sharpening unit consisted of a drill sharpener, with clamping dies for both $1\frac{1}{4}$-in. round steel and 1-in. hexagonal steel. The equipment for each set comprised: A collaring device for the hexagonal steel and lug shanking device for the round steel, gaging dies, fuller dies, dollies for sizes from $1\frac{5}{8}$ in. to 3 in. inclusive, a shank and bit punch, an oil forge, and fuel-oil tank. No trouble was encountered with the operation of the blacksmith and drill-sharpening plants beneath the decking; the natural draft as a result of the difference in temperature between the street and the confined volume beneath the decking were found sufficient to take care of ventilation.

Excavation of Single-Track Tunnels

The nature of the rock exposed in the open-cut work made it necessary to begin tunnel excavation 10 ft north of the theoretical tunnel portal and only after a concrete beam protection had been placed above the tunnel portal (see Fig. 8). The purpose of the beam was to retain any loosened rock and thereby prevent damage not only to the street surface but to the subsurface structures,

Fig. 8.—Entrance at North Portal of Queens Southbound Tunnel

as well as the foundations beneath the elevated columns a short distance to the south of the tunnel portal. In the preliminary excavation, it was found that, because of the seamy nature of the rock, carrying the drilling holes 6 in. outside of the net lines on all sides, including the bottom, was necessary in

Fig. 9.—Effect of Wild Cut Shot on Bracing

order to reduce the amount of trimming. The cut shot normally did not follow the cut holes but broke away along the nearest seam (see Fig. 9).

The general plan of operation was to drill one round sufficiently to give an advance of 6 ft and then blast and muck all in one shift. The typical cycle of operations in both the single and double tunnels is shown in Fig. 4. It must be kept in mind that the blasting operation was governed by contract requirements that not more than 100 lb of dynamite could be used in any one round fired. As a result, it was necessary to load and shoot three times in each cycle of operations. The normal procedure was to shoot the cut by itself, using one delay in some instances; the second firing took care of the relievers and some of the trimmed holes with five delays; and the final round cleaned the entire section, using from three to five delays (see Fig. 6).

The Queens southbound single tunnel was the first of the single tunnels excavated. The record for August, 1937, the first full month of tunnel operation, shows 34 rounds averaging 7.62 ft. It was found that, with seven drills operating and with a cross section giving 12 cu yd per ft of advance, an 8-ft advance completely filled the entire shift of seven and one-half working hours. If a greater advance was drilled and shot, the entire cycle of operation was interfered with. On August 4 and August 26 seams of rock were encountered in this tunnel that were so badly shattered as to require steel bracing directly to the face before drilling could continue. This was necessary as a safety precaution not only for the men in the tunnel but to avoid failure of street surface and elevated columns above the tunnel.

During the entire month of September, 1937, this tunnel was in soft and seamy rock, the seams containing a slippery chlorite. Even with steel frames placed 3 ft on center, the pressure of the rock deformed the steel bracing.

In a number of areas the heading was progressed only after steel outriggers had been extended to the face at the roof and full timber lining was placed on both the roof and the sides. During the month the total progress was 170 ft, with the rounds averaging from 4 to 8 ft, some of them being greater than the actual depth of holes drilled. This tunnel was holed through on October 1, 5 ft south of the theoretical portal and in rock so soft that the normal quantity of dynamite was cut to less than one half because a large area outside of the cut would move and fall when the cut was shot. When shot, this material fell vertically, with practically no displacement beyond the limits of the drill-hole length.

The Queens southbound tunnel ran parallel to a part of the west track previously completed in the open cut in seamy rock. During the open-cut work the seamy rock was doweled by drilling across the seams and by grouting in 1-in. square rods approximately 4 ft long. Some of these rods were encountered in the tunnel excavation where breakage exceeded the drilled area by more than 1 ft. However, no damage was noticed at any time in the completed open-cut structure.

At the south end, the seams in the rock were so soft, and in a number of instances actually open voids, that shooting was reduced to a few holes at a time to reduce the wild breakage. In this area six 6-in. H-beams were carried as cantilever lagging ("crown bars") above the steel ribs up to the face, and after the top holes were shot the lagging was pulled ahead and blocked before the excavation was continued. During this work the advance in the single-track tunnel was given precedence over advance in the center cut of the double tunnel, and no attempt at any definite cycle was considered.

In the other single tunnel (Washington Heights southbound track), although much higher with less cover than the Queens southbound tunnel, rock conditions at the beginning were much better and holes were collared 3 in. outside of the net lines. However, after advancing about 100 ft, the rock turned to a fine-grained syenite with soft mud seams and chlorite jointing. Large quantities of this material would fall from the roof during blasting, and the bracing steel showed definite signs of rock pressure (see Fig. 10). Where this tunnel came beneath an "el" column footing, steel bracing bents were placed on 5-ft centers for a distance of 25 ft in the schist rock. In the seamy rock, steel bents were placed 5 ft on centers continuously. Where the rock breakage was irregular, steel framing was altered to fit the actual breakage, especially on the side adjacent to the double-track tunnel which was already completed and which was only 2 ft or 3 ft away from the actual breakage. Near the end of this tunnel the rock was so poor that no attempt was made to make more than one round in twenty-four hours, especially since additional steel framing was found necessary as intermediate bents when the original framing started to show a sag. Soft ground methods could not be used for tunneling because the individual syenite rocks between the joints were hard and irregular. In January, 1938, the last 50 ft of progress was made in one week, and the remainder of the month was used in rebracing to clear the concrete lines and to place concrete in back of the side-wall sheeting to prevent movement of the rock.

The proximity of adjacent tunnels was shown clearly when a duct manhole was excavated in the side of the Washington Heights southbound tunnel where a theoretical clear distance between the tunnels was 5 ft. Because of the over-breakage of the two adjacent tunnels, and the fact that concrete very

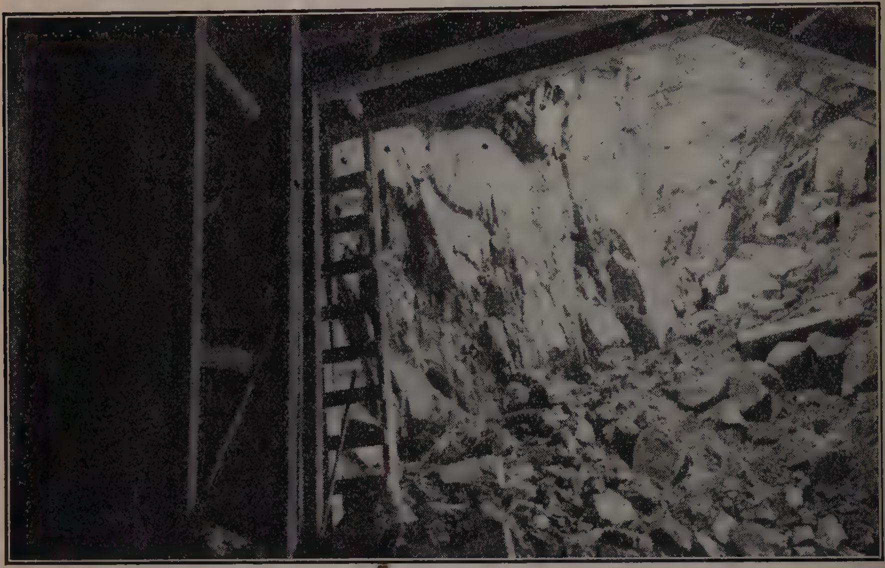


FIG. 10.—ROCK BREAKAGE AT SOUTH END OF TUNNELS AT CHLORITE SEAMS

often ran through the seams, the concrete of the Washington Heights southbound tunnel manhole was encountered in three places in the double tunnel excavation.

The Washington Heights southbound single tunnel was started on November 8, 1937, with special shift periods. It was the only tunnel being excavated and was planned for 2 rounds in twenty-four hours, the drilling and blasting crews coming to the job at 8:00 a.m. and 8:00 p.m. and the mucking crews at 2:00 p.m. and 2:00 a.m. At first it was found difficult to push the drillers so that blasting was completed before the mucking crew came to the job. However, when the drilling crew was given an assignment to drill for a 6-ft round and the entire crew with the exception of one scaler was permitted to go home after the entire round had been shot out, the required progress was obtained easily. As a matter of fact, it was determined by the blaster that the one scaler who remained for the full shift was the driller who finished last. When this rule was instituted, the remainder of the crew usually went home in six to six and a half hours. Generally, between 100 and 110 holes were drilled, each 6.5 ft long, except the ten cut holes which were between 7 and $7\frac{1}{2}$ ft long. Seven drills were operated on the jumbo, using the 4-in. drills for the cut holes and the 3.5-in. drills for the remainder. The average production per twelve hours was 5.42 ft up to December 20 when the seamy and blocky rock was encountered.

Excavation of the Double Tunnel

The general procedure in the double tunnels was to excavate the center drift for the full height, and about 14 ft wide, as a separate tunnel operation. Bracing was designed for future extension to incorporate the bracing of the enlargements on each side. After the center tunnel excavation had been completed, the two sides were ring drilled, and this was followed with concreting of the center wall footing. The structural steel of the center wall was then placed, and the umbrella, consisting of the center wall and parts of the two roof arches, was poured as a unit, incasing the steel beams of the temporary roof support. The side enlargements were then made by shooting the ring holes previously drilled, taking each side as a separate operation.

The center cuts of the double tunnel and the Queens southbound tunnel were started simultaneously from the same face in the open-cut excavation. During August, 1937, 36 rounds were taken with an average progress of 7.34 ft in fairly hard mica schist with some quartz intrusions. Near the south end of the tunnel the rock became quite seamy, and tunneling was stopped 40 ft south of the theoretical tunnel portal, although it had been planned to carry

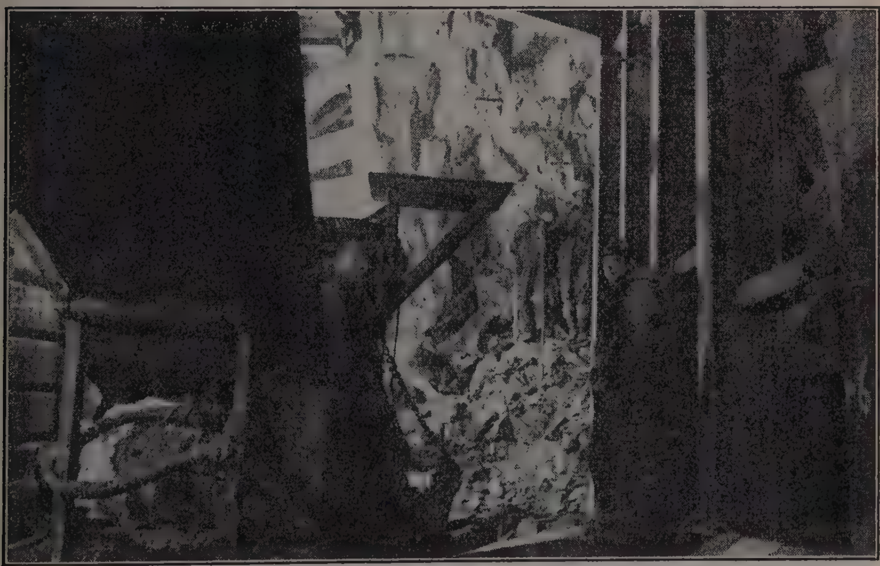


Fig. 11.—Excavation of the Widening in the Double Tunnels

this tunnel 80 ft into the open-cut area not yet started south of 43d Street. Ring drilling was begun immediately thereafter, using the drillers who were working in the Queens southbound tunnel at such times as the latter operation permitted. Ring drilling was completed on October 16 and was followed by steel erection and concrete center bench construction, working from 46th Street southward.

FIG. 12.—WEST WIDENING IN DOUBLE TUNNEL

Blasting for the east enlargement for the center tunnel was started at the north end on November 1, with a small crew, including two drillers for trimming in addition to the normal mucking crew (see Fig. 11). This work was done on a two-shift basis until the progress reached a point at which it was too close to the already excavated Washington Heights southbound tunnel, at which time the east widening was continued on a one-shift basis and the west widening was stopped until the adjacent tunnel was concreted and grouted.

Progress in the east widening during the first month was 288 ft on a two-shift basis. During the same month, beginning November 15, the west widening was extended 210 ft on a two-shift basis with separate and distinct crews and equipment. The intent was to keep the two widenings moving simultaneously but at least 75 ft apart north and south so that the blasting of one would not interfere with work on the other. Because of the dip of the rock seams, the east widening required much more additional longitudinal drilling than the west widening.

As each widening progressed, the timber supports placed in the center cut were removed and used as a protection for the concrete umbrella (see Fig. 12). It also was found that an excellent protection for the upper edge of the concrete umbrella from flying rock was the cushioning effect of discarded rubber belts hung from the temporary roof steel and almost in contact with the concrete.

The east widening of the double tunnel was completed on December 28, progress in December being 316 ft. Until December 22 work was done on three shifts, and after that one shift, per day, making an average progress per eight hours of 4.71 ft. The west widening of the double tunnel was closed

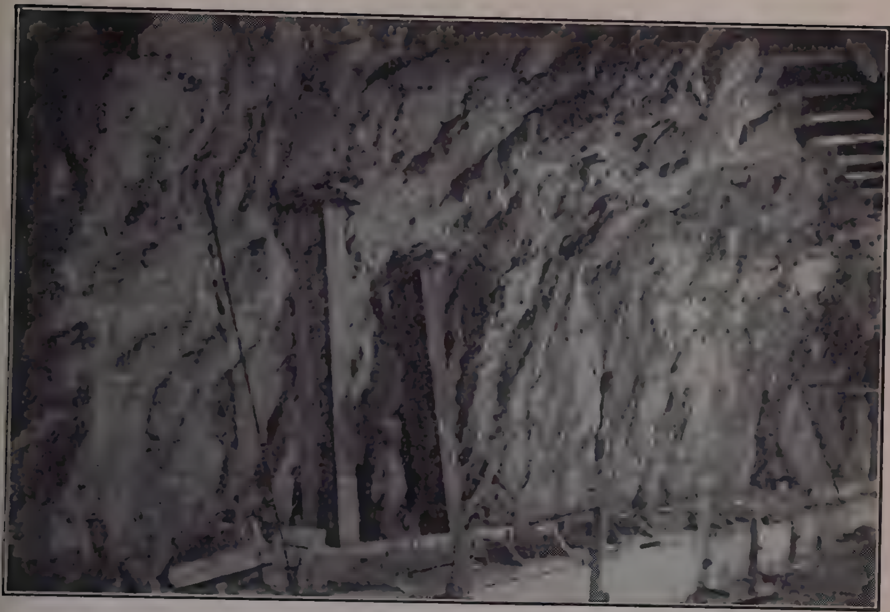


FIG. 13.—INCIDENTAL SEAM BRACING IN THE DOUBLE TUNNEL; WALL BENCH CONCRETED

FIG. 14.—STEEL AND WALL FORMS AT DUCT MANHOLE IN TUNNEL

on December 17 with 193 ft still to go because of the proximity of the adjacent tunnel. In this tunnel progress of 334 ft in December showed an average of 7.79 ft per shift. The much better progress on the east side shows the favorable direction of the rock seams.

In both of the widening operations some steel bracing was found necessary because of the open seams, and such bracing was tied in to the roof steel previously concreted into the umbrella. The temporary steel bracing placed in the tunnels varied considerably, depending upon the rock conditions encountered. In some areas no steel was placed, and in others steel frames were placed about 2 ft apart (see Figs. 13 and 14). Such variation corresponded to the variation in the rock, which was from an extremely hard igneous syenite with seams of feldspar and quartz to soft disintegrated schist and chlorite which in some cases was so soft that the material could be dug out by hand. Seams in the rock were generally almost vertical, with a slight dip from 5° to 15° to the west, and the run of the seams was almost to south, or directly in line with tunnel progress. Worse rock conditions under the small cover and closeness of tunnels scarcely can be imagined.

In the single tunnels steel bracing frames consisted of four pieces of H-beams, which were usually 12-in. column sections cut to approximate shape and length with clip angles attached to each end of each piece. Connections were made by bolting the legs of the angles in contact and wedging with steel wedges as found necessary to take up the shape of the excavated tunnel. Typical sections of the type of bracing used, and details, are shown in Fig. 15. The steel bracing as well as temporary posts and lagging was placed by timbermen crews brought into the tunnel as found necessary (see Fig. 15). During July, 1937, working seventy-four shifts, 70 bents were placed in the single tunnel and 71 bents in the double tunnel. These bents were placed in the total tunnel length of 479 ft.

In February, 1938, after the single tunnels were concreted, the west widening of the double tunnel was completed in seventeen days on a one-shift basis with an average progress of 7.77 ft per day. The rock was soft mica schist with open seams, and steel bracing was placed every 5 ft. The average production for widening of the double tunnel for both sides was 6.50 ft per shift, which includes progress in 90 ft of bad rock at the south end of the west widening, where twenty-seven shifts averaged 3.36 ft.

At approximately the middle of the double tunnel where the vent shaft and emergency exit were to be built, 30 ft of the center umbrella arch was omitted and a crossover track was installed to connect the operations in the east and west widenings. This crossover expedited the mucking problem when both of the widening operations were south of this future shaft. To eliminate any possible damage to buildings in the vicinity of the shaft, the plan was not to break through to the street surface until all tunnel operations were completed.

In the three manholes excavated by widening the tunnels, 10-in. steel pipe connections were provided with the street surface to act as cable feed pipes for signal and other cables to be pulled in the ducts placed along the east sides of the Washington Heights southbound tunnel and of the double-track tunnel (see Fig. 14). These pipes were inserted in 12-in. cored borings

taken from the street surface into the excavated tunnels. Two of these holes measured from rock elevation about 6 ft below street surface, and were each $22\frac{1}{2}$ ft long; the third hole was $13\frac{1}{2}$ ft long. A shot drill operated by a gasoline engine was started on December 14 at the first hole and all work was completed by January 10 for all three holes. Considerable difficulty was encountered because of the large amount of the shot that was lost in the seams of the rock.

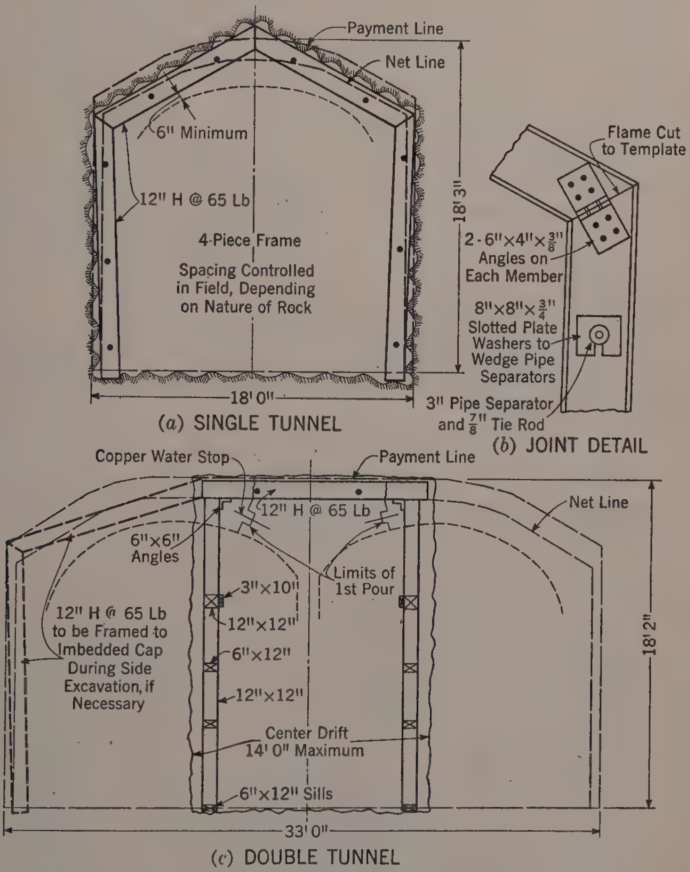


Fig. 15.—Details of Timber Bracing

However, the operation was quite successful, and the total cost of all labor, materials, and equipment was \$11 per foot of core removed. There was no difficulty in placing a 10-in. (inside diameter) galvanized steel pipe in the cored holes. These pipes were grouted in and capped at the street surface.

Labor Schedules, Progress, and Costs

The typical labor setup for tunnel excavation is shown in Table 1. The weekly summary and averages of tunnel progress, consisting of the completion of the Queens southbound and the completed center cut for the double-track tunnel, are shown in Table 2 (see Fig. 4(*a*)).

TABLE 1.—LABOR SCHEDULE ON TUNNEL EXCAVATION

Discription (1)	Single tunnel[a] (2)	Queens south-bound[b] (3)	Washington Heights south-bound[c] (4)	Double tunnel widening[d] (5)	Remarks[e] (6)
Blasters	1	1	1	1	....
Drill runners.........	9	9	{2 at 7 1 at 3	2	Col. 4: 7 runners each for 2 shifts 3 runners for 1 shift
Drill runners' helpers ..	8	9	2 at 7	....	Col. 4: 7 runners each for 2 shifts
Apprentice engineman .	1	1	1	1 at 1	Col. 5: One shift only
Powder monkey.......	1	1	1	1	....
Muck foreman........	1	1	2 at 1	1	Col. 4: Two shifts, 1 foreman each
Tunnel labor.........	7	7	{2 at 9 1 at 5	6	Col. 4: 9 laborers, 2 shifts 5 laborers, 1 shift
Laborers.............	3	3	3	2	....
Hoist engineman......	1	1	1	1 at 1	Col. 5: One shift only
Apprentice engineman..	1	2	1	1	Col. 1: 1 extra on day shift only
Dinkey operator.......	2	2	2	1 at 2	Col. 1: Only one during first month
Electrician...........	1	1	1	1 at 1	{All columns: On day shift only; part time on others
Tunnel boss..........	2	2	2	0	Cols. 2, 3, 4: Two bosses covered all 3 shifts
Powder watchman.....	1	1	1	1	{All Columns: One per shift and additional men for Sundays
Totals for three shifts	113	117	92	35	

[a] Also center drift of double tunnel; three shifts at one round per shift. [b] Three shifts. [c] Also drilling two widening; three shifts. [d] Two shifts, 8:00 a.m. to 4:00 p.m. and 2:30 p.m. to 11:00 p.m. [e] Does not include timber crew, which usually comprised a foreman, 3 timbermen, and 3 helpers, for one or two shifts, as the tunnels required it.

TABLE 2.—SUMMARY OF TUNNEL EXCAVATION PROGRESS

Week ending (1937):	QUEENS SOUTHBOUND TUNNEL		WASHINGTON HEIGHTS AND QUEENS, NORTHBOUND		Totals, in ft	Average progress, in ft per shot
	No. of shots	Progress, in ft	No. of shots	Progress, in ft		
June 5..................	3	16.6	4	22.0	38.6	5.51
June 12.................	7	38.5	8	41.1	79.6	5.38
June 19.................	8	41.5	9	48.9	90.4	5.32
June 26.................	6	35.0	8	49.0	84.0	6.00
July 3..................	9	47.5	8	42.0	89.5	5.27
July 10[a]................	8	44.0	8	49.0	93.0	5.81
July 17.................	7	45.0	8	56.0	101.0	6.73
July 24.................	9	59.0	9	61.0	120.0	6.66
July 31.................	8	56.0	9	61.0	117.0	6.88
August 7................	8	60.0	8	57.0	117.0	7.31
August 14...............	8	57.0	9	65.0	122.0	7.18
August 21...............	9	69.0	8	60.5	120.5	7.62
August 28...............	8	66.0	8	63.0	129.0	8.06
September 4.............	7	45.0	9	64.5	109.5	6.84
September 11............	4	22.0	1[b]	7.0	29.0	4.15
September 18............	7	36.0	....	....	36.0	5.14
September 25............	6	40.0	....	....	40.0	6.67
October 2...............	7[b]	41.0	....	....	41.0	5.85
Total..................	129	819	114	747	1,566	....
Average progress, in ft per round......		6.35	....	6.55	....	6.45
No. of calendar shifts..............		294	....	237	....	294[c]
Progress, in ft per shift:						
To September 7, 1937............		2.90	....	3.15	....	6.05
After September 7, 1937..........		2.34	....	....	....	....
Total[d]......................		2.76	....	....	....	....

[a] Five days. [b] Completed. [c] From June 1. [d] Beginning September 7, 1937, all three shifts, rin drilling in two tunnels during spare time averaged 1.08 shifts per round.

Costs.—The average labor cost of all the tunnel rock removed in 1937 was \$5.75 per cu yd. During August, with two tunnels being excavated, the total labor cost was \$5.11, distributed as follows:

Operation	Dollars per cu yd
Blasting and mucking	4.23
Compressor plant operation	0.12
Blacksmith shop operation	0.35
Equipment repair	0.25
Cleaning up	0.16

The labor cost of ring drilling the double tunnel widenings was \$26.20 per lin ft of tunnel, ring drilled on two sides.

A study was made of the comparative costs of tunnel excavation for the full face running two tunnels at one time, as occurred when the Queens southbound and the center cut of the double tunnel were worked, and the cost of excavating the widening tunnels of the double tunnel in the manner in which it was done with ring drilling, or as separate tunnels with longitudinal drilling. In general, the labor cost of driving two tunnels together is practically the same as the labor cost of the drilling and mucking in widening the double tunnel (see Table 3). However, considerable time was saved by the ring-drilling process. There is an indirect saving (which is difficult to estimate) in the continuous use of drillers while the Queens southbound tunnel was being finished, and there is a saving of 50% in the dynamite cost in the excavation of the widening cuts over that used in the other tunnels.

TABLE 3.—LABOR COSTS OF RING DRILLING VERSUS DRIFTING, IN DOLLARS PER CU YD

Operation	ACTUAL[a]		ALTERNATE[d]		
	Rock excavation[b]	Ring drilling[c]	1.5 shots per shift	1 shot per shift	Two, 6-ft shots per shift
Drillers required for 4.5 shots at 6 ft, in three shifts	....	....	6	4	....[e]
Drilling	4.23	1.82	5.08	5.86	3.82
Blasting, mucking, and trimming		2.46			
Compressor plant operation	0.12	0.12	0.12	0.12	0.12
Sharpening steel	0.35	0.35	0.35	0.35	0.35
Repairs to equipment	0.25	0.25	0.25	0.25	0.25
Total	4.95	5.00	5.80	6.58	4.54

[a] Cost of two tunnels as driven. [b] Comparative labor cost of rock excavation. [c] Ring drilling and widening as of December 13, 1937. [d] Probable costs of driving two drifts instead of using the ring-drilling method. [e] Very unlikely condition.

For the entire tunneling operation, the total labor costs were as follows: Charged to excavation of 30,000 cu yd, \$190,000; charged to timber, placing 155 million fbm, \$16,224; charged to steel, placing 578 tons, \$23,306; and charged to grouting 2,300 bbl of cement, \$3,235. To the foregoing should be added labor costs in the blacksmith shop; equipment repair outside of drills;

compressor-plant operation; the proportion of the cost for setting up the compressor plant and the mucking shaft; and the pay roll of the master mechanic, superintendent, timekeepers, storeroom keeper, and engineering overhead.

Dust Control

After excavation was begun, instructions were received to comply with the then latest regulation of the New York State Industrial Code concerning dust control in rock excavation work. Such a code did not exist at the time the contract was awarded, and, with the exception of normal ventilation and wet drills, no special provisions had been made for such compliance. On June 16, 1937, the New York State Industrial Commission sent representatives, together with observers from the New York Labor Department and the U. S. Bureau of Mines, to take dust count samples during the drilling operation. Although no definite results were reported to the contractor, it was made known indirectly that compliance with the Code would be required and that some method must be submitted for approval unless the strict requirement of the Code (which at that time permitted only dust collectors) was provided.

Immediately thereafter, tests were run in the excavation with wet drills of various types (ventilation being both by blowing and by sucking within the maximum capacity of 4,000 cu ft per min available). Tests were also made to determine the effect of several types of wetting agents mixed in the drill water. Private dust counts were made under each of several combinations of the foregoing possible remedies.

As a result of all of this research, a method or formula for dust control was submitted to the New York State Industrial Commission and after approval was incorporated in the Code. The rules set up for compliance with the amended regulation included the following seven items:

(1) Wet drifters shall be fitted for $1\frac{1}{2}$ to 2 gal per min of water applied to the drill steel;

(2) The exhaust port of the drifter shall be directed away from the face;

(3) A minimum amount of air shall pass through the hole in the drilling steel;

(4) No holes shall be directed upward except a minimum at the top of the tunnel;

(5) Air shall be exhausted at the rate of 1,200 cu ft per min per each drifter—maximum size, 4 in.;

(6) The half-throttle, air-bleeder port shall be plugged;

(7) All holes must be collared wet.

The quantity of water that flowed through the drill needles was tested under various pressures, both for the $\frac{1}{16}$-in. standard needle and for the $\frac{1}{8}$-in. special needle. It was found that the rate of flow varied between 1 and 2 gal per min between the pressures of 20 and 70 lb per sq in. for the $\frac{1}{8}$-in. needle, which was adopted for all drills.

Before any special control methods were used, dust counts were made of the air in the tunnel without drilling and also on the street surface. It was

not unusual to find a dust count more than twice as high in the air at the street surface as in the tunnel without drilling.

A complete report of all these tests has been published by F. B. Flinn and P. S. Miller,[2] Assoc. M. Am. Soc. C. E.

In connection with compliance with the Code (which provides one permissible dust concentration for rock containing less than 10% free silica and a different permissible concentration for rock containing more than 10% free silica), a study was made to determine whether the average rock encountered came in the classification of more than, or less than, 10% free silica. Except for the approximate petrographic methods, the only reliable chemical method for this determination, found after a thorough detailed study by the experimental laboratories of the DuPont Company, was that described by A. Shaw[3] in 1934. This method is so detailed that only the best equipped laboratories could perform the test and then only after about a week of work. Another method reported by Sartorius and Jotten[4] is a combination of chemical separation and centrifuging the free silica from the solution. Neither of these methods was considered practicable enough for attempting a determination of the true amount of free silica in the rocks encountered. When the question was presented to the New York State Industrial Commission, and samples of rock were sent for inspection, it was reported that the rocks contained more than 10% free silica without giving the exact percentage or advising what method had been used.

The Effect on the "El" Structure

During tunnel operations, daily readings were taken on level points fixed on all elevated columns above the excavated parts of the tunnels and for some distance beyond the headings. Practically no deviation in elevations was found in any of the columns except the one column directly above the Washington Heights southbound single-track tunnel, near the south end. During excavation a slight settlement was noticed in this column. Since the excavation was through soft material that broke very poorly, steel bents were placed to support the roof. After the tunnel was completed, some of this steel bracing required readjustment to clear the concrete lines, and during this operation a considerable mass of loose material that had been resting on the roof steel fell down. This happened directly below the elevated column footing, which, however, was not a dangerous condition because the elevated column load had been jacked up on two temporary steel beams above the street surface as soon as the settlement in the column had reached 1 in. After all the loose material had been allowed to fall, exploratory drill holes were started from the street surface, and it was found that this footing was resting on a layer of rock about 5 ft in thickness with soft material below it. This rock was consolidated by grouting from the street surface, as well as through the concrete after it was poured. The column itself was loosened from its footing and jacked up to proper elevation, and the intervening gap was filled with a dry-packed grout.

[2] *Engineering and Mining Journal*, Vol. 139, July, 1938, pp. 38–43.

[3] *The Analyst*, Vol. 39, 1934, p. 449.

[4] *Zentralblatt für Gewerbehygiene*, Vol. 21, p. 65.

Dynamite

Contract specifications limited the use of dynamite to not more than 100 lb of 60% gelatin strength in any one blast, including delayed explosions. As stated previously, the rock was of such nature that 40% gelatin dynamite was found more suitable.

The total quantity of dynamite used for excavating these tunnels was 2.98 lb per cu yd, net measurement, of 40% gelatin strength. Based on actual excavation, due to over-breakage, the total was actually about $2\frac{1}{4}$ lb per cu yd. On the average one cap was used for each six sticks of dynamite, or 3 lb.

The weight of dynamite per cubic yard moved varied from month to month, depending upon the type of rock encountered. The maximum was in June, 1937 (3.12 lb); the minimum was in January, 1938 (1.04 lb). The average dynamite consumption for straight tunnel excavation was approximately 3.10 lb per cu yd, and the average for shooting ring holes was 1.65 lb per cu yd.

All detonation was electrical, a separate blasting line being carried into each heading from a switch located at the portal.

Concreting

Concrete in all arches, including the umbrella portion in the double tunnel, was placed by concrete pumps, using steel forms. Total forms purchased for the job included a 30-ft section of umbrella form, a 30-ft section of side arch that was usable in either of the enlargements for the double tunnel, and a 30-ft section of single complete arch tunnel. The umbrella form consisted of two side panels and part of the arch with a trussed framework that supported the wheels. The form collapsed by sliding it on the wheel shaft. This was done by turning the shaft with a ratchet handle, thereby pulling the form away from the concrete. By means of screw jacks built into the wales, the form was collapsed vertically. In this manner the form tended to lean toward the concrete, and rollers were provided to run on the concrete while the form ahead was moved. Bulkheads at each stage were built up with wood. For the remainder of the double tunnel a usual type of traveling form was used, with jacks at the bottom, and it covered approximately three quarters of the arch. In detail it was very similar to the complete tunnel form.

The complete form for the single-track tunnel was 30 ft long built up in multiples of 6 ft. Two openings in the side-wall were spaced 15 ft apart for niches. The trussing and framework for the form, including the traveler, were so arranged that a muck car could be run through the form, and in a number of locations such a procedure was necessary. Usually, timber braces were placed across the legs of the traveler to increase the stiffness of the form. Because of the fact that the tunnel is wider on curves, a slip lap joint was provided for the top plate which, in the wider part, resulted in a flat spot at the top of the arch. This panel was really the key plate, placed last as the concrete came up to each section. The form was held in place by means of screw anchors embedded at accurate spots in a short height of the wall built with the benches. The same rail was used as for mucking, additional rail

lengths being moved ahead to accommodate the wide gage of the traveler. Collapsing of the form was by lowering the screw jacks on to the axles. The concrete record and progress are shown in Fig. 5 and the location of equipment in Fig. 16.

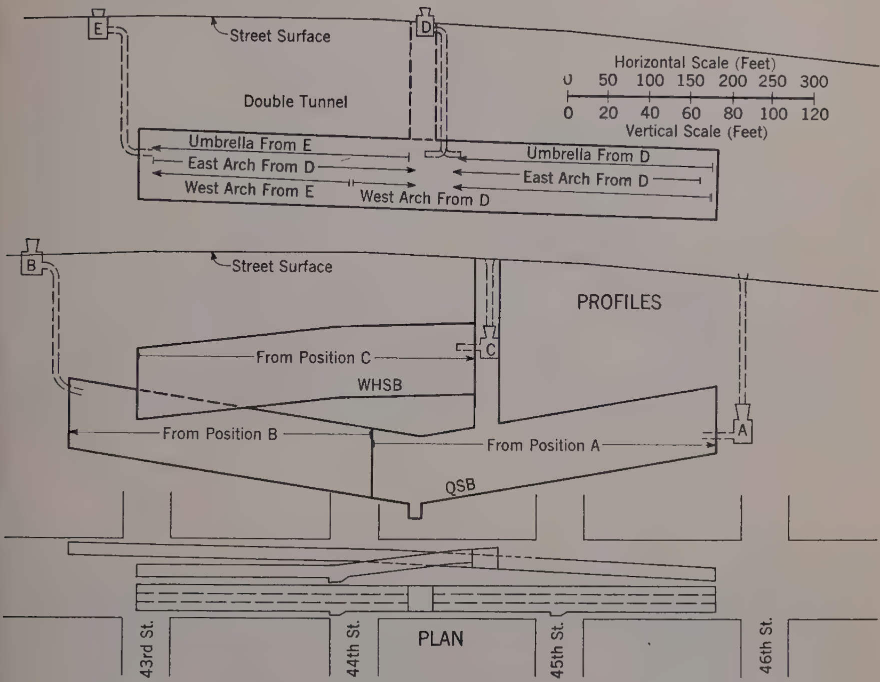


FIG. 16.—CONCRETING SETUPS

In the single-track tunnel, concrete arches were poured at the rate of 30 ft each day, using the larger pump. In the double tunnel, the umbrella section was poured at the rate of 30 ft a day with the smaller pump, such a pour being between 40 and 45 cu yd. The complete arch pour was about 135 cu yd.

No difficulty was experienced in pouring once every day. The early morning shift collapsed the form and moved it ahead, and carpenters starting about 6:00 a.m. would build the bulkhead and prepare for the next pour to start at 10:00 a.m. After the first few days of operation, and after all the idiosyncrasies of the concrete pump apparatus were fairly well understood, the 135 cu yd of pour would be finished by 4:30 p.m. The concrete pumps were placed at approximately subgrade level, and the concrete was chuted down from mixing trucks at the street surface through circular 8-in. elephant trunk chutes. The rate of delivery was controlled by bell signals between the pump operator and the street surface. Pump operation was controlled by the engineers stationed at the pump in accordance with electric bell signals from the foreman at the form. Very little difficulty was encountered because of the rule that all signals, when received, must be repeated by the receiver,

and the little additional expense of providing return signals from each line was found much worth while.

In some instances in the double tunnel it was found possible to place the pump in the shaft area at approximately the middle of the length of the tunnel and at the street surface, using a down-feed pipe to the tunnel level. With fairly uniform delivery of concrete, and starting each day's pour with approximately 3 cu yd of cement grout to lubricate the pipe, no difficulty was found in pumping concrete approximately 500 ft horizontally and 20 ft vertically.

At a point about 30 ft from the form an air-connection tee was added to the concrete pipe to be used as a booster for filling up the form. When the concrete had filled up the entire form and the end bulkhead had been completed, the short pipe sections inside the form were removed as the concrete filled up the volume. Air at 100-lb pressure was admitted into the concrete pipe, and the fluid concrete ahead of the air inlet, about 30 ft in length, was shot into the concrete mass inside the form. This operation, it was found, could not be repeated safely until the normal pumping had again filled up the entire pipe length; otherwise the back pressure clogged the pipe between air connection and pump.

With this procedure the concrete could be pushed up 6 ft above the outlet pipe inside the form where rock crevices were that high; and, as far as visual inspection was concerned, the concrete seemed to be tight against the rock. During pumping operations, it was found practically safe (although not unduly comfortable because of the high humidity and temperatures) for men to work inside the form when placing and tying grout pipes, spading concrete, and operating vibrators.

The typical labor setup for concreting, including stripping and shifting steel form, building bulkheads, setting and brazing copper water stops, setting up concrete pipe, setting and making up grout pipes, revising steel bracing, concreting, and cleaning up pipe and concrete pump, was as given in Table 4.

TABLE 4.—LABOR SCHEDULE FOR CONCRETE WORK

Description	Timbermen	Timbermen helpers	Laborers	Carpenters	Concrete foremen	Concrete laborers	Maintenance enginemen
Single Tunnel Form:							
Queens southbound tunnel	4	4	5	10	1	14	3
Washington Heights southbound tunnel	5	6	....	14	1	11	3
Double tunnel, west arch	6	7	2	16	1	11	5

The labor cost for stripping, moving, and resetting forms and building bulkheads was $0.23 per sq ft contact area of tunnel surface (not including areas of bulkheads).

The total labor cost of tunnel concrete in 1937 was $3.80 per cu yd, to which must be added the cost of the forms, insurances, overhead, and concrete, including the value of the excess concrete outside of pay lines. This labor cost includes all of the men involved in preparing, setting, and removing the forms, and the placing of the concrete inside the form, placing the grout pipes

FIG. 17.—SOUTH END OF DOUBLE TUNNEL

FIG. 18.—DOUBLE TUNNEL COMPLETED EXCEPT FOR CATWALK

and water stops, and any incidental work in connection with the concrete both for the arches and the benches.

Steel Erection

In the double tunnel, structural steel columns and base and top grillages were erected in the center wall. Top grillages were continuous over two columns in the form of bents independent of each other. The structural steel was brought into the tunnel on the mucking tracks and set at the side of the tunnel until all the steel had been delivered. Erection was by hand, using the roof bracing for attaching block and falls. As the structural steel was set in place, the track had to be removed. A total of 102 tons of steel was brought in with a total labor cost of \$22.37 per ton, including supervision, erection, labor, riveting, and trucking from lighters about 2 miles away.

Completion of Tunnels

After the main arches were poured, in which grout pipes had been installed, the track remaining in place, a grouting gang was sent through to grout all the holes that would take it, at 50-lb to 60-lb pressures. This was followed by a second run at 60-lb pressure (and in some cases more), after which the grout holes were filled. At the same time, any finishing that needed to be done on the concrete, such as removal of fins, was completed (see Figs. 17 and 18). Grout pipes were 2 in., with a coupling set flush on the form. Grout cocks were connected by nipples, which were removed after the completion of grouting.

The grout used was approximately of 1 : 1 mix. The grouting machine ran on the track, and the sand and cement were brought up in bags on flat cars. The Queens southbound single tunnel, 700 ft long, required 1,210 bbl of cement, or approximately $1\frac{3}{4}$ bbl of cement per linear foot of tunnel. The total cost of this grouting was as follows: Labor, \$1.24; insurance, \$0.24; cement, \$2.10; $\frac{1}{6}$ cu yd of sand, \$0.25; sales tax, \$0.05; and air and equipment, \$0.12—a total of \$4 per bbl of cement. In the other tunnels the labor cost ran quite uniformly the same, at \$1.25 per bbl of cement. The average day's production was 250 bags of cement, and the maximum day's production was 356 bags.

At convenient times the inverts were graded by hand, removing the track after grading was completed. The invert concrete slab was 12 in. thick, and in some locations covered cast-iron drain pipe was set slightly below the invert. This concrete was placed by pump, keeping the pipe on the side bench. One shift could easily do 250 ft of invert, or 125 cu yd, using the small pump.

In two of the tunnels a catwalk, or emergency walkway, was built in sections of about 30 ft, re-using a set of forms for the entire length of two tunnels. In the other two tunnels a bank of 30 clay ducts was laid against the wall and concreted in, the top becoming the catwalk.

Shafts

After the tunnels were concreted completely, two shafts were excavated down from the top. The west shaft required removal of only the thickness of rock between the two tunnels which crossed each other at that point—approximately 5 ft. Mucking was by crane and buckets from the street surface.

In the east shaft, where the cover was about 20 ft above the tunnels, earth and some of the rock were removed from the street surface to expose and protect the water main and utility ducts, and also to deck the opening in the street surface; then by means of long drilling, holes were driven through to the tunnel. The rock was then blasted into the tunnel by loading the bottom of the holes. Mucking, again, was by crane and buckets from the street surface.

In the east shaft niches were cut to provide for the exit stairways connecting the catwalk with the street surface, in this way avoiding open excavation immediately adjacent to the building. In spite of the seamy rock in the east shaft, steel bracing in the tunnel and in the shaft prevented any movement of the rock, and the operation was completed without hazard to the pedestrian traffic as well as the buildings immediately adjacent to the shaft.

Accidents

In general, the tunnel operation was remarkably free of accidents, although there was one fatal case. As far as equipment was concerned, at one time a mucking machine tipped over in the tunnel causing a loss of three hours in schedule but no injuries. At another time a piece of roof fell out on the boom of the mucking machine and broke the beam connection, without any personal injuries. The fatal accident occurred on the last night of tunneling when a small piece of rock fell from the roof and injured the mucking machine operator, who at the time was watching the track gang install the last piece of track for the final cleanup; the man died some days later.

To reduce accidents, every man in the tunnels was required to wear a safety hat and safety boots that were provided with nonskid bottoms and aluminum tips. The foremen were held personally responsible for accidents to the men in their crews. No separate record was kept of accidents in the tunnels as against those on the remainder of the contract. The total record of lost-time accidents from July, 1937, to April 1, 1938 (during which period most of the tunnel work was done), shows a total of thirty-seven accidents for the nine months, of which thirty-two were lost-time accidents. The number of men engaged in that period averaged between 400 and 500 per day. In this period of nine months the maximum number of lost-time accidents per month was eight in the month of July and the minimum was one in the month of February. There was no accident involving any one outside of regular employees. Alleged damage to neighboring property along the line of the tunnels was practically nil.

Personnel

Contract for the construction of Section 10 of the Sixth Avenue Subway, which included these tunnels, was awarded to the Rosoff-Brader Construction Corporation, which, in August of 1938, became the Brader Construction Corporation. The chief engineer of the Rosoff-Brader Construction Corporation was Fred W. Stiefel, Assoc. M. Am. Soc. C. E.

The work was done under the supervision, and in accordance with the design, of the Board of Transportation of the City of New York, of which Jesse B. Snow, M. Am. Soc. C. E., is chief engineer. During the tunnel-construction period, the late Byron Houghtaling, M. Am. Soc. C. E., was first division engineer, with Charles M. Madden, M. Am. Soc. C. E., as section engineer at first. Later, Mr. Madden replaced the late A. M. Mayell as subdivision engineer. This work was completed under Francis V. Hayes, Assoc. M. Am. Soc. C. E., as section engineer. The design was prepared by Albert Goertz as design engineer for this section.

In the contractor's organization, all of the construction procedure and operations were under the control of the writer, who was in charge of the construction. The engineering staff not only designed the methods of procedure but ordered the materials, laid out the work for the various shifts, and in general scheduled the operation of construction. In the field one engineer was assigned to each shift in the tunnels, the three shifts being covered by S. M. Marks and Sidney Philip, Juniors, Am. Soc. C. E., and James Lyttle. The dust-control work, as well as some of the office work in connection with the planning of the drilling, equipment, etc., was done by Philip S. Miller, Assoc. M. Am. Soc. C. E., aided by Sol Rusitzky and W. Mortensen in the office work.

The superintendent was Jacob Siebert, to whom too much credit cannot be given for the success of the operation. Joseph Tinley was master mechanic and Walter Reed was master electrician. Shift bosses were Harvey Urey and Due Carroll. Credit must be given not only to these men but to all the loyal mechanics and laborers in their respective gangs.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

MODEL TESTS, BRIDGE PIER SUPPORTED ON LONG STEEL PILES

Discussion

BY THOMAS F. COMBER, JR., M. AM. SOC. C. E., AND JOHN M. COAN, JR., JUN. AM. SOC. C. E.

THOMAS F. COMBER, JR.,[6] M. AM. SOC. C. E., AND JOHN M. COAN, JR.,[7] JUN. AM. SOC. C. E. (by letter).[7a]—The writers are indebted to Mr. Feld for his discussion and for his general approval of the paper. Mr. Feld questions the safety of such long piles in actual construction. The purpose of making these tests was to answer this question and to determine by actual measurements the stresses in, and the action of, such unprecedented piles. However, in the actual construction, the pier caps were extended to a level just below the mud line, and this considerably reduced the unsupported length and the deflections of the prototype pier.

Mr. Feld also suggests the possibility that, in such soft material, caisson construction might have been feasible. In this particular case, with more than 100 ft of soft mud in some places, the cost would have been out of the question if the piers were to be extended to solid material or rock.

The writers made clear, in the paper, their reasons for neglecting the restraining action of the soil. Mr. Feld agrees that this will give a more critical stress condition than actually would exist in the prototype.

In discussion, the writers had hoped to develop some mathematical analyses for such pile groups, using the test data as a check. This was the main purpose of submitting the results of this research to the profession, and no claims were made as to the practicability of the proposed construction.

NOTE.—This paper by Thomas F. Comber, Jr., M. Am. Soc. C. E., and John M. Coan, Jr., Jun. Am. Soc. C. E., was published in June, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by Jacob Feld, M. Am. Soc. C. E.

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[7] Stress Analyst, The Glenn L. Martin Co., Middle River, Md.

[7a] Received by the Secretary March 18, 1941.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

CAVITATION IN OUTLET CONDUITS OF HIGH DAMS

Discussion

By Messrs. V. E. Leman, P. S. O'Shaughnessy and E. S. Randolph, and Carroll F. Merriam

V. E. Leman,[11] Esq. (by letter).[11a]—Efforts, made in recent years, to arrive at a clear conception of the processes relating to the behavior of water in the cavitation zone, have not met with much success. J. C. Hunsaker[5] has referred to this condition, as follows: "We cannot describe the exact mechanism by which wall damage results, but there must be some connection with the fluctuation of pressure resulting from the collapse of water vapor which periodically becomes unstable in such vapor phase."

In the light of such developments, Messrs. Thomas and Schuleen have rendered a most valuable service in giving an analytical demonstration of the hammering power of a slug of water traveling into a confined space.

The writer feels, however, that the full scope of the analysis has not been exploited by the authors, who limited themselves to a single aspect of the process—namely, to the action of a slug of water sandwiched in between two vapor cavities, without taking into consideration other factors instrumental in producing pitting in the cavitation zone.

Under the heading, "Behavior of Original Madden Dam Conduit Entrances in Models," the authors describe the character of flow at the collapse zone as one "where especially intense impacts are caused by collisions between the swiftly moving water of the streamline filaments and the broken masses of water and vapor hurled against these filaments from the interior of the pocket."

It seems to be an established fact that, when a vapor cavity collapses, water rushes into it, eventually reaching the wall, not in the shape of broken masses but usually retaining, in substance, its pattern of continuity. It is the writer's contention that both the broken masses and the streamline filaments contribute to the destruction of the wall.

Note.—This paper by Harold A. Thomas, M. Am. Soc. C. E., and Emil P. Schuleen, Assoc. M. Am. Soc. C. E., was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Jerome Fee, Assoc. M. Am. Soc. C. E.

[11] Civ. Engr., New York, N. Y.

[11a] Received by the Secretary March 11, 1941.

[5] "Cavitation Research," by J. C. Hunsaker, *Mechanical Engineering*, Vol. 57, April, 1935, p. 211–216.

To substantiate his point of view, the writer presents a "physical picture" of what, he considers, happens when water enters a crevice, either in the form of an isolated slug or of a solid column. The picture may be helpful also in visualizing the processes inherent in the travel of water into converging spaces under ideal conditions (as assumed by the authors), these processes being, ordinarily, somewhat puzzling. The case of a slug, in accordance with the authors' analysis, will be first presented.

For the sake of simplicity the crevice is assumed to be not a pyramid, as used by the authors, but wedge shaped, as shown in Fig. 15. To facilitate the

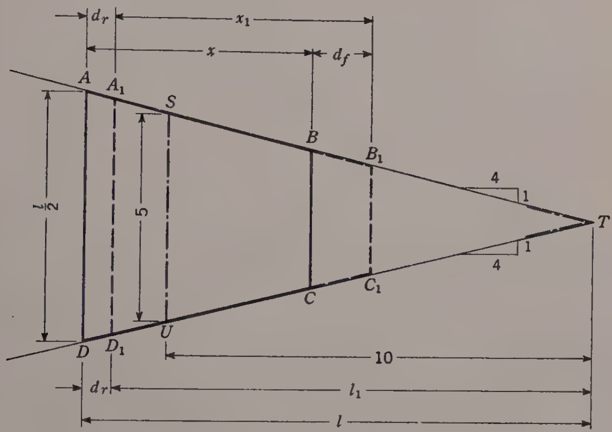

FIG. 15

calculations the sides of the wedge are assumed to slope at a rate of 1 on 4, which makes the spread of the wedge equal to one half the distance to the tip T.

A slug of water (of a width 1 normal to the plane of the drawing) moves from its position $A\ B\ C\ D$ to a new position $A_1\ B_1\ C_1\ D_1$, with its rear traveling a distance d_r, and its front a distance d_f. Due to convergence of space, d_f is larger than d_r, as volume $A\ A_1\ D_1\ D$ must be equal to volume $B\ B_1\ C_1\ C$. This faster movement of the front in relation to that of the rear becomes more and more pronounced as the slug travels toward the tip of the wedge. The following analysis will express this algebraically:

Suppose the slug $A\ B\ C\ D$ moves from a position, where its rear is l units away from the tip T, to its final position when its front hits the tip—this final position being represented by a triangle $S\ T\ U$, with a base 5 and a height 10. As the volume $A\ B\ C\ D$ is equal to the volume of the triangular prism $S\ T\ U$:

$$\tfrac{1}{2}\left(\frac{l}{2}+\frac{l-x}{2}\right)x = \frac{5 \times 10}{2} = 25;$$

or

$$x = l - \sqrt{l^2 - 100} \qquad \text{(24)}$$

in which x represents the changing length of the slug as it travels toward the tip.

From Fig. 15:

$$d_r = l - l_1 \qquad \text{(25a)}$$

and

$$d_f = d_r + x_1 - x \qquad (25b)$$

in which l and x refer to the initial positions, and l_1 and x_1 to the new positions of the slug in its various stages of travel toward the tip.

Noting that expression $\frac{d_f}{d_r}$ represents the relation of average velocity of the front V_f to that of the rear V_r, the movement of the front and the rear occurring in the same interval of time:

$$\frac{V_f}{V_r} = \frac{d_f}{d_r} = \frac{l - l_1 + x_1 - x}{l - l_1} \qquad (26)$$

and, as the energy changes with the square of velocity:

$$\frac{E_f}{E_r} = \left(\frac{V_f}{V_r}\right)^2 \qquad (27)$$

in which E_f and E_r represent the energies of particles at the front and the rear of the slug, respectively.

Eqs. 24, 26, and 27 have been used to plot the curves in Fig. 16, the values of l being selected in such a way as to give the more characteristic points shown. One can readily see that, for an ideal case of motion (no viscosity, no elastic deformations and absolute vacuum), the relations of velocities and energies of front and rear may become infinite, with practically all energy concentrated at the tip of the crevice. This is in complete agreement with the authors' statement to that effect in presenting Eqs. 3 and 4.

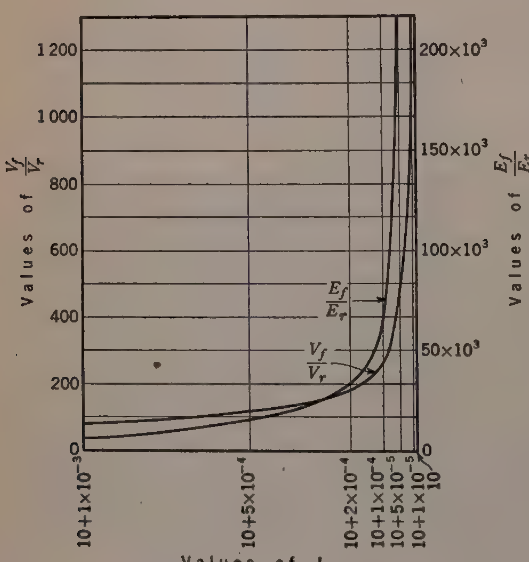

FIG. 16

The case of streamline flow involves conditions differing in many ways from those of a slug; whereas the latter is comparable to a flying projectile, the former is characterized by a water column reaching back to the free water surface of the reservoir.

The purely kinetic energy formulas (Eqs. 1 and 2) must now yield to the total energy equations that would include the pressure energy as well, no matter how small this factor appears to be in the low-pressure zone.

Fortunately, however, these considerations need not be taken in account as far as the "physical picture" of the process within the crevice is concerned.

Referring again to Fig. 15, imagine that the space to the left of line $A\ D$ is not a vacuum but is filled with water extending to the free surface of the reservoir, the line $A\ D$ being merely a dividing line, separating a volume $A\ B\ C\ D$ from the column of water to the left. The reasoning used in establishing the curves in Fig. 16 can now be applied, without reservations, to the case of a solid water column entering a crevice; in spite of a difference in absolute values of velocities and energies in the two cases, the process of their respective increase remains identical. The final result (accumulation of energy at the tip) is also identical.

A comparison of the actions of the two flow patterns from the point of view of destruction may be proper.

The writer is unable either to agree with the authors' theory of water-hammer action in respect to the slug, or to deny it. He would not be willing to go far beyond the assumption that, a pressure wave within the slug itself being a certainty, its propagation farther out, over the vapor cavities, remains an open question.

This uncertainty, coupled with insufficient information as to the actual velocity of the slug, precludes any accurate estimation of the relative destruction capacities of the two flow patterns. The following, therefore, is merely an attempt to evaluate such capacities in a rather general manner, under the assumption that some kind of water-hammer action does exist in the case of a slug.

The familiar expression for the head due to water hammer (in excess of the static head) is:

$$h = \frac{V_w}{g} V \qquad (28)$$

Since V_w and g are practically constant, h would depend on V alone. This means that, in order to discover which pattern of flow creates a greater water hammer, reasons for the occurrence of higher velocities should be investigated.

For lack of experimental data, an accurate solution of this problem cannot possibly be given at the present time; one may feel, however, that, in the case of a solid water column reaching the wall, its velocity, being dependent on the static head in the conduit, is certain to be higher than that of a minute slug working under the handicap of collisions with broken masses of water in the vapor pocket. Besides, the momentum of such a column is certain to support the forward motion of the water particles in the crevice, helping them to cope with the counter action of viscous resistance, elastic stresses, or condensation of vapor in the tip. In that case, the flow has a better chance of retaining its original velocity at the point of contact with the wall.

These considerations lead the writer to the belief that, provided the water column has a chance to reach the wall as a solid mass, its eventual effect is certain to be more violent than that of a slug. This does not necessarily mean that all destruction is the result of such action. The fact that the collapse of cavities occurs periodically, whereas the slugs are continuously at work, makes the problem rather indeterminate. Experiments by P. De Haller,[12] in Switzer-

[12] *Schweizerische Bauzeitung*, May, 1933, p. 243.

land, are worth being mentioned in this connection: Metal specimens have been subjected to heavy hammering by water slugs. The results are indicative of the fact that, to a high degree, such slugs are instrumental in producing erosion which, in its sponge-like appearance, is strikingly similar to the pitted surfaces resulting from collapse of cavities.

The writer has one more remark to make: The authors state (under the heading "Model Studies of Various Methods Proposed to * * *: Group 2") that the reason for not using vent pipes in their final design was their unwillingness to reduce the discharge capacity of the conduits. No matter how important this consideration, the writer is under the impression that vent pipes have not been given a chance to prove their worth. The three 3-in. pipes, installed in connection with the modified design of 1935, could not help being more of a nuisance than a remedy, owing to their quite insufficient size. Judging by the standards of the U. S. Bureau of Reclamation,[13] the cross-sectional area of the pipes should have been about 8 times larger than the one used.

P. S. O'Shaughnessy,[14] Esq., and E. S. Randolph,[15] M. Am. Soc. C. E. (by letter).[15a]—In 1928, when plans were first made for the Madden Dam, sluiceways near the level of the Chagres River bed were specified. These outlets were to permit lowering of the reservoir for purposes of inspection, cleaning out debris, and routine maintenance and repairs. The sluiceways became a part of the flood-control plan, to be used when and if floods occurred in excess of the spillway capacity. During construction of the dam, the six 5-ft 8-in. by 10-ft sluiceways proved to be a valuable aid in diverting the flow of the river.

The flare provided at the sluiceway entrances in the original design was limited to permit the construction of slots to accommodate stop logs of reasonable size. When the laboratory tests were made, one of the objects was to preserve the stop-log slots and thereby confine any required modifications to the part of the conduit downstream from the slots. This was accomplished in the case of sluiceways 1 to 4, where the model studies showed that the entrance could be reconstructed in such a manner that cavitation was eliminated with very slight changes in the stop-log slots. However, for sluiceways 5 and 6, the most satisfactory means developed to reduce cavitation consisted of providing covers at the stop-log slots.

Between September and December, 1934, as part of a comprehensive program of hydraulic tests on the Madden Dam, the sluiceways were operated at various heads from 71.5 to 138.5 ft during the rise of the reservoir. After December, 1934, the sluiceways (except sluiceway 3 in which the testing equipment had been installed) were used at times to control the reservoir level.

From the beginning of the sluiceway operation, it was noticed that the flow was accompanied by popping and crackling noises that were very loud within the sluice-gate galleries and were audible even on the parapet of the dam. Apprehension was felt at the time that damage was occurring, but

[13] "Dams and Control Works," U. S. Dept. of the Interior, 1938, p. 180.

[14] Asst. Engr., Special Eng. Div., The Panama Canal, Balboa Heights, Canal Zone.

[15] Cons. Engr., The Panama Canal, Balboa Heights, Canal Zone.

[15a] Received by the Secretary March 13, 1941.

definite evidence was lacking until the cavitation had progressed to such a depth that a porous tile drain began to discharge into the dam on March 15, 1935 (see heading "Introduction," in the paper). The knowledge that cavitation had occurred coincided closely with the appearance of the paper by Mr. Hunsaker,[5] which was of assistance in understanding the matter. Remedial measures were undertaken immediately. It was hoped that small air vents would prevent or diminish the cavitation by serving as a cushion, but they did not produce the desired result of eliminating cavitation completely.

After the collapse of the air-vented filler in sluiceway 6, which had been fitted with a bellmouth inlet, attention was directed to the great intensity of the force acting downward through the slot. Consideration was given to a stronger filler for closing the four sides of the slot, but sufficient confidence existed in the efficacy of a tight-fitting cover, alone, to result in its selection as a means of closure. This was done prior to making the model tests, which eventually justified the selection of this method. The covers were placed on the two sluiceways provided with bellmouths—5 and 6. These covers were designed to withstand full hydrostatic head in addition to atmospheric pressure. Steel cables were attached to the covers to permit their removal without the services of a diver.

Messrs. Thomas and Schuleen state that the Madden Dam sluiceways operate under a maximum head of 155.3 ft. This head corresponds to El. 250, the top of the spillway gates, but the computed reservoir level for the greatest anticipated flood is at El. 263, which is equivalent to a hydrostatic head at 168.3 ft. It was for this reason that tests at higher heads were requested in the second series of tests. The fact that 168.3 ft was the maximum anticipated head was not transmitted to the authors.

During the second season of operation of sluiceways 5 and 6, after installation of the bellmouths, there was a noticeable difference in the volume of concrete removed. As shown in Table 2, the operating periods at high heads were, respectively, 1,248 and 1,421 hours. The volumes of concrete removed were, respectively, 30.5 and 16.4 cu ft. It appears that the vented stop-log slot filler in sluiceway 6 remained in place for some time after its installation but collapsed at some unknown date. Vacuum measurements were made at the entrance to sluiceway 6 on December 11 and 16, 1935. On the former date, with the sluiceway operating under a head of 134.3 ft, a vacuum of 24 in. of mercury was recorded. On December 16, under a head of 124.3 ft, no vacuum was developed.

It seems reasonable to assume that the collapse of the frame occurred between these two dates. Since most of the sluiceway operations, both in point of time and of high heads, had occurred by December 16, it would appear that the lesser damage to sluiceway 6 may have been due to the beneficial action of the vented filler frame before its collapse. That the frame did not collapse under the higher heads prevailing at the beginning of operation can be explained by assuming that the force acting upon it was not constant but of a pulsating character. It was noticed throughout the sluiceway tests that the stream issuing from the conduits was of this type.

During the first season of operation, sluiceways 1 and 2 were badly damaged by 138 and 130 hours, respectively, of operation under high heads. After these sluiceways were repaired, they were operated 134 and 107 hours, respectively, at high heads without suffering damage. It is probable that the concrete used in the repairs was denser and, since it was mixed and placed in smaller batches, its placing was done more carefully than that of the original concrete. It is possible, also, that the air vents installed after repairs were made were of some use in decreasing the effect of cavitation collapse.

It is not made clear in the paper how the air measured in the model tests on the diverging tube apparatus can be related quantitatively to the air that would be required in venting the prototype. Even if the quantity of air can be expressed in terms of the prototype, it appears doubtful that venting is the best approach to most problems of eliminating cavitation in outlet conduits.

Mr. Hunsaker[5] showed that for a given hydraulic system the product of the frequency and the length of the cavitation pockets is equal to the product of the throat velocity and a substantially constant non-dimensional number. If the velocity remains constant, venting will lengthen the cavitation pocket and decrease the frequency in direct proportion. In this case, the pockets will form farther downstream and the immediate effect of cavitation will be decreased. However, this is not a satisfactory solution to the problem since cavitation is not eliminated unless a large volume of air is introduced; and, in this case (as stated by Messrs. Thomas and Schuleen), a reduction in discharge will result.

Difficulties similar to those at the Madden Dam may have been encountered in other structures. These, if reported and discussed, may serve to focus attention on the importance of considering cavitation in designing.

Since the completion of the model tests, it has been determined to limit sluice-gate openings on sluiceways 1 to 4 to 70% until they have been repaired. Sluiceways 5 and 6, provided with bellmouths and cover plates over the stop-log slots, may be operated at full capacity for the passage of extreme floods.

Acknowledgments.—The designs for the Madden Dam were prepared by the U. S. Bureau of Reclamation, in collaboration with engineers of the Panama Canal. The chief engineer and the chief designing engineer of the Bureau of Reclamation were engaged as consulting engineers by the Governor of the Panama Canal Zone. The bellmouth entrances that were attached later to sluiceways 5 and 6 were designed also by engineers of the Bureau of Reclamation.

CARROLL F. MERRIAM,[16] ESQ. (by letter).[16a]—There is much food for thought in the similarities and the differences to be noted in the work of experimenters in the field of cavitation. The striking point is that interest in cavitation is not confined to the builders and users of hydraulic machinery. It proves that the research conducted primarily through interest in the mechanical aspects finds application in a problem that is essentially structural. At the present stage of development at least, there is one outstanding difference in approach. In the case of discharge conduits, which may be operated at full

[16] With Pennsylvania Water & Power Company, Baltimore, Md.

[16a] Received by the Secretary March 17, 1941.

capacity but a relatively short period of time, the idea of the designer is to avoid all danger of cavitation, whereas the builder of high-speed hydraulic machinery is seeking the economic balance between operating revenue and cost of repairs. Consequently, he must strive for quantitative measurement, always seeking to evaluate risk of damage and resistance of materials, together with the value of increased capacity. The former is more interested in qualitative tests which give reasonable assurance that the design proving best in the model is the most suitable for the prototype. Under these conditions it would appear that the authors' experimental results have greater immediate practical application than their mathematical analysis.

The mathematical analysis affords an interesting approach that is not to be discredited by any means, for, even though exact agreement with practice can never be expected, it is true that only through mathematics does the investigator gain insight into the complicated play of factors which influence the results.

Obviously, the action that takes place is far too involved to permit prediction of the exact pressure, at any given region, in model or prototype, under any given conditions. However, the authors' development of the conception of a slug of water striking a converging passage explains, excellently, why it is that, when pitting has once started and has gained a little foothold, further damage is likely to progress at an accelerated rate. The test pieces removed from the Holtwood stand[17] showed deep pitting along the edge where the sides of the flume make a right angle.

It should be emphasized in addition that this multiplication of impact leads to tremendously high concentration of stress over an exceedingly limited area. This is well illustrated by the experiments cited in the paper,[5] in which ordinary methods of measuring high pressures such as are used in internal combustion engines, gun barrels, etc., failed completely as criteria of cavitation severity. Finally, the use of thin diaphragms, backed up by perforated plates, proved in vain because they were punctured merely by accidental hits like bullets rather than by a widespread blow.

The principal point of disagreement with the authors lies in their not entertaining the theory advanced by W. Watters Pagon,[18] M. Am. Soc. C. E., in 1935, at least as an alternative explanation. His conception of eddies as the fundamental form of the cavities demonstrates not only the possibility of extremely high local pressure concentrations, resulting from bombardment of the walls by particles propelled at terrific velocity generated by progressive but sudden collapse, but also explains the location of the more seriously pitted areas with relation to the form of the cavitation pocket.

The authors recognize that any mathematical treatment is beset with factors that are extremely difficult to evaluate since they enumerate the many things that must be neglected in order to justify equating kinetic energies. To this number should be added the rotational energy in the eddies.

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[17] "Pitting Resistance of Metals Under Cavitation Conditions," by J. M. Mousson, *Transactions*, A. S. M. E., July (1939), Vol. 59, No. 5, pp. 399–408.

[18] "Cavitation and Erosion Investigated as a Problem in Fluid Mechanics," by W. Watters Pagon (unpublished papers, A. S. M. E., 1935). On file for reference in Engineering Societies Library, 29 W. 39th St., New York, N. Y.

Although the action within the cavitation pocket, particularly at the downstream fringe, is extremely complicated, as shown by the high-speed moving pictures to which the authors refer,[5] the reader should not gain the impression of a confusion of slugs of water being hurled about promiscuously, these slugs striking with destructive violence upon the walls of the conduit. Careful study of these moving pictures, together with stop-motion stills and examination of the models in operation illuminated by ordinary light as well as by electric spark, reveals that at the upstream edge there seems to be an orderly leaping of the water away from the walls. Behind this region, when cavitation is well developed, there is undoubtedly a part of the channel boundary that is never wetted. The most important part of the pocket, from the point of view of damage, is where the stream apparently returns to the walls. If the phenomenon were perfectly continuous and could take place smoothly and uniformly, the stream would merely deliver a glancing blow with little probability of doing any harm.

Instead, the action is far from stable and is characterized by an alternate lengthening and shortening of the pocket so that the downstream boundary is a fringe of extreme turbulence. In this region slugs of water can be imagined as being hurled through an environment of water vapor; or the opposite, which would mean bubble-like cavities filled only with condensable water vapor tossed about in a seething mass of turbulent currents of water. The chances are that neither of these conceptions is entirely correct.

The point is that eddies are set up by viscous sheer and are shed by passing around the lip at the entrance where the water finally leaves the surface of the conduit. It has been shown mathematically, by Mr. Pagon and others, that at the heart of such eddies the pressure is much lower than in the surrounding environment. Consequently, when the absolute pressure drops, these eddies are open and can remain as open cavities even after the pressure is partly restored; but they are extremely unstable. It seems more likely that the final collapse of these cores is the direct cause of the destruction rather than any pelting by slugs of water as assumed by the authors. In support of this idea there is the observation that in two-dimensional, venturi-shaped passages the damage comes where the axes of the eddies would terminate against the side walls rather than on the profiles, which would be more likely to receive the onslaught of slugs of water.

Regardless of the theory of the collapse, there is an excellent field for further research. In the first place the authors' pressure gradient curve in Fig. 8 is well confirmed by other experiments and shows a rising pressure in a channel of uniform cross section. This cannot be explained by the principle of Bernoulli. What is the origin of this pressure increase?

Furthermore, a strong similarity has been noted between this pressure rise and the surface of the hydraulic jump. The action appears to be similar, at least from visual observation. In addition, it should be noted that in expanding passages both phenomena are stable. In passages having parallel sides both are particularly unstable. In one model which consisted of a venturi-shaped passage, terminating in a parallel section, there was a sharp increase in noise

as well as damage just as soon as the cavitation pocket was made to extend into the parallel section. This phenomenon was reported by W. Spannhake.[19] The disruptive effort seems to be definitely associated with the instability of flow.

A most important precaution taken by the authors was to free the water from dissolved air as much as possible. It is doubtful whether they were completely successful although the small volume of air remaining probably had no appreciable effect upon their results. It should be emphasized, however, that even a small amount of dissolved air is sufficient when very low pressures are encountered to upset the laws of hydraulic similitude. Under these conditions the water expands as an emulsion with the air coming out of solution. The result is that Q is no longer equal to $A \times V$. In hydraulic model testing, in which at any point there are pressures less than a half an atmosphere, there is danger of trouble from this source.

With further reference to the aforementioned hydraulic grade line, detailed exploration will show that the valley has a flat bottom rather than rounded as reported.

Although the authors are correct in stating that in practice it usually requires months to develop deep pitting in metal surfaces, it should not be overlooked that in certain laboratory tests it is possible to see the first signs of pitting after several seconds and to produce satisfactory results in the course of an hour.[20] Nevertheless, attempts to force metal to pit in models as it would be expected to do in service have not been particularly successful. Perhaps one of the most fruitful fields for further investigation is to establish relations between actual service and the results of various accelerated tests.

Many investigators have been forced to rely upon indications of surface damage, such as the removal of paint, and consequently the authors are to be congratulated for attaining any success at all in finding a material to simulate pitting. The principal difficulty would seem to lie in there being a threshold value of severity below which no damage is done but above which failure takes place with surprising rapidity. Quantitative measures of the extent of pitting either by loss of weight or volume are also very difficult.

In indicating an interest in cavitation outside of the field of hydraulic machinery, this paper suggests the possibility of even more extended application. So far, cavitation has always been regarded as the "villain in the plot"—the bane of the designer. The word cavitation itself carries a bad connotation. It would be interesting if some day a useful purpose could be found for it. Experimenters have noticed a similarity between cavitation and the hydraulic jump; furthermore, the hydraulic jump has been used to produce an intimate mixture of liquids.[21] Do not these two statements suggest that cavitation

[19] "Cavitation Research," Progress Report, by Carroll F. Merriam based on work of Wilhelm Spannhake, New England Hydraulic Power Committee of the New England Electrical Utilities Engrs., 1933. Filed for reference in Engineering Societies Library, 29 W. 39th St., New York, N. Y.

[20] "Progress Report on Cavitation Research at Massachusetts Institute of Technology," by J. C. Hunsaker, *Transactions*, A. S. M. E., Vol. 57, 1935, pp. 423–424; and "Determination of the Relative Resistance to Cavitation Erosion by the Vibratory Method," by S. Logan Kerr, M. Am. Soc. C. E., *loc. cit.*, Vol. 59, 1937, No. 5, pp. 373–397.

[21] "Computation of the Tailwater Depth of the Hydraulic Jump in Sloping Flumes," by Robert W. Ellms, *loc. cit.*, HYD–50–5.

might be used to secure an intimate mixture of two chemicals when, for some reason or other, it was desired to have the time of mixing reduced to an extremely small fraction of a second? Another possible application may be found in the fact that cavitation is capable of extracting dissolved gases from liquids. Air content in water is reduced in the cavitation zone faster than it can be dissolved in the following turbulent zone. When such gases have harmful effects, such as the oxidation of fats in milk, there is at least a possibility of the content being quickly and materially reduced by cavitation.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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DISCUSSIONS

RELIABILITY OF STATION-YEAR RAINFALL-FREQUENCY DETERMINATIONS

Discussion

BY EUGENE L. GRANT, M. AM. SOC. C. E.

EUGENE L. GRANT,[32] M. AM. SOC. C. E. (by letter).[32a]—The value of this excellent paper lies not only in its specific contributions but also in its implied suggestion that engineers dealing with problems of hydrology should be interested in statistical measures of the reliability of the conclusions of their studies. The station-year method presents two questions which constitute a challenge to the student of statistical technique. One question is whether or not the stations used are too far apart to be subject to the same cause system with regard to storms of the duration and intensity under consideration. The other question is whether the stations used are so close together that the same storm has been counted more than once. The Bartels' technique, explained in this paper, provides a satisfactory method of answering this second question.

One generalization which seems justified on the basis of the evidence now available is that the station-year method is particularly well adapted to dealing with high-intensity storms of short duration. The greater the intensity and the shorter the duration, the larger will be the area that seems to be statistically homogeneous and the smaller will be the area covered by an individual storm. It has already been shown[27] that a large area of Mid-Western United States is statistically homogeneous with regard to the frequency of certain extreme storms of 5-min to 120-min duration, although not so with regard to the frequency of less intense storms of the same duration. Table 2 shows how, in the Iowa and Carolina qudarangles studied by Mrs. Clarke-Hafstad, the value of N_d decreases rapidly with an increase in intensity; although a 1-in., 24-hr storm in Iowa affected more than 7 stations, a 4-in., 24-hr storm affected only one station. An examination of storm maps such as Fig. 1 serves to confirm the

NOTE.—This paper by Katharine Clarke-Hafstad was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by Paul V. Hodges, M. Am. Soc. C. E.; February, 1941, by Messrs. C. S. Jarvis, and Howard W. Brod; and March, 1941, by Messrs. Merrill Bernard, and Charles F. Ruff.

[32] Prof. of Economics of Eng., Dept. of Civ. Eng., Stanford Univ., Stanford University, Calif.

[32a] Received by the Secretary March 14, 1941.

[27] "Rainfall Intensities and Frequencies," by A. J. Schafmayer, M. Am. Soc. C. E., and B. E. Grant, *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 344.

view that, the greater the intensity, the smaller will be the area that a storm is likely to cover.

As the author states, the dense networks of rain gages now established in several areas may be expected to provide better estimates of the areas associated with storms of various degrees of intensity. With regard to the very intense storms that cover only a few square miles, these networks will provide many independent station years of record in a very short time.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

PLASTIC THEORY OF REINFORCED CONCRETE DESIGN

Discussion

By Messrs. Paul Andersen, and R. A. Caughey

Paul Andersen,[49] Assoc. M. Am. Soc. C. E. (by letter).[49a]—The conventional method of designing reinforced concrete is unsatisfactory. Attempts have been made to eliminate from the analysis the modular ratio, n. At present, columns concentrically loaded are analyzed by means of formulas that carefully avoid this quantity. If, however, the load is moved slightly off center so as to produce combined flexure and compression, the analysis is immediately changed to one that makes extensive use of the modular ratio. Therefore, the author should be congratulated upon his efforts in correcting these contradicting procedures and presenting methods of analysis consistent for all types of members.

The plastic theory encourages the use of much higher steel percentages than the usual straight-line theory. According to the former theory, the critical percentage of steel required to develop the full compressive strength of the concrete is expressed by Eq. 13.

According to the straight-line theory this percentage can be found by equating total tension and total compression; thus: $\frac{1}{2}\,k\,j\,b\,d^2\,f_c' = A_s\,f_s\,j\,d$, and

$$p_0 = \frac{A_s}{b\,d} = \frac{k}{2} \times \frac{f_c'}{f_s} = \frac{n\,f_c'}{2(n\,f_c' + f_s)} \qquad (43)$$

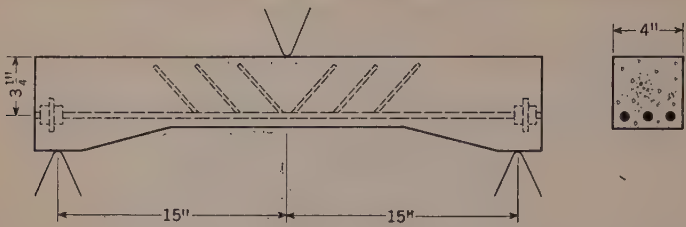
To compare Eqs. 13 and 43 to actual tests, six beams with varying steel percentages were made up and tested at the age of 28 days in the Materials Testing Laboratory of the University of Minnesota, Minneapolis, Minn. In each beam the bottom reinforcement differed, as shown in Table 12. The compressive strength of the concrete (average of two cylinders) was 4,060 lb per sq in.; the secant modulus of elasticity at 75% of ultimate strength was 3.67×10^6 lb per sq in.; and the yield point of the reinforcement was 47,000

Note.—This paper by Charles S. Whitney, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by Messrs. L. E. Grinter, and Basil Sourochnikoff; and March, 1941, by Messrs. R. W. Stewart, George C. Ernst, Homer M. Hadley, and Robert W. Abbett.

[49] Associate Prof., Structural Eng., Univ. of Minnesota, Minneapolis, Minn.

[49a] Received by the Secretary February 20, 1941.

TABLE 12.—DETAILS AND SCHEDULE OF REINFORCEMENT, TEST DATA PLOTTED IN FIG. 14



Description	Beam No. 1	Beam No. 2	Beam No. 3	Beam No. 4	Beam No. 5	Beam No. 6
Number of bars	2	3	2	3	2	3
Diameter of bars, in inches . . .	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$

lb per sq in. Bending moments at failure plotted against steel ratios are shown in Fig. 14. Critical steel percentages according to Eqs. 13 and 43 are indicated.

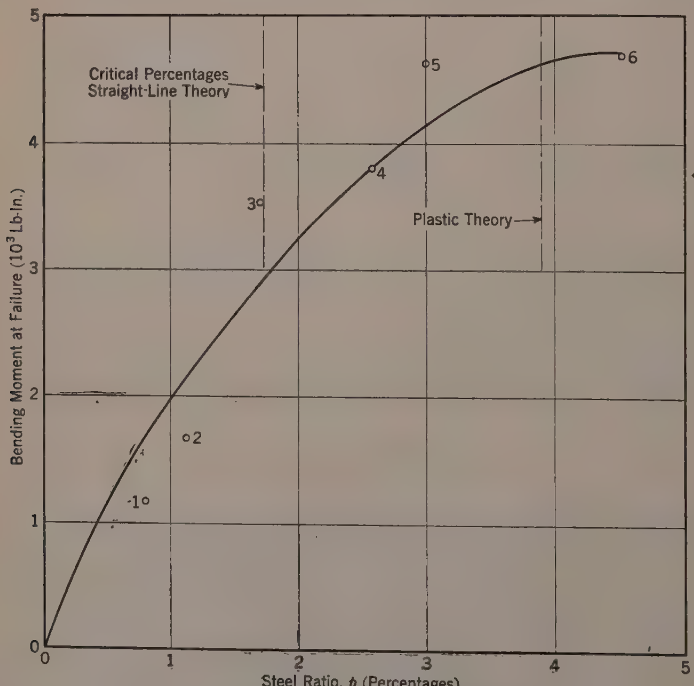


FIG. 14.—TESTS ON BALANCED REINFORCEMENT

These tests, together with those cited by the author, support the contention that the conventional theory does not permit percentages of steel sufficient for balanced design.

The question that will arise immediately after the adoption of a plastic theory for the proportioning of reinforced concrete sections is whether the determination of moments and shears should not be by a procedure based on limit design rather than elastic theory. If the different parts of the structure are designed for forces existing prior to failure it appears logical to determine the distribution of these forces on the same basis.

R. A. Caughey,[50] M. Am. Soc. C. E. (by letter).[50a]—The attention of the engineering profession has been directed to a problem that should receive serious attention and that should call forth various suggestions for the improvement of certain phases of reinforced concrete design. The writer, however, feels that the problem is not finally solved and that certain details should, and will, receive attention before the proposed method is considered for general use.

From the viewpoint of an engineering teacher, the writer feels that, in beam design, it would be very difficult to substitute a rectangle for the present stress diagram and to tell students that "The use of the rectangle is merely a mathematical device to approximate the effect of the true distribution." Most students in reinforced concrete classes have tested cylinders in concrete materials courses and know that the relation between stress and strain in concrete is a difficult problem; but they might reasonably expect a stress-strain curve closer to the results of the laboratory than would be afforded by this "device." Table 1 seems to suggest that, outside of the method suggested, a return to the parabolic formula would be a solution in the case of beams; but designers generally, recognizing the variations in the material, uncertainties in the flexural behavior of the beam, etc., would scarcely desire such a change.

It should be noted in the case of beams that the author's comparisons are made between ultimate-strength values (see Tables 1 and 2) when what the engineer desires to consider in concrete design is what the concrete does under working stresses. Fig. 1 seems to indicate that, up to half the ultimate strength, the stress-strain curve approaches a straight line. True enough, plastic flow is not taken into account, but neither does the author's method consider the effect of plastic flow under working loads. It seems to the writer that designers have had enough test data and experience with design, by the method of straight-line variation and working stresses, to warrant confidence in design but they do need some information on the effect of plastic flow under working loads. Mr. Whitney attempts to make allowance for the plastic flow by use of the rectangle but confines his attention to consideration of ultimate loads.

It will be noted that all of the formulas of the paper are based on ultimate strengths. At first this would seem to be desirable in these days of rigid frames but the designer still has the moments of inertia of the members of frames to deal with in determining bending moments; and it would seem to offset, to some extent, the advantages of an ultimate-strength method to use

[50] Prof. of Structural Eng., Dept. of Civ. Eng., Iowa State Coll., Ames, Iowa.

[50a] Received by the Secretary March 3, 1941.

moments of inertia which, in turn, infer straight-line variation in concrete stresses.

Dropping the use of working stresses would also be objectionable because members of all other structural materials are designed on that basis. Most designers have regarded the abandonment of the ultimate-strength method of design as a very desirable step and would scarcely welcome a return to that method. Concrete design at present, with available curves, tables, etc., compares well in simplicity with design of other materials; and this simplicity with similarity in design methods to methods used in other materials is very desirable.

What the writer has stated thus far applies principally to beams. It is a generally recognized fact that structural engineers have gone far in considering plastic flow in formulas for columns[51] carrying concentric loads. Stress distribution over the section of a concentrically loaded column, however, is a problem that differs from the case of a beam. The unit stress in the concrete is practically the same over the entire section whereas, in the case of the beam, it presents a major problem. The writer feels that more tests of columns under combined direct stress and moment should be made, with the idea of determining strain distribution under working loads. With the distribution of strains determined, it might be possible to extend the work of J. R. Shank,[52] M. Am. Soc. C. E., and others to the determination of stresses with plastic flow taken into account.

The aforementioned method might be used in the case of beams, possibly by arriving at some method of adjusting the value of n so that the present convenient method of design might be more satisfactory.

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[51] Final Report of Committee 105, Reinforced Column Investigation, *Proceedings*, Am. Concrete Inst., Vol. 29, p. 275.

[52] "The Mechanics of Plastic Flow of Concrete," by J. R. Shank, *Proceedings*, Am. Concrete Inst., Vol. 32, pp. 149–180.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

EXPANSION OF CONCRETE THROUGH REACTION BETWEEN CEMENT AND AGGREGATE

Discussion

BY MESSRS. HUBERT WOODS, AND N. T. STADTFELD

HUBERT WOODS,[11] ESQ. (by letter).[11a]—Unquestionably, this paper provides, for the first time, an adequate explanation of a number of baffling cases of concrete deterioration that have occurred in the coastal region of California. It also provides new incentive for a more comprehensive study of aggregates in their relation to cement composition. There is great need for further studies on the chemical nature of the expansive reaction between certain natural minerals and the alkalis in cements. Only when this reaction is well understood can intelligent steps be taken to forestall or prevent it.

It would be as unreasonable to conclude that all aggregates will react expansively with alkalis in cement as to conclude that only those aggregates peculiar to the coastal area of California will do so. The widespread satisfactory behavior of concrete, in general, attests the fact that such troubles are not omnipresent, and where local aggregates and cements have had a long record of satisfactory performance there is no reason for alarm. On the other hand, with aggregate from a new source, without adequate history of performance, a thorough study is justified, and there is great need of a rapid test by which such aggregates may be judged with respect to possible expansive reaction with alkali.

In cases where the only aggregates economically available are likely to give trouble through reaction with alkalis in cement, it might be concluded that a simple solution is to put a limit on the alkali content of cement. As a matter of fact, at present that is about all that can be done; but it is not a satisfactory final solution. It is not economically desirable. All cements made in the United States contain more or less alkali, not by intent but by reason of the fact that nature has put alkali-containing minerals in practically

NOTE.—This paper by Thomas E. Stanton, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by R. W. Carlson, Assoc. M. Am. Soc. C. E.; and March, 1941, by Bailey Tremper, Esq.

[11] Chf. Chemist, Riverside Cement Co., Riverside, Calif.

[11a] Received by the Secretary March 7, 1941.

all clays and shales used in cement making. The alkalis are partly, but not wholly, volatilized in the rotary kilns. In order to reduce the alkalis in his cement, the manufacturer may have to use other raw materials containing less of the alkali minerals, and these may be very difficult to find and very expensive to use. It would be highly desirable to find some inexpensive means, perhaps by additions to the cement, whereby the expansive reaction would be circumvented, and a diligent search for such means would seem to be essential.

N. T. Stadtfeld,[12] M. Am. Soc. C. E. (by letter).[12a]—In this paper Mr. Stanton has made a valuable contribution, based on a painstaking investigation. He eliminated all other possible factors to tie in definitely the disintegration of concretes containing certain aggregates with the alkali content of the portland cement used.

Although they are present in small quantity, the alkalis, nevertheless, have been under suspicion for a number of years. In 1929 the late Thaddeus Merriman,[13] M. Am. Soc. C. E., gave the results of an extensive investigation, which had led him to conclude that the rate of formation of the hydration products obtained when water is added to cement is dependent on the alkalinity of the solution. Within a certain zone of alkalinity, hydration is slow and orderly; but alkalinities either above or below this range were found to increase the rate of hydration. Thus the addition of calcium chloride to the mixing water, which lowers the alkalinity, will accelerate the hardening process. Similarly, the addition of sodium hydroxide, which increases the alkalinity, not only speeds up the hardening process but increases the maximum hydration temperature, as measured by a calorimeter in which a cement paste is hardening. After several years of investigation and numerous tests on the sulfate-resistant cement used in the tunnels and spillway of the Fort Peck (Mont.) Dam, Mr. Merriman finally embodied a restriction on the soluble alkali content in the specifications for portland cement of the Board of Water Supply (BWS) of the City of New York.[6] Cement to the extent of three million barrels made at 10 plants, complying with these specifications, has now been delivered to the various sections of the Delaware aqueduct. The principle involved in restricting the alkali content to obtain a uniform rate of hydration and to protect men against alkali burns when they work the wet concrete is different from the principle involved in restricting alkali content because of the deleterious effect on the concrete when used in conjunction with certain aggregates. Nevertheless, the fact remains that a well-defined beginning has been made to produce, consistently, a portland cement of a low-alkali type; and therefore, experience gained during four years of procedure may be of interest.

It is true that, whereas the BWS specifications confine their restrictions to water-soluble alkalis, the author uses the total alkali content, computed as

[12] Div. Engr., Board of Water Supply, New York, N. Y.

[12a] Received by the Secretary March 10, 1941.

[13] "Cement," by Thaddeus Merriman, Paper No. 308, *Proceedings of the World Engineering Congress*, Tokyo, 1929, Vol. XXXI, p. 345.

[6] Specifications for Cement, Board of Water Supply, City of New York.

sodium oxide (Na_2O), as the criterion of fitness. In his investigation these two did not always keep pace with one another, but records from plants producing BWS cement do show a close correlation. In addition, some records from plants not making BWS cement, which were made available to the writer, also show substantial correlation. In every case, cement passing BWS specifications would have been under the 0.5% Na_2O limitation, and cement failing in one would have failed in the other.

Table 6 gives the results of a series of tests run at one of the plants which consistently meet the BWS specifications. The raw mix was kept constant and, as the temperature was lowered, samples were removed and alkali determinations made. Although no accurate record of the temperatures was made, the samples were arranged in accordance with descending temperature.

TABLE 6.—Comparison of Total Alkalis with Free Alkalis (Water Soluble)

Method	Sample Numbers:									
	1	2	3[a]	4[a]	5[a]	6[a]	7	8	9	10
Free alkali[b] (cc)	1.20	1.70	1.80	1.80	3.95	3.95	6.40	6.80	11.30	11.40
Smith[c] (%):										
Na_2O	0.16	0.20	0.17	0.18	0.23	0.19	0.32	0.19	0.31	0.29
K_2O	0.26	0.27	0.29	0.28	0.36	0.36	0.53	0.55	0.72	0.79
Computed as Na_2O	0.38	0.42	0.41	0.41	0.53	0.49	0.76	0.65	0.91	0.95

[a] Sample 4 is a duplicate of Sample 3; and Sample 6 is a duplicate of Sample 5. [b] Merriman method. [c] J. Lawrence Smith method (see Scott's "Standard Methods of Chemical Analysis," 5th Ed., 1939, p. 1612).

It is apparent that potassium can be more readily eliminated by temperature increases than sodium; this was to be expected as potassium compounds volatilize at lower temperatures. Note the decrease in potassium content from 0.79% to 0.26% and the decrease in sodium content from 0.29% to 0.16%, a ratio of more than 3 to 1 for the potassium to a ratio of less than 2 to 1 for the sodium. Also note that, as the author's limit of 0.5% (total alkalis) is approached, the titration value of 3.5 for the BWS specifications is similarly reached. The method of determination is described in the specifications, as follows:

> "The alkalinity of the cement shall not be greater than 3.8 and its content of free alkali shall not exceed 3.5. These characteristics shall be determined as follows: Weigh out 800 grams of cement and put into an enameled saucepan with 500 c.c. of distilled water. Stir frequently for two hours, then filter through large folded filter paper for 10 minutes. If filtrate is not clear, refilter. Titrate 25 c.c. of the filtrate with N/2 HCl, using methyl orange as the indicator. The number of c.c. of acid required to neutralize the filtrate to the methyl orange end point is the measure of the alkalinity.
>
> "The free alkali content of the cement shall be determined as follows: Measure out 100 c.c. of the filtrate obtained in the alkalinity test of the preceding paragraph into a small beaker, and add 30 to 35 c.c. of a saturated filtered solution of $Ba(OH)_2$. Let stand, filter and wash with H_2O. Pass CO_2 into the filtrate for five minutes. Let stand, filter and wash the

precipitate with H_2O. Heat to boiling and, if a precipitate forms, filter it out. Then boil the total filtrate plus wash water down to about 50 c.c., filter it and make it up to 100 c.c. with distilled water. Now take 25 c.c. of the solution and titrate it with N/2 HCl in the presence of methyl orange. The number of c.c. of the acid required to neutralize the 25 c.c. to the end point of the methyl orange is the measure of the free alkali content. Both of these tests are to be made at room temperature."

Close correlation holds true at other plants meeting the specifications. The opinion may be ventured that this is due to the fact that in all cases the so-called "flue dust" is discarded. "Flue dust" is the fine material blown from the rear end of a rotary kiln and caught under waste-heat boilers, in precipitators, or by other means. In many cases it is fed back to the raw mix and sometimes to the finished cement. In the former instance it builds up the alkali content, occasionally to such an extent that even with high temperatures it becomes impossible to clinker the mix properly, thus causing waste in fuel and lowering of quality. In the latter case it tends to increase the alkali content of the cement and mainly the non-water soluble alkali content.

Had the paper included such items in the history of the clinker as burning-zone temperature, speed of kiln rotation, subsequent treatment of the clinker, and a notation as to the disposition of the "flue dust," additional and valuable deductions might have been made.

A plea is made here for placing restrictions on water-soluble alkalis rather than on the total alkalis, because the determination of the former is extremely simple and excellent checks are obtained between laboratories, whereas determination of the latter is laborious ($3\frac{1}{2}$ days for an experienced chemist) and checking between laboratories is difficult.

Regardless of the form which the restrictions on alkalis may take, however, some additional burden is likely to be placed on producers, as disintegration of parts of the Parker Dam[14] in southern Arizona has been traced recently to the destructive action of the alkalis in the portland cement on a hydrous silica occurring in the aggregate, an aggregate which had passed all known tests for durability. A trend may well be expected toward the incorporation of low alkali content in future specifications for cement as a safeguard against possible disintegration. With this in mind a review of some of the difficulties encountered in manufacturing such cement may be of value.

All BWS cement is made from well-burned clinker assured by:

(1) Insistence on continuous high temperature (about 2,700° F) in the burning zone of the kiln, as recorded by pyrometers placed outside and in front of the kilns;

(2) Insistence on regularity of kiln rotations coordinated with the occasional rise and fall of the temperature; and

(3) Insistence on satisfactory performance in the "sugar test" which indicates the degree of completion with which the various constituents in the clinker have been combined.

[14] See minutes of December 6, 1940, meeting of Committee C-1 on cement, A.S.T.M., p. 5.

The BWS test for sugar solubility is described in the specifications as follows:

> "The sugar solubility of the cement shall not be greater than 8.0 to the phenolphthalein end point, nor greater than 10.0 to the final clear point. These values shall be determined as follows: A sample of about 100 grams of the cement is passed through the 200-mesh screen and put into a glass bottle closed with a rubber stopper. From this bottle 15 grams are then weighed out and placed into a Nessler tube containing 100 c.c. of a 15-percent. solution of cane sugar in distilled water (commercial granulated sugar such as 'Jack Frost'); this solution shall not be more than three days old. The tube and its contents are then quickly shaken by hand and placed on a wheel revolving about 60 times per minute. At the end of about 1 hour and 50 minutes the mixture is poured into a filter paper and funnel and allowed to filter for ten minutes when the beaker containing the filtrate is removed. (In case the filtration time of 10 minutes is too short to produce a volume of filtrate of 30 c.c., it may be lengthened by shortening the time of shaking, but the total time from the putting of the sample into the solution to the end of the filtration must be exactly 2 hours.) Twenty-five c.c. of the filtrate is now titrated with N/2 HCl in the presence of phenolphthalein and the number of c.c. of acid to the end point is the first measure of the sugar solubility. At this stage of the test, in the case of a thoroughly burned cement which has been kept dry, the solution will be practically clear and only traces of ferric oxide and alumina will be in suspension. In the case of an under-burned cement, or one which has been exposed to moisture, the solution will be heavily clouded and the end point must be approached slowly and with caution. When so performed, the phenolphthalein end point can be definitely determined, as the color changes from light pink to yellow. The titration is then continued until the solution is crystal clear and nothing remains in suspension. The total number of c.c. of acid from the beginning of the titration to this final clear point is the second measure of the sugar solubility. This test shall be made at room temperature. In addition to disclosing the quality of the cement as above stated, this test further indicates the character of the hydration products which will be realized in the completed concrete."

In most cases adherence to principles (1) to (3) resulted in a cement of satisfactory alkali content. However, it was found in a few of the plants that with proper burning and low sugar-test results the alkali content was higher than allowed by the specifications. After some experimentation it developed that, where potassium was present, shortening of the burning flame and consequently impinging it on the hot clinker bed would drive off this element. This could be noted through blue glasses, which showed up the unmistakable purple hue of potassium vapors.

Where even this technique was insufficiently effective, probably due to the presence of larger proportions of sodium (which, as previously mentioned, does not respond as well as potassium), addition up to 0.5% by weight of calcium chloride was made to the raw mix, which changed some of the alkalis, in whatever form present, to alkali chlorides. These volatilize easily and appear to be driven off completely, as none of the resultant cement showed any trace of chlorides, but did show a large reduction in alkali content.

To summarize, therefore, with present knowledge based on four years of experience, the following is suggested for the consistent production of cement low in alkali content:

(*a*) Discard all "flue dust" and incidentally save on fuel;

(*b*) Install pyrometers and kiln-rotation recorders and note the alkali content with varying burning temperatures;

(*c*) Experiment with length of flame in the burning zone; and

(*d*) As a last resort, add calcium chloride up to 0.5% by weight to the raw mix.

In conclusion it may be stated that, although the disintegration of concrete, as found by the author and in the Parker Dam, may well be due to the action of alkalis in cement on localized aggregates not generally encountered, the limitation on the alkali content should be encouraged in any case for additional safeguard and the production of a decidedly better product, as outlined in the first part of this discussion.

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DISCUSSIONS

ANALYSIS OF STATICALLY INDETERMINATE TRUSSED STRUCTURES BY SUCCESSIVE APPROXIMATIONS

Discussion

BY MESSRS. CHARLES A. ELLIS, AND DAVID J. PEERY

CHARLES A. ELLIS,[8] M. AM. SOC. C. E. (by letter).[8a]—There are three principles by which a statically indeterminate structure may be analyzed: (1) Principle of work; (2) principle of least work; and (3) principle of deflections.

The first two are essentially mathematical abstractions of horse-and-buggy days. They do not permit physical conception or space perception, so helpful in the comprehension of stress analysis. The principle of deflections is far

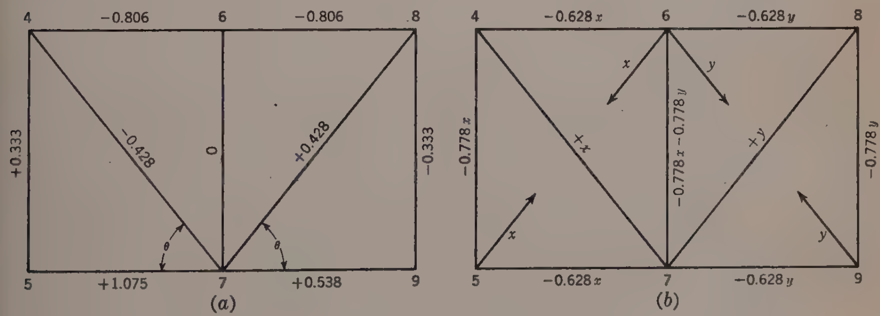

FIG. 9

superior, and analysis may be made by at least three methods: (a) Area-moments; (b) expression of moments in terms of angular rotations of joints and members (generally known as the "slope-deflection method"); and (c) moment distribution. The two latter methods are most easily derived by the area-moment method which the writer considers the closest to fundamentals.

NOTE.—This paper by O. T. Voodhigula, Jun. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Francis L. Castleman, Jr., Assoc. M. Am. Soc. C. E.

[8] Prof., Structural Eng., Purdue Univ., West Lafayette, Ind.

[8a] Received by the Secretary February 3, 1941.

The principles of work and least work were the only ones available for an algebraic solution of statically indeterminate trusses until the "Williot Strain Equations"[9] were discovered by the writer, when computing the tower stresses in the Golden Gate Suspension Bridge. These equations will now be used in solving the author's Example 1.

The stresses in the necessary members (redundant members removed) are shown in Fig. 9(*a*); and the stresses due to stresses x and y, respectively, in the redundant members are shown in Fig. 9(*b*). The combined stresses for each member are given in Table 4, in which s represents the stress and Δ the strain. Hence,

$$E\,\Delta = \frac{s\,l}{A} \quad \ldots\ldots(12)$$

TABLE 4.—COMBINED STRESSES FOR MEMBERS IN FIG. 9

Member	s	$\frac{l}{A}$	$E\,\Delta = \frac{s\,l}{A}$
4–6	$-0.806 \quad -0.628\,x$	5.39	$-\,4.344 \quad -\,3.385\,x$
5–7	$+1.075 \quad -0.628\,x$	5.97	$+\,6.418 \quad -\,3.749\,x$
4–5	$+0.333 \quad -0.778\,x$	18.15	$+\,6.044 \quad -14.121\,x$
6–7	$-0.778\,x - 0.778\,y$	18.76	$-14.595\,x - 14.595\,y$
4–7	$-0.428 \quad +\,x$	24.10	$-10.315\,x + 24.100\,x$
5–6	$+\,x$	24.10	$+24.100\,x$
6–8	$-0.806 \quad -0.628\,y$	5.39	$-\,4.344 \quad -\,3.385\,y$
7–9	$+0.538 \quad -0.628\,y$	5.97	$+\,3.212 \quad -\,3.749\,y$
8–9	$-0.333 \quad -0.778\,y$	18.15	$-\,6.044 \quad -14.121\,y$
7–8	$+0.428 \quad +\,y$	24.10	$+10.315 \quad +24.100\,y$
6–9	$+\,y$	24.10	$+24.100\,y$

In any rectangular panel,[9] the sum of the strains in the diagonals multiplied by cosec θ equals the sum of the strains in the horizontals multiplied by cot θ plus the sum of the strains in the verticals, or

$$(\Delta_{47} + \Delta_{56}) \operatorname{cosec} \theta = (\Delta_{57} + \Delta_{46}) \cot \theta + \Delta_{45} + \Delta_{67} \quad \ldots\ldots(13a)$$

and

$$(\Delta_{69} + \Delta_{78}) \operatorname{cosec} \theta = (\Delta_{79} + \Delta_{68}) \cot \theta + \Delta_{67} + \Delta_{89} \quad \ldots\ldots(13b)$$

Substituting from Table 4,

$$(-10.315 + 48.2\,x)\,1.285 = (2.074 - 7.134\,x)\,0.8065 + 6.044 - 28.716\,x - 14,595\,y$$

and

$$(+10.315 + 48.2\,y)\,1.285 = (-1.132 - 7.134\,y)\,0.8065 - 6.044 - 14.595\,x - 28.716\,y$$

whence

$$96.407\,x + 14.595\,y = +\,20.972 \quad \ldots\ldots(14a)$$

and

$$96.407\,y + 14.595\,x = -\,20.212 \quad \ldots\ldots(14b)$$

or $x = +\,0.2551$; and $y = -\,0.2483$. Eqs. 13 may be written in the form

$$x = +\,0.2175 - 0.1514\,y \quad \ldots\ldots(15a)$$

and

$$y = -\,0.2097 - 0.1514\,x \quad \ldots\ldots(15b)$$

Any one with leisure time may solve for x and y by successive approximations, as follows: Assume $y = 0$ in Eq. 15*a* and $x = 0$ in Eq. 15*b*; then:

[9] "Simplified Analysis of Indeterminate Frames," by Charles A. Ellis, *Engineering News-Record*, April 26, 1934, p. 534; also "Williot Equations for Statically Indeterminate Structures," by Charles A. Ellis, *Transactions*, Am. Soc. C. E., Vol. 100 (1935), p. 580.

$x = +0.2175$; and $y = -0.2097$. Substitute $y = -0.2097$ in Eq. 15*a* and $x = +0.2175$ in Eq. 15*b*; whence: $x = +0.2492$; and $y = -0.2426$. Repeat this process three times and the following successive values appear:

x	y
+0.2175	−0.2097
+0.2492	−0.2426
+0.2542	−0.2474
+0.2550	−0.2482
+0.2551	−0.2483

These same numerical values appear in Fig. 3.

Comment on solutions by successive approximations seems relevant. This process has received much attention since Professor Cross published his moment-distribution method[4] and used successive approximations as a tool; and it is an exceedingly efficient tool, especially when the unknowns are numerous. Moment distribution has no monopoly, however. Slope-deflection equations invariably have the same peculiarity that renders them susceptible to the same rapidity of convergence when solved by successive approximations, as when moment distribution is used.

Consider any problem in which rotation of joints, but no rotation of members, is assumed and solve by moment distribution. Then solve by slope deflection using successive approximations in the solution of the equations and note that identical numerical values appear in both methods.

When the unknowns are numerous, the method of successive approximations is an exceedingly efficient tool; but for many writers it has become a toy to be used whenever and wherever possible. For example: In a box culvert having large fillets with increasing moments of inertia at the ends of the members, wherein the carry-over factor is as large as three fourths, resulting in relatively slow convergence, the simultaneous solution of four slope-deflection equations is the method to be preferred.

Perhaps the most risible example is the so-called rigid frame—the two-hinged rectangular reinforced concrete arch. One and only one elastic equation[10] is required for finding the corner moment for each condition of loading, whether dead or live. The solution of such a problem, containing but one unknown quantity by any method where successive approximations may or can be used, is something fancy, and, if one reveres mathematical accuracy, the elastic equations can take into account the curvature of the deck axis, whereas the slope deflection and moment distribution are impotent to do so.

Finally the writer expresses doubt in the author's conclusion that the principle of work combined with successive approximations is useful in a better understanding of the structure.

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[4] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 96 (1932), p. 1.

[10] "Two Hinged Rectangular Arches Having Variable Moments of Inertia," *Bulletin No. 79*, Eng. Experiment Station, Purdue Univ., Lafayette, Ind.

David J. Peery,[10] Jun. Am. Soc. C. E. (by letter).[10a]—A method of analyzing statically indeterminate trussed structures is presented in this paper that is similar to modern methods of analyzing continuous beams and rigid-frame structures. The author points out that for trussed structures it seems better to solve simultaneous equations set up by classical strain-energy methods than to use direct methods of successive approximation in distributing stresses. The simultaneous equations are set up by customary methods, therefore, and a method of successive approximations is applied to their solution.

The method of solving the simultaneous equations is essentially the method of iteration, or successive substitutions. It has been in use for some time by structural engineers for the solution of similar types of equations obtained by the use of three-moment equations, slope-deflection equations, or moment distribution.[11,12] For equations in which the convergence of the proposed method is rather slow, the method proposed by George H. Dell,[13] Assoc. M. Am. Soc. C. E., in which a series summation is used after the first few cycles of substitution, will materially reduce the labor involved.

The examples chosen by the author yield systems of simultaneous linear equations in which the coefficients belonging to the principal diagonal of the matrix are much greater than the other coefficients. The solution of such equations by successive approximation converges rapidly. Most problems in indeterminate analysis are of this type, or can be made so by a proper selection of the redundants. In Example 2, for instance, the redundants are such that the equations are much simpler than for the general structure with eight redundants. If the expressions of Eqs. 3 are expanded, they yield, for the general case of a structure with eight redundants:

$$\left.\begin{aligned}
-\delta_{a0} &= X_a\delta_{aa}+X_b\delta_{ab}+X_c\delta_{ac}+X_d\delta_{ad}+X_e\delta_{ae}+X_f\delta_{af}+X_g\delta_{ag}+X_h\delta_{ah}\\
-\delta_{b0} &= X_a\delta_{ba}+X_b\delta_{bb}+X_c\delta_{bc}+X_d\delta_{bd}+X_e\delta_{be}+X_f\delta_{bf}+X_g\delta_{bg}+X_h\delta_{bh}\\
-\delta_{c0} &= X_a\delta_{ca}+X_b\delta_{cb}+X_c\delta_{cc}+X_d\delta_{cd}+X_e\delta_{ce}+X_f\delta_{cf}+X_g\delta_{cg}+X_h\delta_{ch}\\
-\delta_{d0} &= X_a\delta_{da}+X_b\delta_{db}+X_c\delta_{dc}+X_d\delta_{dd}+X_e\delta_{de}+X_f\delta_{df}+X_g\delta_{dg}+X_h\delta_{dh}\\
-\delta_{e0} &= X_a\delta_{ea}+X_b\delta_{eb}+X_c\delta_{ec}+X_d\delta_{ed}+X_e\delta_{ee}+X_f\delta_{ef}+X_g\delta_{eg}+X_h\delta_{eh}\\
-\delta_{f0} &= X_a\delta_{fa}+X_b\delta_{fb}+X_c\delta_{fc}+X_d\delta_{fd}+X_e\delta_{fe}+X_f\delta_{ff}+X_g\delta_{fg}+X_h\delta_{fh}\\
-\delta_{g0} &= X_a\delta_{ga}+X_b\delta_{gb}+X_c\delta_{gc}+X_d\delta_{gd}+X_e\delta_{ge}+X_f\delta_{gf}+X_g\delta_{gg}+X_h\delta_{gh}\\
-\delta_{h0} &= X_a\delta_{ha}+X_b\delta_{hb}+X_c\delta_{hc}+X_d\delta_{hd}+X_e\delta_{he}+X_f\delta_{hf}+X_g\delta_{hg}+X_h\delta_{hh}
\end{aligned}\right\}\quad\ldots(16)$$

Substituting the numerical values from Example 2:

$$\left.\begin{array}{lllllllll}
-140.08 = & 79.96\,X_a & +20.45\,X_b & +\;0 & +\;0 & +\;0 & +\;0 & +\;0 & +\;0\\
-169.60 = & 20.45\,X_a & +93.86\,X_b & +\;20.45\,X_c & +\;0 & +\;0 & +\;0 & +\;0 & +\;0\\
-193.39 = & 0 & +20.45\,X_b & +112.86\,X_c & +\;20.45\,X_d & +\;0 & +\;0 & +\;0 & +\;0\\
-206.90 = & 0 & +\;0 & +\;20.45\,X_c & +123.68\,X_d & +\;20.45\,X_e & +\;0 & +\;0 & +\;0\\
-206.90 = & 0 & +\;0 & +\;0 & +\;20.45\,X_d & +123.68\,X_e & +\;20.45\,X_f & +\;0 & +\;0\\
+\;11.30 = & 0 & +\;0 & +\;0 & +\;0 & +\;20.45\,X_e & +112.86\,X_f & +20.45\,X_g & +\;0\\
+508.60 = & 0 & +\;0 & +\;0 & +\;0 & +\;0 & +\;20.45\,X_f & +93.86\,X_g & +20.45\,X_h\\
+336.40 = & 0 & +\;0 & +\;0 & +\;0 & +\;0 & +\;0 & +20.45\,X_g & +79.96\,X_h
\end{array}\right\}(17)$$

[10] Stress Analyst, Curtiss-Wright Corp., Robertson, Mo.; in Chg. Eng. Defense Training Courses in Airplane Stress Analysis, Washington Univ., St. Louis, Mo.

[10a] Received by the Secretary February 26, 1941.

[11] "On a Simple Method for Solving Simultaneous Linear Equations by a Successive Approximation Process," by J. Morris, *Journal*, Royal Aeronautical Soc., April, 1935, p. 349.

[12] "Analysis by Moment Distribution Aided Through Use of Iteration," by A. Floris, *Engineering News-Record*, June 25, 1936, p. 922.

[13] "Solution of Equations in Structural Analysis by Converging Increments," by George H. Dell, *Transactions*, Am. Soc. C. E., Vol. 104 (1939), p. 1543.

It should be noticed that 42 of the 64 coefficients of the unknowns are zero, resulting in an enormous simplification of the solution.

The solution of Eqs. 17 is quite easy by the common methods of elimination or determinants, whereas, if none of the coefficients of Eqs. 16 were zero, the solution would be quite laborious by successive approximations, and practically impossible by elimination or determinants. It seems obvious, therefore, that it is more important to select the redundants in such a manner as to simplify Eqs. 3 than it is to simplify the actual solution of the equations.

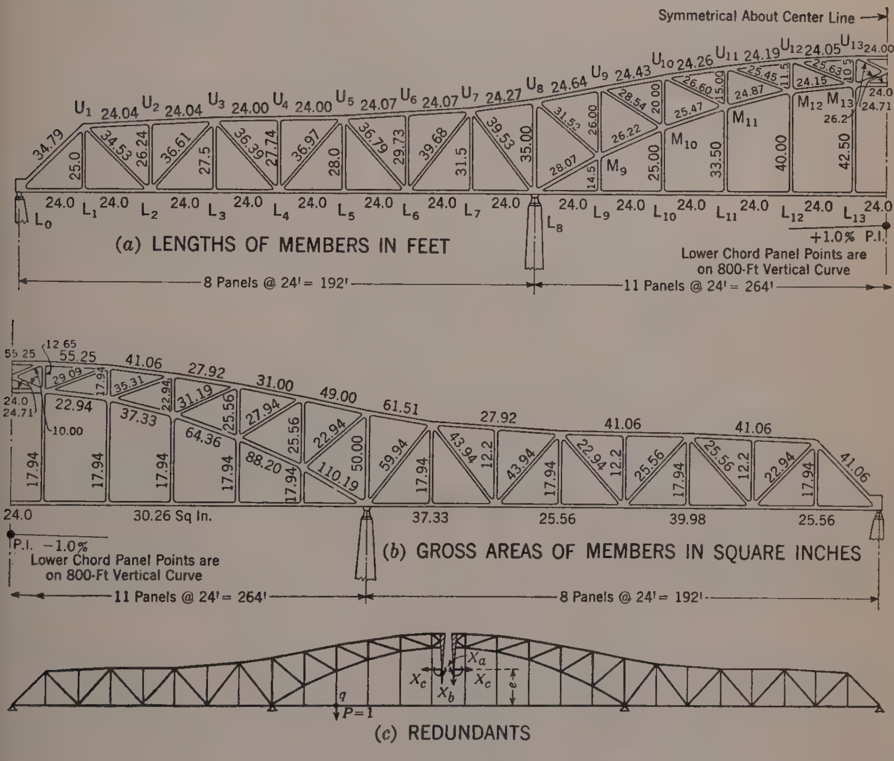


FIG. 10.—CONTINUOUS TIED-ARCH BRIDGE

If the redundants are selected so that all the coefficients δ_{mn} having unlike subscripts are zero, Eqs. 3 will have the following solutions:

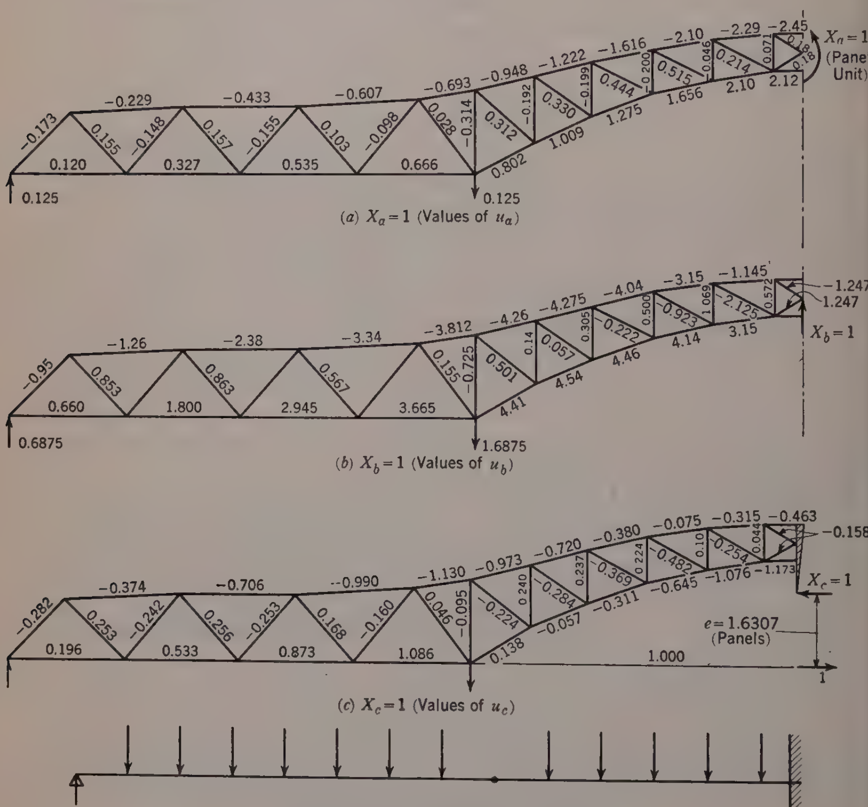
$$X_a = -\frac{\delta_{a0}}{\delta_{aa}} \qquad (18a)$$

$$X_b = -\frac{\delta_{b0}}{\delta_{bb}} \qquad (18b)$$

and

$$X_c = -\frac{\delta_{c0}}{\delta_{cc}} \qquad (18c)$$

This simplification eliminates the necessity for solving simultaneous equations, and makes it possible to obtain the influence lines for the redundants directly as simple deflection curves. The influence lines shown in Figs. 7 and 8 are not for the redundants X_a, X_b, and X_c, but give only the terms S_d', δ_{aa}, and δ_{ab} to substitute in the simultaneous equations, which must then be solved for the redundants.



(a) $X_a=1$ (Values of u_a)

(b) $X_b=1$ (Values of u_b)

(c) $X_c=1$ (Values of u_c)

(d) Conjugate Beam Loaded With Elastic Weights

FIG. 11.—STRESSES

It is possible to select the redundants for any statically indeterminate structure in such a manner that Eqs. 3 are satisfied.[14] Although this procedure is often used for the analysis of fixed arches, many engineers do not realize that it may be used for the analysis of statically indeterminate trussed structures. The influence lines for the redundant forces may be found directly by the method of elastic weights, without solving simultaneous equations.

The continuous tied arch shown in Fig. 10 will be used to illustrate the method of analysis. This is a threefold statically indeterminate structure symmetrical about the center line of the main span. The statically determinate

[14] "Simplified Calculation of Statically Indeterminate Bridges," by G. G. Krivoshein, Prague, 1930.

base system is formed by cutting the structure at the center line of the arch. If the redundant forces X_a, X_b, and X_c are applied at the ends of rigid brackets, as shown in Fig. 10(c), the conditions,

$$\delta_{ab} = \sum \frac{u_a\, u_b\, L}{A\, E} = 0 \quad \ldots\ldots\ldots\ldots (19a)$$

and

$$\delta_{bc} = \sum \frac{u_b\, u_c\, L}{A\, E} = 0 \quad \ldots\ldots\ldots\ldots (19b)$$

are evidently satisfied. The stresses u_a and u_c, resulting from $X_a = 1$ and $X_c = 1$, respectively, are symmetrical with respect to the center of the structure. The stresses u_b, resulting from $X_b = 1$, are antisymmetrical. The summations of Eqs. 19a and 19b, therefore, will have the same absolute values for the left half of the structure as for the right half, but will have different signs.

The distance e may be computed from the equation,

$$\delta_{ac} = \sum \frac{u_a\, u_c\, L}{A\, E} = 0 \quad \ldots\ldots\ldots\ldots (19c)$$

Substituting the numerical values of the stresses u_a (Fig. 11a) and u_c (Fig. 11c), in terms of e, into the summation of Eq. 19c, the following values are obtained: $\sum \frac{u_a\, u_c\, L}{A} = 46.570\, e - 75.942 = 0$; or, $e = 1.6307$ panels.

Since Eqs. 19 are now satisfied, the value of the couple X_a for a load $P = 1$ at any point q on the structure may be found as follows:

$$X_a = -\frac{\delta_{aq}}{\delta_{aa}} = -\frac{\sum \frac{u_a\, u_a\, L}{A\, E}}{\sum \frac{u_a^2\, L}{A\, E}} \quad \ldots\ldots\ldots\ldots (20a)$$

The numerator of Eq. 20a represents the deflection of any point q, resulting from a couple $X_a = 1$. This numerator is evaluated for all positions of the load P by the method of elastic weights. The elastic weights, or angle changes, for the load $X_a = 1$ panel unit are computed as shown in Table 5(a), Col. 1. The shears and bending moments in the conjugate beam of Table 5(a), Cols. 2 and 3, represent the slopes and deflections of the actual structure when it is loaded with the couple X_a. The terms used in Eq. 20a may be in any units as long as the same units are used in the numerator and in the denominator. In this discussion the moment arms are used in panel units, in order that the moments in the beam of elastic weights (Table 5(a), Col. 1) may be obtained by a summation of the shears. The numerator and denominator of Eq. 20a both may be multiplied by E.

The denominator of Eq. 20a represents the rotation of the cut ends of the structure relative to each other, resulting from the couple $X_a = 1$. This term may be evaluated from the shear in the conjugate beam loaded with the elastic

weights for $X_a = 1$. Thus, from Table 5(*a*), Col. 2: $\delta_{aa} = 2 \times 97.054 = 194.108$.

Therefore, the influence line for X_a is found by dividing the bending moments in the beam of elastic weights (Table 5(*a*), Col. 1) by 194.108, as shown

TABLE 5.—COMPUTATION OF INFLUENCE ORDINATES (IN PANEL UNITS)

(*a*) ORDINATES FOR X_a

Panel point	Elastic weights[a] (1)	Conjugate Beam: Shears (2)	Conjugate Beam: Moments[b] (3)	Influence ordinate[c] (4)
0	−5.776		0	0
		+5.776		
1	0.399		+5.776	−0.0297
		+5.377		
2	1.068		+11.153	−0.0575
		+4.309		
3	1.372		+15.462	−0.0797
		+2.937		
4	1.658		+18.399	−0.0948
		+1.279		
5	4.487		+19.678	−0.1014
		−3.208		
6	3.484		+16.470	−0.0849
		−6.692		
7	3.084		+9.778	−0.0504
		−9.776		
8	−0.676[d]		0[d]	0[d]
		−9.100		
9	4.028		−9.100	0.0469
		−13.128		
10	7.780		−22.228	0.1145
		−20.908		
11	17.474		−43.136	0.2222
		−38.382		
12	32.522		−81.518	0.420
		−70.904		
13	26.15		−152.422	0.785
		−97.054		
C[e]	....		−200.949	1.035[f]

(*b*) ORDINATES FOR X_b

Panel point	Elastic weights[a] (1)	Conjugate Beam: Shears (2)	Conjugate Beam: Moments[b] (3)	Influence ordinate[c] (4)
0	−31.918		0	0
		+31.918		
1	2.180		+31.918	−0.0221
		+29.738		
2	5.892		+61.656	−0.0428
		+23.846		
3	7.550		+85.502	−0.0593
		+16.296		
4	9.380		+101.798	−0.0706
		+6.916		
5	24.707		+108.714	−0.0754
		−17.791		
6	19.187		+90.923	−0.0631
		−36.978		
7	16.965		+53.945	−0.0374
		−53.943		
8	4.553[d]		0[d]	0[d]
		−58.496		
9	19.948		−58.496	0.0406
		−78.444		
10	28.107		−136.940	0.0950
		−106.551		
11	46.088		−243.491	0.1688
		−152.639		
12	58.192		−396.130	0.2750
		−210.831		
13	17.064		−606.961	0.4210
		−227.895		
C[e]	....		−720.908	0.5000

(*c*) ORDINATES FOR X_c

Panel point	Elastic weights[a] (1)	Conjugate Beam: Shears (2)	Conjugate Beam: Moments[b] (3)	Influence ordinate[c] (4)
0	−9.422		0	0
		9.422		
1	0.650		9.422	−0.0746
		8.772		
2	1.743		18.194	−0.1440
		7.029		
3	2.236		25.223	−0.1996
		4.793		
4	2.705		30.016	−0.2376
		2.088		
5	7.317		32.104	−0.2543
		−5.229		
6	5.683		26.875	−0.2126
		−10.912		
7	5.030		15.963	−0.1263
7	5.030		15.963	−0.1263
		−15.942		
8	2.893[d]		0[d]	0[d]
		−18.835		
9	2.184		−18.835	0.1491
		−21.019		
10	1.852		−39.854	0.3155
		−22.871		
11	−0.896		−62.725	0.4965
		−21.975		
12	−11.766		−84.700	0.6705
		−10.209		
13	−10.210		−94.909	0.7510
		0		
C[e]	....		....	....

[a] Elastic weights for X_a, X_b, or $X_c = 1$. [b] Values of δ_{aq}, δ_{bq}, or δ_{cq}. [c] Values of $-\frac{\delta_{aq}}{\delta_{aa}}$, $-\frac{\delta_{bq}}{\delta_{bb}}$, or $-\frac{\delta_{cq}}{\delta_{cc}}$. [d] Hinged. [e] Center cross. [f] Slope of the influence line at this point $= \theta = \tan \theta = 0.5$.

in Table 5(*a*), Col. 4. This influence line is symmetrical with respect to the center line of the structure. The influence-line ordinates are shown in panel

units, or they represent the value of X_a in panel-kips for a load of one kip. The moments may be converted to foot-kips by multiplying by 24, since the panel length is 24 ft.

The value of X_b for a unit load at any point q is found from the equation:

$$X_b = -\frac{\delta_{bq}}{\delta_{bb}} = -\frac{\sum \frac{u_b\, u_q\, L}{A\, E}}{\sum \frac{u_b^2\, L}{A\, E}} \quad \text{(20b)}$$

The numerator of this equation represents the deflection of point q resulting from a unit value of X_b, and is evaluated for all positions of q by the method of elastic weights. These computations are shown in Table 5(b), Col. 1. The denominator of Eq. 20b is found as the deflection in the direction of X_b, for a load of $X_b = 1$; or as twice the bending moment in the conjugate beam at the center line of the structure. The influence-line ordinates for X_b are given in Table 5(b), Col. 4, for half the structure. They are antisymmetrical with respect to the center line.

The value of X_c for a unit load at any point q of the structure is found from the equation:

$$X_c = -\frac{\delta_{cq}}{\delta_{cc}} = -\frac{\sum \frac{u_c\, u_q\, L}{A\, E}}{\sum \frac{u_c^2\, L}{A\, E}} \quad \text{(20c)}$$

The numerator of Eq. 20c may be evaluated by the method of elastic weights, as in the computation of the influence lines for X_a and X_b. The denominator, however, represents the horizontal deflection of the point of application of the redundants, and must be evaluated by summation. Substituting the values of u_c from Fig. 11(c), and the lengths and areas from Fig. 10, in this summation:

$$\sum \frac{u_c^2\, L}{A\, E} = \frac{126.34}{E}\left(\frac{\text{panels}}{\text{kip}}\right) \quad \text{(21)}$$

If the values shown in Table 5(c), Col. 3, are divided by 126.34, the ordinates of the influence line for X_c are obtained. This influence line is symmetrical with respect to the center line of the structure, and is shown for half the structure in Table 5(c), Col. 4.

The stresses in all the members of the structure may be found from the influence lines of Cols. 4, Table 5, by simple statics. Obviously, it is not necessary to solve simultaneous equations at any point in the analysis of this structure. G. G. Krivoshein[14] shows that all statically indeterminate structures may be analyzed without the solution of simultaneous equations for the redundants. Although the symmetrical continuous tied arch is readily analyzed in this manner, there are some types of structures in which it may be as laborious to select the redundants so that they satisfy Eqs. 18 as it is to solve the simultaneous equations. The redundants for the structure in Example 2, for instance, may be chosen by inspection so that 42 of the coefficients

vanish, and it probably would not be desirable to compute the redundants so that 56 of the coefficients vanish in order to satisfy Eqs. 18.

Acknowledgment.—The continuous tied-arch bridge analyzed in this discussion is probably the first structure of its type in America. This bridge was constructed in 1940 over the Meramec River near St. Louis, Mo., by the Missouri State Highway Department. The data on this structure were furnished through the courtesy of Howard Mullins, designer of the bridge, and N. R. Sack, bridge engineer of the Missouri State Highway Department.

Corrections for *Transactions:* January, 1941, *Proceedings*, in Table 1(a), values of s, change "-0.807 and -1.074" to "-0.806 and $+1.074$," respectively.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

CONCRETE IN SEA WATER: A REVISED VIEWPOINT NEEDED

Discussion

By Messrs. W. F. Way, and Glenn S. Paxson

W. F. Way,[14] M. Am. Soc. C. E. (by letter).[14a]—It is the writer's impression that Mr. Hadley's paper is timely. Too long it has been thought that all concrete, good or bad, deteriorates in sea-water structures, the deterioration being due to chemical reaction between the sea water and the various compounds in the hydrated cement of the concrete. The writer agrees quite heartily with the conclusions in the paper. In fact, one cannot come to any other conclusion when observing in the same structure both sound and unsound concrete in close proximity. Concrete that was placed, probably, from the same bucket or buggy—that is, concrete that has been under the same exposure—is found partly sound and partly unsound. Surely the difference is not due to chemical action. True it is that concrete structures in sea water do not enjoy a flawless record; but it is also true that the real causes of unsound concrete have not been explained by the allegations of sea-water attack.

The writer once had occasion to repair the substructure of a reinforced concrete pier whose supports were cylinders 4 ft in diameter, belling out to bases of 13-ft diameters. In the original cylinder construction, a timber shell form was made that was placed over a pile cluster previously driven, and when in place the marine ooze was pumped out and the base was sealed with a tremie plug. The next step was to dewater the form and to place the reinforcing steel. The base and shaft were completed with a concrete supposedly placed "in the dry." For various reasons a good job was not obtained, and years later after the teredos had eaten away the wooden forms (which had all been left in place) the concrete began to ravel away. The repair contract required all work to be done in the dry, which necessitated the use of a pneumatic caisson. This caisson enclosed the cylinders successively, one at a time, from high water to the mud line—about 45 ft. Thereby an excellent opportunity was afforded to study this concrete after its fifteen years of service. The typical case

Note.—This paper by Homer M. Hadley, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Thomas E. Stanton, M. Am. Soc. C. E.

[14] Vice-Pres. and Supt., Stuart Cameron & Co., Ltd., Stillwater, B. C., Canada.

[14a] Received by the Secretary February 19, 1941.

showed lenses of mud through the base of tremie concrete, above which was a very lean concrete in the shaft capped off with a plug of solid laitance varying from 2 to 4 ft thick, or sections of lean concrete separated by bands of laitance 6 to 12 in. thick. Many local areas of good solid concrete were encountered. It can be appreciated that this "failure of concrete" was due solely to poor workmanship, although the impression gained by many was that an inherent weakness of concrete when exposed to sea water had something to do with this failure.

The design of marine structures is a most important factor in determining the amount of maintenance that later will be required. Generally, designers have the option of using concrete piles or concrete cylinders in supporting pier decks. With concrete piles it is almost certain to be specified that all cracked piles shall be pulled and replaced with sound piles. Considerable controversy may arise over this point and therefore a brief description of a project in which 3,500 piles were driven may be apropos.

The piles on this job were 16 in. by 16 in. for lengths as great as 60 ft, and 18 in. by 18 in. for 60-ft to 80-ft lengths. All piles were driven with a double acting hammer on a floating rig. These piles were carefully designed; the materials used were certainly of the highest quality; and the workmanship throughout, including curing, was in accordance with the very best practice (1933). When the piles were placed in the leads of the driver, no signs of cracks were visible. After they were driven each of the 3,500 piles was inspected by inspectors in diving suits, and it was found that practically every one had developed cracks during driving. Only at the inshore and in shallow water where the piles were shorter (about 40 ft), and the unsupported length during driving was 20 ft or less, were no cracks observed. Since it would have been beyond reason to expect the contractor to pull these piles and drive others, a distinction was made on this project between a "cracked" pile and a "broken pile," under which it was considered that only such piles as were incapable of acting as elastic members would be rated "broken." Of the total number driven, about 60 piles were definitely "broken" and had to be replaced.

In reaching this decision about "cracked" piles an investigation was made of the piles in an adjacent existing structure, built fifteen years previously. The marine growth was removed and the faces of the piles were inspected carefully in an effort to locate cracks similar to those developing on the present job. As expected they were found; the concrete was then chipped back along the cracks, and in all cases the cracks were found to be opposite the reinforcing ties binding the vertical bars. They were all tight, and no signs of corrosion of either the bands or the main steel were observed.

Since it was definite that these cracks, which occurred opposite the bands, had developed during the driving it was decided that the new piles should be watched during the actual driving process. Because the water was warm, a shallow-water helmet was used, being safer and permitting freer movements around the pile. Furthermore, the large face plate affords excellent vision. After a pile was in the leads, the inspector, in a shallow-water suit, climbed on the pile, keeping under the barge as much as possible to protect himself from falling spalls. After a few blows the cracks would be observed forming, and as

driving proceeded the cracks would start to "spit" out the crushed material ground up by each blow. In many of the piles there would be a crack opposite each reinforcing band. The conclusion was that as long as the bond on the main vertical steel was practically unimpaired there would be sufficient ground-up material remaining in each crack to re-cement itself and to protect the steel, and that the pile was as good as could be obtained. It is the writer's opinion that it is impossible to drive a pre-cast reinforced concrete pile, as now being made, without developing these small fine cracks, especially when the length is around 35 to 40 ft from the mud line to the water surface. It is his further opinion that such fine cracks are not harmful if the concrete of the pile is of good quality, but that they are dangerous if the concrete is of poor quality. It all depends on "what kind" of concrete cracks.

Many instances of poor concrete have come to the writer's attention, occurring both in fresh and salt water substructures, and also many examples of good concrete have likewise been observed. One that might be mentioned is a small sea wall built along the beach of Elliott Bay at Seattle, Wash. This wall is 5 in. thick at the top and 7 in. at the base, and 5 ft high. The elevation of the top is at extreme high water. The wall is reinforced at the center with $\frac{3}{8}$-in. round bars 15 in. on centers both ways. The concrete was made with the sand and gravel shoveled up from the beach, which at times contained a considerable quantity of sea water. The cement content was 6 sacks per cubic yard. The exposure is very severe and the wall has had to stand much pounding from both waves and drift. The wall is now (1941) thirteen years old and the only signs of distress are those due to erosion; no chemical deterioration can be observed.

Glenn S. Paxson,[15] Assoc. M. Am. Soc. C. E. (by letter).[15a]—Mr. Hadley's excellent paper on the resistance of concrete to disintegration in sea water should find an appreciative audience among engineers who are working along the seacoasts. He should, indeed, be commended on this summary of his observations and its clear presentation.

It is noted that the list of representative structures given in the paper includes some in British Columbia, Washington, and California. The writer would add to this list the structures along the entire 400-mile length of the Oregon Coast Highway. Frequent inspections of all highway structures are made for maintenance purposes, and no example of disintegration due to chemical action of sea water has been found. These structures are not as old as those cited by Mr. Hadley—the oldest dating from 1920—but, if chemical action is taking place, some evidence should be apparent after 20 years of exposure.

Every indication points to abrasion rather than chemical action as the cause of any disintegration. It is noted that concrete members in deep water, where little or no sand is carried by wave action, retain the surface film of rich mortar and show the form marks. Piers and walls in shallow water or in the tidal range have this film worn away and show a granular texture. The

[15] Bridge Engr., State Highway Comm., Salem, Ore.

[15a] Received by the Secretary February 26, 1941.

concrete is hard and firm, and it can be assumed that this slight wear is due to the sand carried in the water rather than to any solvent or disruptive action of the sea water itself.

The enlargement of gravel pockets, laitance seams, and construction joints is the result of this same abrasive action. Examples of such structural defects that have not changed after 20 years of exposure to quiet sea water are common.

The excessive rusting of inadequately protected reinforcing steel in sea water with the concomitant spalling of the concrete incasement is often apparent. Adjoining bars with the same exposure, but with adequate protection, show no rusting. This disintegration is certainly not chargeable to any solvent or disruptive action in the concrete itself.

The writer is in complete accord with Mr. Hadley's conclusions that there is no evidence of any solvent or disruptive attack by sea water on properly made and placed concrete.

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DISCUSSIONS

HYDRAULICS OF SPRINKLING SYSTEMS FOR IRRIGATION

Discussion

By Arthur F. Pillsbury, Assoc. M. Am. Soc. C. E.

Arthur F. Pillsbury,[8] Assoc. M. Am. Soc. C. E. (by letter).[8a]—In California, sprinkling systems for irrigation often have failed to perform satisfactorily because of faulty design, failure to obtain good distribution characteristics for the sprinklers with the spacings provided, and failure to interpret, properly, the limitations of the soil in absorbing the precipitation. Therefore, the excellent dissertation by Mr. Christiansen, based on a wealth of experimental evidence, is timely.

Sprinkling systems are useful where: (1) A high water level exists, and less uniform surface applications would cause that level to rise; (2) the topsoil is shallow and pervious, and less uniform surface applications would create a temporary water table above a subsoil of different texture; (3) the soil is extremely pervious resulting in waste of water in parts of a planting; (4) irrigation is infrequent, the good topsoil is shallow, and for other reasons it is not practical to establish satisfactory grades for surface irrigation; and (5) extremely light and frequent applications are desired, as with some pastures, lawns, and gardens.

In short, sprinkler irrigation is useful where surface methods cannot be economically and efficiently utilized. Its use with field crops is limited to areas of high water table, or where extremely light frequent or infrequent supplemental irrigations are desired, and with orchards to areas of shallow soil, extremely pervious soil, or unfavorable topography.

In orchard irrigation, under-tree systems are unquestionably preferable to the overhead type. The essential sprinkler objective is to improve distribution of water. With the under-tree system, the distribution is less affected by wind and by the trees, it is better adapted to operation by the usual farm laborer, and it should be less expensive.

Note.—This paper by J. E. Christiansen, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*.

[8] Asst. Prof. of Irrig., Univ. of California, Los Angeles, Calif.

[8a] Received by the Secretary February 17, 1941.

Mr. Christiansen's data on distribution of water from sprinklers are of greatest importance with the overhead type used for field crops. With under-tree orchard irrigation, sprinklers are generally spaced only 18 to 24 ft apart, and the greatest essential is to insure approximately equal discharge of each sprinkler, rather than absolutely uniform distribution in each space bounded by four trees, which is seldom attained. A good pattern is nevertheless desirable.

For under-tree work the slow revolving sprinklers and perforated pipe systems generally are used for low-pressure (4 to 15 lb per sq in.) operation with medium to high rates of discharge (2 to 8 gal per min, or equivalent for perforated lines). Whirling sprinklers are used for intermediate pressures (10 to 20 lb per sq in.) with all usual rates of discharge (1 to 8 gal per min) and for higher pressures with low to medium rates of discharge (1 to 4 gal per min). Fixed heads are used for high pressure (20 lb per sq in. and up) with high rates of discharge (4 to 8 gal per min) and for lower rates of discharge when spacing is closer than usual or when exceptionally high pressure is available. Intermediate and high-pressure systems usually result in economies in material where sufficient pressure is available without pumping. On the other hand, low-pressure systems may eliminate the necessity of pumping, or may minimize the pumping cost.

The rate of application depends upon general farm practices or upon the maximum infiltration rates permissible without appreciable runoff and erosion. Infiltration rates decrease somewhat with increase in the proportion of the finer particles of a soil. More important is the physical condition of the soil. Generally, infiltration rates increase with decrease in original moisture content, but powdery dry soils may not take water readily. Organic matter, such as cover crops and manure, increases infiltration rates. Chemically "soft" waters (relatively low in calcium or with a high percentage of sodium in relation to other cations), and some inorganic fertilizers, may tend to leave a top sodium-saturated crust on the surface which is relatively impervious. Similarly, impervious crusts may be formed by physical compaction. Cultivation will improve infiltration by breaking up and mixing such crust with other soil.

Of frequent practical importance in sprinkler design is the fact that the size of sprinkler drops (with equivalent spray trajectories) is closely correlated with physical compaction ("puddling") of the surface crust. For instance, on a bare soil classified[9] as Yolo loam, with sprinkler precipitation at a constant rate of 0.59 in. per hr, a 5.4-in. depth of fine spray was applied without runoff, while with a somewhat coarser spray only 2.5 in. could be so applied. Relative size of drops was roughly checked with the Bentley technique.[10] Detailed investigations are now (1941) being conducted by the U. S. Soil Conservation Service on the effect of drop size on erosion and infiltration.[11]

In general, size of drop with a given sprinkler increases with distance from the sprinkler, and, comparing different sprinklers producing equivalent patterns, drop size decreases as pressure increases. This point is stressed because many

[9] "Soil Survey of the Los Angeles Area, California," Bureau of Soils, U. S. Dept. of Agriculture in cooperation with the Univ. of California Agricultural Experiment Station, 1916.

[10] "Studies in Raindrops and Raindrop Phenomena," by W. A. Bentley, *Monthly Weather Review*, Weather Bureau, U. S. Dept. of Agriculture, October, 1904, pp. 450–456.

[11] "Recent Studies in Raindrops and Erosion," by J. Otis Laws, *Agricultural Engineering*, Am. Soc. of Agri. Engrs., Vol. 21, No. 11, November, 1940, pp. 431–433.

TABLE 1.—FOR SELECTING SIZES OF PORTABLE UNDER-TREE SPRINKLER UNITS

Spacing (ft)	Maximum Recommended Discharge per Sprinkler, at 20 Lb per Sq In., in Gal per Min, for Units with the Following No. of Sprinklers:														
	3	4	5	6	7	8	9	10	11	12	14	16	18	20	22
(a) ¾-In., Type M, Copper Pipe—Solder Fittings															
16	7.6	4.5	3.1	2.3	1.8	1.4	1.2	1.0	...	...	...	...	...	...	...
20	6.8	4.0	2.7	2.0	1.6	1.2	1.0	0.9	...	...	...	...	...	...	...
24	6.0	3.6	2.5	1.8	1.4	1.2	0.9	0.8	...	...	...	...	...	...	...
(b) 1-In., Type M, Copper Pipe—Solder Fittings															
16	...	9.1	6.2	4.5	3.5	2.8	2.3	2.0	1.7	...	...	...	...	...	...
20	...	8.0	5.4	4.0	3.1	2.5	2.1	1.7	1.5	...	...	...	...	...	...
24	...	7.2	4.9	3.6	2.8	2.2	1.9	1.6	1.3	...	...	...	...	...	...
30	...	6.4	4.3	3.2	2.5	2.0	1.6	1.4	1.2	...	...	...	...	...	...
(c) 1¼-In., Type M, Copper Pipe—Solder Fittings															
16	...	...	10.9	7.9	6.1	4.9	4.1	3.4	...	2.5	2.0	...	...	...	...
20	...	...	9.4	7.0	5.4	4.4	3.6	3.0	...	2.2	1.7	...	...	...	...
24	...	...	8.6	6.3	4.9	3.9	3.2	2.7	...	2.0	1.6	...	...	...	...
30	...	...	7.6	5.6	4.3	3.4	2.9	2.4	...	1.8	1.4	...	...	...	...
(d) 1-In. Electrical Conduit (1³/₆₄-In. Inside Diameter), Galvanized, with Constricting Airhose-Type Couplings															
16	9.1	5.5	3.8	2.8	2.2	...	...	...	...	...	...	...	...	...	...
20	8.1	4.9	3.4	2.5	2.0	...	...	...	...	...	...	...	...	...	...
24	7.4	4.5	3.1	2.3	1.8	...	...	...	...	...	...	...	...	...	...
(e) 1-In. Electrical Conduit (1³/₆₄-In. Inside Diameter), Galvanized, with No Constricting Couplings															
16	12.7	7.6	5.2	3.9	3.0	2.5	...	1.7	...	1.3	...	...	...	...	...
20	11.2	6.8	4.6	3.4	2.7	2.2	...	1.5	...	1.1	...	...	...	...	...
24	10.1	6.1	4.2	3.1	2.4	1.9	...	1.4	...	1.0	...	...	...	...	...
(f) 1¼-In. Electrical Conduit (1⅜-In. Inside Diameter), Galvanized, with No Constricting Couplings															
16	...	...	10.6	7.9	6.2	5.0	4.1	3.5	...	2.6	2.1	...	...	...	...
20	...	...	9.5	7.1	5.5	4.4	3.7	3.1	...	2.3	1.8	...	...	...	...
24	...	...	8.6	6.4	5.0	4.0	3.3	2.8	...	2.1	1.6	...	...	...	...
30	...	...	7.7	5.7	4.4	3.6	2.9	2.5	...	1.9	1.5	...	...	...	...
(g) 1½-In. Galvanized Sprinkler Pipe (1.40-In. Inside Diameter) with Quick Couplings															
16	...	...	...	8.6	6.7	5.4	4.5	3.8	...	2.9	2.2	1.8	...	...	...
20	...	...	...	7.7	6.0	4.8	4.0	3.4	...	2.5	2.0	1.6	...	...	...
24	...	...	...	7.0	5.5	4.4	3.6	3.0	...	2.3	1.8	1.5	...	...	...
30	...	...	...	6.2	4.8	3.9	3.2	2.8	...	2.1	1.6	1.3	...	...	...
(h) 2-In. Galvanized Sprinkler Pipe (1.90-In. Inside Diameter) with Quick Couplings															
16	...	...	...	...	...	12.1	...	8.4	...	6.4	4.9	4.0	3.4	2.8	2.4
20	...	...	...	...	...	10.7	...	7.5	...	5.6	4.4	3.6	3.0	2.5	2.2
24	...	...	...	...	...	9.7	...	6.8	...	5.1	4.0	3.2	2.7	2.3	2.0
30	...	...	...	...	...	8.6	...	6.0	...	4.5	3.5	2.9	2.4	2.0	1.7

sprinkler systems have failed due to serious runoff and erosion that developed before sufficient water could be applied. Later investigations have indicated: (1) That the soil was too heavy for satisfactory sprinkler use under the cultural practices followed, and (2) that the condition might have been prevented had sprinklers with a finer spray been used. It must be recognized, then, that low-pressure sprinklers (with a coarse spray) are sometimes not suitable for irrigation of bare soils, whereas higher pressure sprinklers would be suitable.

Mr. Christiansen's data on evaporation losses indicate that loss from the sprinkler spray is insignificant. Some increase in evaporation, as compared with furrow irrigation, may result, however, from the greater ground surface wetted and from water intercepted by plant foliage. His data on the hydraulics of sprinkler lines are quite complete. Comments on the hydraulics, having in mind the small under-tree units commonly used in Southern California, follow:

(1) The recommendations of Mr. Pigott[6] for friction loss in new steel and copper pipes are very satisfactory for use in design of $\frac{3}{4}$-in. to $1\frac{1}{2}$-in. units, as indicated by recent tests of friction losses in such pipes.

(2) Generally, the under-tree units have sprinklers at each end of the line, and are uniformly spaced. Sufficient accuracy is attained if it is conservatively assumed that friction losses will be one third the loss that would occur if the full flow of all sprinklers was carried to the end of the line.[12]

(3) Table 1 shows the maximum recommended sprinkler discharge in gallons per minute at a pressure of 20 lb per sq in. and an over-all pressure ratio of 1.2. The sprinklers are assumed to be all similar, equally spaced, and placed at each end of a unit. Spacings that are not included can be interpolated. In Table 1, approximate lengths of pipe that can be dragged readily by one man, under favorable conditions, are underlined. Assuming a maximum allowable over-all pressure ratio of 1.2, Table 1 is satisfactory for use in designing the portable units. Included are all the various types of pipe now used for such portable units. The usual steps in designing a unit are: (*a*) Select the application rate desired (generally 0.2 to 1 in. per hr); (*b*) compute the desired sprinkler discharge by the formula

$$\text{Sprinkler discharge in gal per min} = \frac{\text{Area (sq ft) per sprinkler} \times \text{rate of application (in 1 hr)}}{96.3} \quad \ldots\ldots(17)$$

(*c*) select sprinkler and operating pressure; (*d*) determine what the discharge per sprinkler would be at 20 lb per sq in. (discharge proportional to square root of pressure); and (*e*) select pipe and length of unit from Table 1. Usually sprinklers with devices for manually adjusting pressure at each sprinkler are not satisfactory for under-tree systems, but whirling sprinklers are being developed which automatically regulate discharge if pressure is greater than 8 lb per sq in. If these prove satisfactory, it will not be necessary to limit the pressure ratio when using them but simply to insure adequate pressure at the distal end of each unit.

[6] "The Flow of Fluids in Closed Conduits," by R. J. S. Pigott, *Mechanical Engineering*, August, 1933, pp. 497–501, and 515.

[12] "The Design of Overhead Irrigation Systems," by E. S. West and A. Howard, *Pamphlet No. 50*, Council for Scientific and Industrial Research, Commonwealth of Australia, Melbourne, 1934.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

VALUE OF PUBLIC WORKS

Discussion

BY CLARENCE W. POST, M. AM. SOC. C. E.

CLARENCE W. POST,[4] M. AM. SOC. C. E. (by letter).[4a]—In this paper, Major Hallihan places sewage treatment plants and water purification plants in the category of projects that naturally fall into a class handled by the PWA. In New York State this has not been true; some of the largest and best projects that the WPA has constructed in the state have been for water treatment, sewer systems, and treatment plants. These range from a small complete sewer system and treatment plant at the Village of Ft. Ann (population in 1940, 437) to the interceptors and treatment plant for the Ley Creek Sanitary District, Syracuse, which cost more than \$3,000,000, and which was designed for a population of 30,000 and a daily average flow of 4½ million gal.

Under the WPA program, thirty-four treatment plants have been constructed and twenty-seven plants enlarged and improved. Projects for interception and treatment plants have also been started in many more communities. In addition to these plants, 1,900 miles of sanitary and storm sewers have been installed. Two storm relief tunnels driven through rock have been built in the City of Rochester, and a third submitted for approval.

The plans and specifications for this type of work must be prepared by either regular civil service employees of the engineering bureaus of the various cities, or by consulting engineers. All plans and specifications must be approved by the Division of Sanitation, New York State Health Department, before any work may be performed.

When a project is placed in operation the selection of a superintendent to have charge of the operation is of utmost importance. Applicants' personnel records and construction background are examined and the final determination and approval for employment comes from the director of operations or the chief engineer of the New York State Work Projects Administration.

One other factor helps in this type of work in New York State: Each community signs an agreement to furnish the necessary skilled labor if such

NOTE.—This paper by J. P. Hallihan, M. Am. Soc. C. E., was published in February, 1941, *Proceedings*.

[4] Deputy State Administrator & Chf. Engr., New York State WPA, Albany, N. Y.

[4a] Received by the Secretary March 14, 1941.

skills are not available from the relief rolls. Small buildings have been, and are being, built successfully by the cooperation received from the sponsors who, in many instances, have furnished the bricklayers and any other skilled artisans who are lacking from the relief rolls. In every instance, however, the federal expenditures on these projects are only about 30% to 40% of the total cost.

The plants can be built with relief labor when care is exercised over the selection of key personnel to supervise the work.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

ON THE METHOD OF COMPLEMENTARY ENERGY

Discussion

BY MESSRS. I. K. SILVERMAN, AND GEORGE R. RICH

I. K. SILVERMAN,[29] ASSOC. M. AM. SOC. C. E. (by letter).[29a]—The ideas presented in this paper throw a great deal of light on a concept that seems to have become axiomatic in many minds. This concept is one that deals with the economy of Nature, and it is expressed in textbooks dealing with the theory of structures by means of the "Principle of Least Work." Perhaps it is because the engineer, in dealing with forces, has come to have a respect for the way Nature behaves. The behavior of liquids, the action of falling bodies, the structure of organic materials such as bones and feathers,[30] the apparent minimal surfaces that may be obtained with thin films, cells of honeycombs, etc., all seem to be conclusive evidence of the economy of Nature.

It is a far cry from the Ptolemaic concept of the universe to the Principle of Least Work of modern structural analysis, but the conflict between the theological and mechanical concepts of the universe, which raged around the Ptolemaic theory, imparted a metaphysical twinge to the ideas of the philosophers who followed Galileo.

In their studies of the phenomena of Nature the investigators of the seventeenth and eighteenth centuries attempted to explain them from the inherent perfection of the Creator. It is not strange, therefore, to find the concepts of minimum arising from the efforts of the philosophers of this age to explain the universe. So great has been the influence of these precursors of modern science that their "minimum principles" have come down through the centuries to be included in modern textbooks with even the argument of the wisdom of the Creator in calling for economy in Nature. Such is the background of the principle of the minimum in the theory of structures.

Jules Henri Poincaré declared that any generalization which is based on experimental results has no right to be called a principle, and an examination of

NOTE.—This paper by H. M. Westergaard, M. Am. Soc. C. E., was published in February, 1941, *Proceedings*.

[29] Asst. Engr., U. S. Bureau of Reclamation, Denver, Colo.

[29a] Received by the Secretary March 12, 1941.

[30] "The Science of Mechanics," by E. Mach, The Open Court Publishing Co., Chicago, Ill., 1893, p. 452.

the assumptions which led to Castigliano's second theorem and thence to what is generally, but incorrectly, called the Principle of Least Work will bear this out. The basic assumptions are as follows, as applied to engineering structures:

1. The deformations produced are infinitesimal and thus it follows that the law of superposition holds; and
2. A linear relation exists between the forces and displacements.

From these two assumptions the energy stored in a body can be expressed as a quadratic in the forces or the displacements. Castigliano's first theorem follows: The partial derivative of the expression for energy containing the forces with respect to any force gives the displacement of the force in the direction it acts; and, for those forces which do not move because of their geometrical or kinematical restraints, the partial derivatives are zero. This latter result can be expressed as a theorem interpreting the vanishing of the derivative as a condition for a minimum. This theorem of "minimum" will not hold for the cases in which (*a*) the law of superposition is no longer true, and (*b*) the material does not follow Hooke's law.

These conditions are purely experimental, and in the light of Poincaré's specifications the term "principle" is inadmissible. Most of the materials used in structural engineering may be considered to satisfy the conditions under which the theorem holds, and the results obtained in practise agree essentially with those predicted by the application of the theorem.

The author has introduced a new term, complementary energy, which is certainly not of anthropomorphic origin. It is nothing more than a mathematical expression involving stresses or forces which, upon differentiation with respect to the stress or force, gives the corresponding strain or displacement. When the forces involved are constrained kinematically or geometrically the ensuing derivative is set equal to zero, which is interpreted as a minimum. The author has shown that in the case of Hooke's law "complementary energy" coincides with "elastic energy."

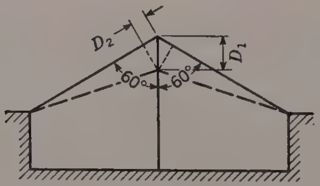

Fig. 6

Fig. 7

The only check on the results obtained by using either the "law of least complementary energy" or the "law of least elastic energy" is a geometrical one. In both cases the equations of equilibrium are first applied and when dealing with a statically indeterminate structure a "minimum principle" is used to determine the statically indeterminate quantities. In the last analysis the results obtained can be checked only by noting that the conditions of continuity are preserved. Thus the three solutions for S_1 of Fig. 2(*a*) can be shown to be correct as follows:

From Fig. 6 the deformation D_1 is connected with the deformations D_2 and D_3 by the following equation:

$$D_1 = \frac{D_2}{\cos 60°} = 2\,D_2 \qquad (128a)$$

For $P < 3\,C$,

$$D_1 = (P - X)\,k = 2\,k\,X = 2\,D_2; \quad \text{and} \quad X = \frac{P}{3} \qquad (128b)$$

For $3\,C < P < 5\,C$,

$$D_1 = 2\,k\,(P - X - C) = 2\,k\,X = 2\,D_2; \quad \text{and} \quad X = \frac{P - C}{2} \qquad (128c)$$

and, for $P > 5\,C$,

$$D_1 = 2\,k\,(P - X - C) = 4\,k\,(X - C) = 2\,D_2; \quad \text{and} \quad X = \frac{P + C}{3} \qquad (128d)$$

The fundamental character of a solution based on consistent deflections is apparent; but it must be admitted that a solution based on a "minimum" principle as presented by the author will prove more attractive to analysts because of its economy of thought.

Stability Problems.—The essential feature in the study of stability problems is that the structure in question is assumed to have a geometrical configuration entirely different from its original state. The equations of equilibrium are set up from this strained state rather than from the unstrained state as is done in all other problems of equilibrium. The question then becomes: What must be the value of the forces so that the strained state may be maintained? Two general methods are available that can be shown to be identical.

The first method involves the problem of the elastica—that is, the solution of the differential equation that describes the possible states of equilibrium. The second method, the energy method, is based on the fact that, for unstable equilibrium, the potential energy of a system is a maximum and any virtual variation from that state involves a decrease in potential energy. The criterion by which the conditions of unstable equilibrium are determined is that, in the transition from the actual state to one infinitely close to it, the variation of the potential energy is zero; that is,

$$\delta T = 0 \qquad (129)$$

in which T is the potential energy of the system. One important feature of this statement is that it is independent of the law of elasticity and holds for large displacements.

Instead of using the potential energy of the structure, the author has substituted for it a "complementary energy" and claims superiority of the method of "complementary energy" over the potential energy by showing that, by the use of Eq. 23, based on "complementary energy," a closer value to the buckling load can be obtained than by using Eq. 27, which is based on potential energy.

The reason for this apparent superiority lies in the fact that the author has satisfied all boundary conditions by using the expression $M = P\,z$ for the moment at any point whereas Eq. 27 utilizes the more general term

$M = E\,I\,\dfrac{d^2 z}{dx^2}$. When Eq. 27 is to be used the approximating function must satisfy all the boundary conditions and using Eq. 24 would naturally give a poor result. The Timoshenko variant takes this fact into account by writing

$$P\,z = M = E\,I\,\frac{d^2 z}{dx^2} \qquad (130)$$

If an expression that satisfies all boundary conditions is used, Eq. 27 furnishes a more accurate result than does Eq. 23 when Eq. 24 is used. For example, assume

$$z = c\,(l^3\,x - 2\,l\,x^3 + x^4) \qquad (131)$$

which satisfies all boundary conditions. From Eq. 27

$$P = \frac{168}{17}\,\frac{E\,I}{l^2} = 9.8824\,\frac{E\,I}{l^2}.$$

Substituting this expression in Eq. 23 or Eq. 28 will yield as a result

$$P = \frac{306}{31}\,\frac{E\,I}{l^2} = 9.8797\,\frac{E\,I}{l^2} \qquad (132)$$

As a matter of fact Eq. 27 has a marked superiority over Eq. 23 in that it is perfectly general and will hold under all boundary conditions, whereas Eq. 23 will have to be modified according to the boundary conditions. Consider the case shown in Fig. 7.

Assume a starting shape

$$z = c\,(3\,l\,x^2 - x^3) \qquad (133)$$

which satisfies all boundary conditions. The exact solution for this case is $P = \dfrac{\pi^2}{4}\,\dfrac{E\,I}{l^2}$. From Eq. 27, $P = 2.5\,\dfrac{E\,I}{l^2}$, whereas Eq. 23 gives the impossible value of $P = -\dfrac{14}{11}\,\dfrac{E\,I}{l^2}$. Evidently Eq. 23 must be modified to take the free end into account.

Eq. 27 was derived by equating Eq. 26 to zero, but an equivalent and (in the writer's opinion) more fundamental expression may be derived by the application of Eq. 129.

From Eq. 26

$$T = \frac{1}{2}\int_0^l \left[E\,I\left(\frac{d^2 z}{dx^2}\right)^2 - P\left(\frac{dz}{dx}\right)^2\right] dx \qquad (134)$$

The variation of this integral is a problem in the Calculus of Variations, and on performing the variation the following result is obtained:

$$\delta T = \int_0^l \left[E\,I\,\frac{d^4 z}{dx^4} + P\,\frac{d^2 z}{dx^2}\right]\delta z\,dx - \left[\left(P\,\frac{dz}{dx} + E\,I\,\frac{d^3 z}{dx^3}\right)\delta z\right]_0^l + E\,I\left[\frac{d^2 z}{dx^2}\,\delta\left(\frac{dz}{dx}\right)\right]_0^l = 0 \qquad (135)$$

The terms in the brackets are called "the terms at the limits," and when they are zero Eq. 135 becomes

$$\int_0^l \left[E\,I \frac{d^4z}{dx^4} + P \frac{d^2z}{dx^2} \right] \delta z\, dx = 0 \ldots\ldots\ldots\ldots\ldots (136)$$

Since the variation δz is arbitrary, Eq. 136 is fulfilled when

$$E\,I \frac{d^4z}{dx^4} + P \frac{d^2z}{dx^2} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (137a)$$

Eq. 137*a* may be integrated to

$$E\,I \frac{d^2z}{dx^2} + P\,z = 0 \ldots\ldots\ldots\ldots\ldots\ldots (137b)$$

which is nothing more than the differential equation given by Leonhard Euler. From this demonstration may be seen the connection between the problem of the elastica and the energy method.

The approximate solution of all problems in mechanics based on the variation of an integral can be simplified greatly, as follows: When the boundary conditions are such that the "terms at the limits" are zero, Eq. 135 becomes Eq. 136. In general, Eq. 136 may be written[31] as

$$\int_0^l \text{[Differential equation of equilibrium]} \times \text{[Virtual change of the function describing the state of equilibrium]}\, dx = 0 \ldots\ldots\ldots (138)$$

When the "terms at the limits" are not zero an equation of the form of Eq. 135 must be dealt with. It must be remembered that Eq. 135 is the result of the variation of the integral of potential energy of a straight column subjected to end loads. Any other condition of loading will require a slightly different expression.

Consider the case shown by Fig. 3(*a*). The application of Eq. 135 to this case is as follows:

Since the ends are held, the terms at the limits vanish leaving Eq. 136 or 138. Assuming the value of z in Eq. 131, which satisfies all boundary conditions:

$$\frac{dz}{dx} = c\,(l^3 - 6\,l\,x^2 + 4\,x^3);\quad \frac{d^2z}{dx^2} = 12\,c\,(x^2 - l\,x) \ldots\ldots (139a)$$

$$\frac{d^3z}{dx^3} = 12\,c\,(2\,x - l);\quad \frac{d^4z}{dx^4} = 24\,c \ldots\ldots\ldots\ldots (139b)$$

and,

$$\delta z = \delta c\,(l^3\,x - 2\,l\,x^3 + x^4) \ldots\ldots\ldots\ldots\ldots (139c)$$

Substituting in Eq. 136

$$\int_0^l [E\,I\,24\,c + 12\,P\,c\,(x^2 - l\,x)][l^3\,x - 2\,l\,x^3 + x^4]\,\delta c\,dx = 0 \ldots (140)$$

[31] "Eine Wichtige Vereinfachung der Methode von Ritz zur angenaherten Behandlung von Variationsproblemen," by H. Hencky, *Zeitschrift für angew. Mathematik und Mechanik*, Vol. 7, 1927, p. 80.

Performing the integrations it will be found that

$$P = \frac{168}{17} \frac{E\,I}{l^2} \qquad (141)$$

The differential equation of the strained column is given by Eq. 137*b*. From Eq. 138

$$\int_0^l \left[E\,I \frac{d^2 z}{dx^2} + P\,z \right] \delta z\, dx = 0 \qquad (142)$$

Substituting Eqs. 139 and performing the integrations will lead to

$$P = \frac{306}{31} \frac{E\,I}{l^2} \qquad (143)$$

For the case shown by Fig. 7, Eq. 138 cannot be used and Eq. 135 including the "terms at the limits" must be used.

The same general method can be applied to vibrating systems.[32]

One point which should be emphasized in the foregoing approximations is that the critical loads and their analogous quantities in vibrating systems are always larger than the actual values and, therefore, are necessary but not sufficient magnitudes. Therefore, these approximate values are on the unsafe side and other methods[33] should be used to determine the lower limit.

Fundamentally there is no difference between the method presented by the author as expressed by Eq. 11 and that as expressed by Eq. 138. The writer believes that the differential equation or the energy expression of a given problem can be "set up" with greater ease than the deflections, displacements, and changes of curvature required by Eq. 10. Furthermore, all reference volumes on the theory of structures present the general theory of equilibrium via the differential equation and the investigator will find Eq. 138 quite easy to handle.

The method based on Eq. 138 also may be used in the approximate solution of problems of stable equilibrium (for example, in the determination of the deflections of beams, etc.). One important problem to which it has been applied is that dealing with the action of a suspension bridge subjected to lateral forces and to the determination of the natural frequencies of this type of structure when vibrating in a horizontal plane.

Under the action of a uniform wind load the deflection of a suspension bridge can be expressed as

$$\delta = \sum_{n=1,3,5\cdots} \frac{a_n\, l^4}{E\,I} \sin \frac{n\,\pi\,x}{l} \qquad (144)$$

The moment and shear in the horizontal system may be determined from $M = E\,I \frac{d^2\delta}{dx^2}$; and shear $= E\,I \frac{d^3\delta}{dx^3}$. Eq. 138 may be used to determine the

[32] "On Forced Pseudo-Harmonic Vibrations," by I. K. Silverman, *Journal of the Franklin Institute*, June, 1934, p. 743.

[33] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., 1936, p. 84; also "Rayleigh's Principle," by G. F. J. Temple and W. G. Bickley, Oxford Press, 1933, p. 29 and p. 72.

values of a_n. A set of simultaneous equations is obtained given by the general formula

$$a_n\left[(\pi\, n)^4 A_1 + (\pi\, n)^2 A_2 - A_3\right] - \sum_i a_i \frac{A_4\,(n\, i)^5}{(n - i)^2\,(n + i)^2}$$
$$= \frac{A_5}{(n\,\pi)^3} - \frac{A_6}{n\,\pi}, \qquad \text{for } n = 1, 3, 5, \cdots \ldots\ldots\ldots\ldots (145)$$

in which i may have any odd integral value except n, depending upon the number of terms assumed in the series, Eq. 144; and A_1 to A_6 are constants of the structure and loading. Thus a direct solution is furnished by use of Eq. 138 to replace the tedious trial and error solution now available.[34]

George R. Rich,[35] M. Am. Soc. C. E. (by letter).[35a]—Dean Westergaard has prepared a valuable and interesting paper which, for maximum benefit, should be read as a sequel to his earlier paper on the "Buckling of Elastic Structures,"[36] from which the somewhat formidable terms "orthostatic action," "astatic action," and "heterostatic action" emerge as the more familiar expressions "simple stress or bending," "pure buckling," and "mixed buckling."

In determining critical buckling loads or natural periods of vibration, it appears from the examples adduced by the author that the method of complementary energy affords much greater latitude than the Rayleigh method in the selection of functions to represent the deflected elastic line. This is particularly striking in the author's use of parabolic loci in the cases of the buckling of hinged-end columns and the lateral vibration of prismatic beams. In the Rayleigh type of investigation, parabolic curves do not ordinarily yield dependable results for the reason that the resultant expression for curvature of the member, $\frac{d^2z}{dx^2}$, is a constant, whereas the curvature should vary with the bending moment. In problems similar to the analysis of vibrations in suspension bridges, in which the choice of functions naturally gravitates to certain standard trigonometric forms or Fourier expressions, the Rayleigh method appears to have the advantages of greater simplicity and directness without any considerable sacrifice of accuracy.

For example, Eqs. 125 and 126 may be verified exactly, Eq. 124 checked very closely, and Eq. 127 checked within 10% by direct substitution, in a single operation, of the functions

$$z = z_n \sin\frac{n\,\pi\,x}{l} \sin\omega\,t, \qquad \text{for } n = 2, 4, 6, \cdots \ldots\ldots\ldots (146a)$$

and

$$z = z_n\left(\sin\frac{n\,\pi\,x}{l} - \frac{1}{n}\sin\frac{\pi\,x}{l}\right)\sin\omega\,t, \qquad \text{for } n = 3, 5, 7, \cdots \ldots\ldots (146b)$$

[34] "Suspension Bridges Under the Action of Lateral Forces," by Leon S. Moisseiff and Frederick Lienhard, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 98 (1933), p. 1080.

[35] Asst. Chf. Design Engr., TVA, Knoxville, Tenn.

[35a] Received by the Secretary March 13, 1941.

[36] "Buckling of Elastic Structures," by H. M. Westergaard, *Transactions*, Am. Soc. C. E., Vol. LXXXV (1922), p. 576.

in the standard Rayleigh form of the energy equation:

$$\int_0^l \frac{w}{2\,g}\left(\frac{dz}{dt}\right)^2 dx = \int_0^l \frac{E\,I}{2}\left(\frac{d^2z}{dx^2}\right)^2 dx + \int_0^l \frac{Q}{2}\left(\frac{d^2z}{dx^2}\right) z\,dx \;. . (147)$$

In fact, Eqs. 125 and 126 may be verified exactly by using the somewhat rougher form of Rayleigh expression:

$$\int_0^l \frac{w}{2\,g}\left(\frac{dz}{dt}\right)^2 dx = \int_0^l \frac{E\,I}{2}\left(\frac{d^2z}{dx^2}\right)^2 dx + \int_0^l \frac{Q}{2}\left(\frac{dz}{dx}\right)^2 dx \;. . . . (148)$$

in which the term $\int_0^l \frac{Q}{2}\left(\frac{dz}{dx}\right)^2 dx$ reflects the assumption that the increased length of cable due to the vibratory motion is equal to the increased length of elastic line of the deflecting stiffening truss. In using the Rayleigh method for this problem, the x-axis is taken as the undeflected axis of the stiffening truss, and all ordinates z are measured to the deflected elastic line of the truss. No ordinates are measured to the cable. The functions adopted for z simply represent the idea that the stiffening truss deflects during vibration into either an even number of alternating equal bay waves or an odd number of alternating unequal bay waves, the configurations in either case being such that the net departure of the cable during vibratory motion from the parabolic form in equilibrium under the dead load is a minimum. This action is implied by Eqs. 93. In addition, these functions represent the familiar normal modes in which the oscillations of all elements of the elastic system are either in phase or in opposition, and the functions chosen have the convenient conjugate orthogonal property

$$\int \sin m\,x \sin n\,x\,dx = 0, \qquad \text{for } m \neq n \;. . . . . (149)$$

In this connection, it is recognized that the author's primary purpose is to give examples of sufficient variety to demonstrate the power and range of the method of complementary energy. He is well aware, of course, that in certain instances alternative methods may prove equally convenient, as is shown by his statement that the problem of buckling of a thin circular disk supported at the edges and loaded by a normal edge load may be solved with equal ease by the standard method of attack using Bessel functions.

Dean Westergaard has rendered continued service to practicing engineers by demonstrating the great economy of time and labor, and the penetrating insight into structural action, that are afforded by the use of the more incisive methods of advanced analysis. Without attempting the mastery of existence theorems, reasonable facility in the use of the variational principles may be acquired for a surprisingly small expenditure of time; and, as remarked by Lord Rayleigh sixty years ago,[18] the difficulty in connection with Fourier series lies not in its practical physical applications, but only in a rigorous demonstration of what the professional mathematician means by the statement that the expansion represents the function "almost everywhere."

[18] "Theory of Sound," by Lord Rayleigh, 1877, 2d Ed., MacMillan & Co., Vol. 1, 1894, pp. 109 and 257; see also "Vibration Problems in Engineering," by S. Timoshenko, Van Nostrand Co., 1928, pp. 55–60.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

AN INVESTIGATION OF STEEL RIGID FRAMES

Discussion

By Messrs. M. Hirschthal, and Jaroslav J. Polivka

M. Hirschthal,[16] M. Am. Soc. C. E. (by letter).[16a]—Tests of large-scale models of indeterminate structures are of the utmost value to the engineering profession as a verification of theory (or its disproof), as well as for the clarification or decision of controversial questions in connection therewith. However, to accomplish such results, meticulous care must be exercised so that neither a predetermined theory nor methods pursued in similar investigations exercise any influence on the researcher in his tests. It is to be feared that the authors permitted themselves to be somewhat influenced in both ways. There is no question of the great advantage to be gained in planning tests that will either confirm or deny results previously obtained by the selection of similar forms for tests, but no attempt should be made to read into the results identity with the prototypes (tests) or to resort to "modified sections" to confirm theoretical analysis in which assumptions must necessarily be made as a basis for logical results. The tests were well planned, yet it is questionable whether the degree of accuracy was sufficiently great when the variations between assumed theory and stresses obtained from strain readings are of approximately the same amount at times as the expected accuracy of 300 lb per sq in. in the instruments used. The authors state that the number of readings or observations was governed by the desirability of limiting the number of holes made in the angles. Could not the remedy have been found in welding and thus this difficulty been eliminated? Gage lines along the gravity lines of each of the angles of the curved inside section might have furnished valuable information. In selecting the configuration for the curved-knee frame (Fig. 1(*b*)), the authors made the crown thickness 6 in., carrying the soffit of the beam parallel to the extrados, whereas the leg is $7\frac{1}{2}$ in. thick at the base. The result is that the sections at the diagonal through the knee are not equal—the vertical section through the beam is $13\frac{7}{8}$ in. deep and the horizontal section through the leg

Note.—This paper by Inge Lyse, M. Am. Soc. C. E., and W. E. Black, Jun. Am. Soc. C. E., was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by C. J. Posey, Assoc. M. Am. Soc. C. E.; February, 1941, by W. J. Eney, Assoc. M. Am. Soc. C. E.; and March, 1941, by Messrs. LaMotte Grover, and William R. Osgood.

[16] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

[16a] Received by the Secretary March 6, 1941.

is $15\frac{3}{8}$ in. deep (Fig. 31). In the case of the square-knee frames, these sections are of the same depth, which is preferable.

An examination of Fig. 18 discloses the fact (and it is pointed out by the authors) that, from the tangent point 22 (opposite 2) around the curve to the tangent point 32 (opposite 19), the observed stresses (computed from the strains) are considerably higher than those computed by the "beam formula." Beyond these points, both in the horizontal beam and in the vertical legs, the stresses are lower than those computed theoretically. The writer differs from the authors on two points as to the reasons and the results of these conditions.

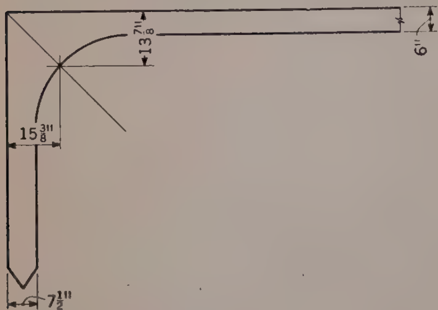

Fig. 31

First: Since points 32 and 22 and the points opposite them are subjected respectively to the maximum compressive and maximum tensile stresses, it is evident that (*a*) the sections through these points (practically the points of tangency) are at the points of maximum negative moment of the respective horizontal and vertical spans; (*b*) that they are the "fixed ends," and (*c*) that the section between, or the knee section, furnishes the restraint for such fixedness. Under such conditions (Fig. 11) the stress distribution along a diagonal across the knee would not follow the "straight-beam" theory, but rather, more nearly, that resulting from a thrust against this diagonal section. The resistance to this thrust is offered by the remainder of the frame (the other half of the knee and the leg) down to the hinged support (Fig. 32). The bending moment at the end of the beam span, resulting in tension in the upper plane and compression in the lower, together with the vertical shear, may be considered replaced by a thrust acting eccentrically on the section, the eccentricity being equivalent to that resulting in the aforementioned bending moment. From the observed location of the zero strain (therefore, zero stress), the resultant thrust is evidently applied very close to the inside edge of the diagonal. The point of zero stress under this assumption would be located at three times the distance from the inside edge to the point of application of the resultant, similar to the condition of a

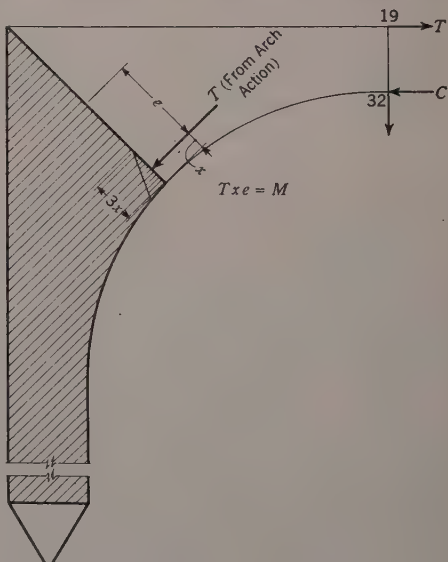

Fig. 32

footing on rock or other unyielding material, in which the resultant of the pressure falls outside of the middle third of the base. As long as there is no yielding under the point of maximum compression (in other words, if the surface resists distortion), there is no possibility of a tensile stress at the outer edge.

Second: From the fact that the extreme fiber stresses between the tangent points of the horizontal and vertical members, as computed from the observed strains, were considerably greater than those calculated from the theoretical assumptions, the authors conclude that the knee was "not stiff enough." The writer differs with this conclusion, and feels that on the contrary the knee, because of its very stiffness, was capable of taking greater stresses than assumed. Thus it relieved the sections of the horizontal beam, and the vertical legs beyond the knee, of part of the stresses or moments, resulting in the lower actual observed stresses noted in those sections. The situation is similar to that occurring at the junction of two members, such as a column and a beam, subjected to a bending moment, in which case the stiffer member takes the larger share of the moment. Likewise, within a member itself the section having the greater stiffness will take the greater part of the moment. Perhaps this is the explanation of the fact that a reinforced-concrete beam never yields a deflection anywhere near that calculated for the particular condition of loading. A well-proportioned beam has about one third of its tensile reinforcement bent up for shear and diagonal tension, and this reinforcement is carried into the opposite plane for proper anchorage, thus providing a cross section at the supports far stiffer than that assumed for the theoretical consideration of a reinforced-concrete beam design from which the deflection is ordinarily computed.

The condition at the knee is not unlike that posed when a deep girder is connected with a relatively light column and the column selected is found to be overstressed. An increase in the column section will result only in an increase in the share of the bending moment it will be required to resist. Thus, it is evident that, to relieve the column of its overstress, it will be necessary to increase the girder section at its junction with the column so as to take a greater share of the moment because of its increased stiffness. This, however, does not end the problem, which cannot be treated herein.

Square Knee.—In Fig. 16, which shows the distribution of stress for the square-knee section, it is to be noted that the compressive stress in the corner (the junction of the soffit of the horizontal beam with the vertical leg), from gage observation, is much higher than that which had been computed—the theoretical stress. F. E. Richart, M. Am. Soc. C. E., in his tests of reinforced-concrete knees at the University of Illinois, found a similar concentration of stress at the inside corner of the specimen. At that time, the writer had suggested that this might have resulted from the fact that the horizontal section through the leg and the vertical section through the beam, at their junction, have a common extreme fiber in compression and such a concentration of stress was to be expected. Where these sections are at right angles to each other, the resultant compressive stress would be the square root of the sum of the individual squares of the individual extreme fiber compressive stresses; and, where the depth of section and reinforcement are the same in both sections,

this resultant stress would be 1.41 times the compressive stress of either. This would also apply to the structural steel section. This assumption appears to be practically verified in Fig. 16 where the observed stress in the horizontal member is about 22 kips, whereas the theoretical stress is only 15 kips per sq in., although the section for the vertical member does not quite corroborate this value. For the foregoing reason, the writer would suggest including this consideration in the recommendation for design that the authors include in recommendation 1.

In the opinion of the writer the results of the tests are not quite conclusive and do not justify the other definite conclusions embraced in the recommendations.

Jaroslav J. Polivka,[17] M. Am. Soc. C. E. (by letter).[17a]—The investigation is a valuable contribution to the analysis of framed structures with variable moments of inertia. In the analysis of such types of structures, certain assumptions are made, and it is important to know how nearly these assumptions conform with the actual behavior of the structure, as proved by reliable tests.

The authors' comparison between the results of the tests and conventional methods of analysis shows that a fair conformity may be obtained. The observed horizontal reactions differ slightly from the computed values, considering the influence of shear on deformation; but the computed center moments and deflections are as much as 10% less than observed values—a fact affecting, seriously, the design of the structure and its safety.

The writer has analyzed, completely, both types of hinged frames subjected to investigation, using the "ellipse-of-elasticity" method, and has found a very good conformity with the results of the tests. The method is essentially graphical. Better accuracy may be obtained by expressing the constants of the ellipse of elasticity by algebraic values. Generally, the method requires some knowledge of geometry (especially projective geometry), but the analysis is greatly simplified in cases where the change of moment of inertia can be expressed by algebraic terms permitting the use of tabular values. Three of the most common of such cases are demonstrated in Fig. 33.

Eqs. 1 and 2 are correct as long as the sections Δs are very small so that the elastic centroid of the section may be properly assumed as coinciding with the geometrical center of gravity. For sections in Fig. 33 (relatively long with respect to the change of the depth of the structural part) the elastic centroid approaches considerably nearer to the smaller cross section. It affects the elastic weight $\frac{\Delta s}{E\,I}$ and other elastic constants of the section considered, such as semi-axes of ellipse of elasticity, angular rotations, and respective centers of elastic rotations.

In Fig. 33, let: s = distance between elastic centroid S and face B; G = the elastic weight of the element $A\,B$, concentrated at elastic centroid S; and d = the distance between center of rotation (produced by a force F acting along the face B) and face B. Then the following equations express the elastic

[17] Research Associate, Univ. of California, Berkeley, Calif.

[17a] Received by the Secretary March 17, 1941.

deformations of the face B with respect to the face A due to pure bending:[18]
Angular rotation—

$$\Delta\delta = F\,G\,s \quad \text{(7)}$$

Displacement along the line of action of the force F—

$$\Delta = d\,\Delta\delta = F\,G\,s\,d \quad \text{(8)}$$

Moment of inertia of elastic weight with respect to the face B—

$$I_G = G\,s\,d \quad \text{(9)}$$

Square of longitudinal semi-axis of ellipse of elasticity—

$$r_1^2 = (d - s)\,s \quad \text{(10)}$$

Considering the effect of shear on deformation, Eq. 9 becomes (see Fig. 33)—

$$I_G' = \int_0^{\Delta s} \left[x^2 + (r_2')^2\,k\,\frac{E}{S} \right] dG \quad \text{(11)}$$

in which: S is the modulus of elasticity in shear; r_2' is the radius of gyration of an element ds; and k is a numerical constant depending on the cross-sectional shape of the member (section). For prismatic members the value of k is $\frac{6}{5}$; and for **I**-shaped members it is approximately $\frac{A}{A'}$, in which A is the full cross-sectional area and A' is the cross section of the web.

The elastic constants of sections having the shapes shown in Fig. 33 are computed as follows:[19]
Straight line—

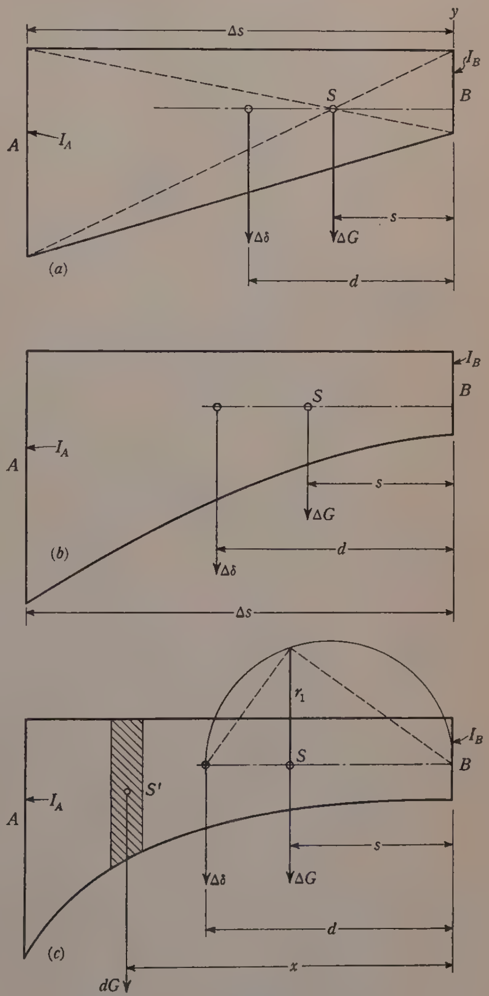

FIG. 33

$$G = G_B\,\frac{2 + C}{2\,(1 + C)^2} \quad \text{(12a)}$$

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[18] "Graphical Analysis of Framed Structures on the Basis of Ellipse of Elasticity," by Jaroslav Polivka, Soc. of Eng. Students, Institute of Technology, Prague, 1919; also, "Kontinuierlicher Bogentraeger auf elastisch nachgiebigen Stuetzen," by Jaroslav Polivka, *Der Brueckenbau*, 1920, Vols. 4, 5, and 6.

[19] "Graphical Methods of Analyzing Statically Indeterminate Structures," mimeographed lectures by Jaroslav Polivka, Berkeley, Calif., 1940 and 1941.

in which I_B is the moment of inertia of cross section at B; and,

$$G_B = \frac{\Delta s}{E\, I_B} \quad \text{(12b)}$$

$$C = \sqrt[3]{\frac{I_A}{I_B}} - 1 \quad \text{(12c)}$$

$$s = \Delta s \frac{1}{2 + C} \quad \text{(12d)}$$

$$d = \frac{\Delta s}{C^3} \left[2 \log_e (1 + C) (1 + C)^2 - C (2 + 3\, C)\right] \quad \text{(12e)}$$

and

$$G\, s\, d = G_B \frac{\Delta s^2}{C^3} \left[\log_e (1 + C) - C \frac{2 + 3\, C}{2 (1 + C)^2} \right] \quad \text{(12f)}$$

Eq. 12*d* is determined exactly by the intersection of diagonals.

Parabolic—

$$G = \frac{G_B}{8} \left[\frac{5 + 3\, C}{(1 + C)^2} + \frac{3}{\sqrt{C}} \arctan \sqrt{C} \right] \quad \text{(13a)}$$

$$G\, s = G_B\, \Delta s \frac{2 + C}{4 (1 + C)^2} \quad \text{(13b)}$$

and

$$G\, s\, d = \frac{G_B\, \Delta s^2}{8} \left[\frac{C - 1}{C (1 + C)^2} + \frac{1}{C \sqrt{C}} \arctan \sqrt{C} \right] \quad \text{(13c)}$$

Sharply curved—

$$G = G_B \frac{1 + b}{2} \quad \text{(14a)}$$

in which

$$b = \frac{I_B}{I_A} \quad \text{(14b)}$$

$$s = \Delta s \frac{1 + 2\, b}{3 (1 + b)} \quad \text{(14c)}$$

and

$$d = \Delta s \frac{1 + 3\, b}{2 (1 + 2\, b)} \quad \text{(14d)}$$

Elastic constants for these three shapes are compiled in tables.[20] For example, using the tabulated values p, q, and u:[20]

$$G = (p + q)\, G_B \quad \text{(15)}$$

$$s = \frac{p}{p + q} \Delta s \quad \text{(16)}$$

and

$$d = (1 - u)\, \Delta s \quad \text{(17)}$$

[20] Discussion by Walter Ruppel, Assoc. M. Am. Soc. C. E., of "Moments in Restrained and Continuous Beams by the Method of Conjugate Points," by L. H. Nishkian and D. B. Steinman, Members, Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. 90 (1927), pp. 167–187.

These formulas are valid for each shape shown in Fig. 33. Only the values of d are affected by shear. Considering the shear, the moments of inertia of the elastic weight with respect to face B are $I_G' = I_G + I_G''$, in which the values of I_G'' (due to shear) are given by the following equations (in which $(r_2')^2 = \frac{I_B}{A_B}$):

Straight line—

$$I_G'' = k\frac{E}{S} \times (r_2')^2 \times \log_e \frac{1 + C}{C} G_B \qquad (18a)$$

Parabolic—

$$I_G'' = k\frac{E}{S} \times (r_2')^2 \times \frac{1}{\sqrt{C}} \text{arc tan} \sqrt{C}\, G_B \qquad (18b)$$

Sharply curved—

$$I_G'' = k\frac{E}{S} \times (r_2')^2 \times \frac{3}{4} \times \frac{1}{1 - b}(1 - \sqrt[3]{b^4})\, G_B \qquad (18c)$$

The framed structure is divided into sections allowing the application of the foregoing formulas (Fig. 34). The redundant horizontal reaction is com-

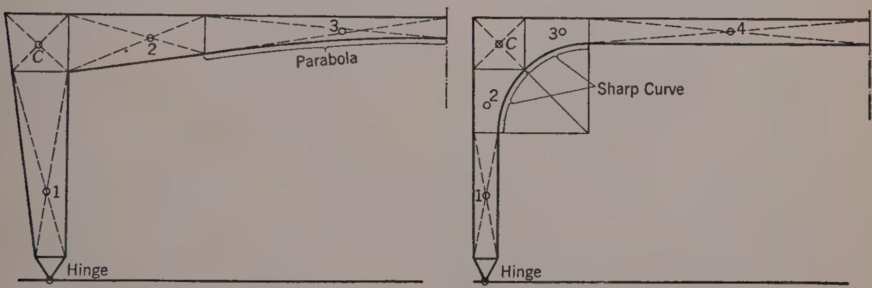

FIG. 34

puted by that one of the following methods of analysis, based on the principles of the ellipse of elasticity, which appears most suitable for a given case:

(1) Compute the angular rotation $(\delta)_M$ for a simply supported beam $A\ B$ due to the given loading, and its center D_M (Fig. 35(a)). The horizontal reaction is then

$$H = \frac{(\delta)_M}{G\, s_H} \times \frac{d_M}{d_H} \qquad (19)$$

in which G is the total elastic weight of the framed structure concentrated at the elastic centroid S; s_H is the lever arm of the elastic weight G with respect to the line of action of the force H (passing through hinges C and D); d_H is the perpendicular distance between the center of rotation D_H of the force H and the line $C\ D$, D_H being the antipole of $C\ D$ with respect to the ellipse of elasticity of the whole structure; and d_M is the perpendicular distance between the center of rotation D_M of a simply supported beam $A\ B$ due to the given loading.

(2) Consider $A\ B$ as fixed at A and B (Fig. 35(b)), and compute the angular rotations of the columns at A and B due to unit moment. These angular rotations define and determine the elastic weights G_A and G_B of supports A and B. Combine the elastic weights of the supports with the elastic weight G_0 of the beam $A\ B$ and determine the end moments M_A and M_B of the restrained beam.

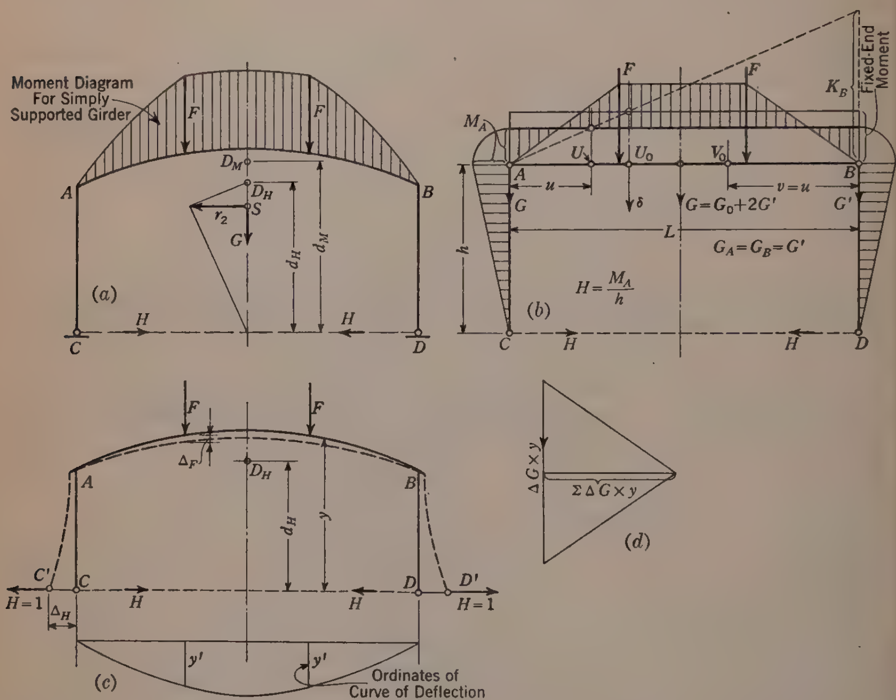

FIG. 35

(3) Determine the deflection Δ_F at the point of application of the force F and the displacement Δ_H along $C\ D$ due to a force $H = 1$. From the basic principle of reciprocity of deformations, the magnitude of the horizontal reaction is found to be:

$$H = -\frac{F\,\Delta_F}{\Delta_H} \qquad (20)$$

(4) When the loading consists of a group of concentrated forces, or of uniformly distributed weights, it is preferable to draw the deflection curve due to $H = 1$, acting along $C\ D$, with the pole distance $\Sigma\Delta G\ y$, which is the influence line of the redundant force H (Fig. 35(c)):

$$H = -\frac{F\,y'}{d_H} \qquad (21)$$

in which d_H has the same significance as in Fig. 35(a). As far as the stress distribution within the knee of the investigated frames is concerned, the results reported by the authors were compared with photoelastic tests, at the University of California,[21] relating to stress concentrations at sharp reentrant angles and curved fillets.

Conclusion.—The tests reported in this paper are checked very closely by the analysis outlined herein.

[21] "Some Stress Relationships in Photoelasticity," by J. J. Polivka and H. D. Eberhart, Assoc. M. Am. Soc. C. E., Proceedings of the Tenth Semi-Annual Eastern Photoelasticity Conference, Cambridge, 1939; also "A New Method of Determination of Separate Stresses from Photoelastic Results on the Basis of First Derivatives of Airy's Function," paper presented by J. J. Polivka before California-Stanford Seminar, March 6, 1940.

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DISCUSSIONS

EXPERIENCES IN OPERATING A CHEMICAL-MECHANICAL SEWAGE TREATMENT PLANT

Discussion

BY FREDERIC BASS, M. AM. SOC. C. E.

FREDERIC BASS,[7] M. AM. SOC. C. E. (by letter).[7a]—The Minneapolis-St. Paul sewage disposal plant was located and designed after a preliminary study extending over a period of seven years, from 1927 to 1934. During this period the science and art of sewage disposal was rapidly advancing, and the design ultimately adopted embodied a number of features not previously available. Perhaps the most notable characteristic of the design is its flexibility, which enables it to function with economy under the varying conditions of flow in the Mississippi River. Among other features which, at the time of design, were considered by some engineers as innovations, but which were included after careful observation and study, were the unusual length of the sedimentation tanks, the elimination of sludge digestion, and the incineration of filtered sludge. The performances and costs of operation, presented in the paper, appear to justify the many studies made by the author.

Although conclusions from the limited period of operation covered by the author must be regarded as tentative, they unmistakably hold special interest for all sanitary engineers. For the purposes of this discussion the author has made available to the writer certain operating data obtained later than April 30, 1939. The records previous to January 1, 1939, because of the usual process of adjustment in a new plant, probably do not represent normal averages of performance and cost as closely as do the subsequent records. Table 15 contains statistics of operation for the year 1940.

The quantity of sewage was somewhat less than expected, the strength expressed in B.O.D. was approximately equal to that expected, but the suspended solids (perhaps because of changing industrial conditions and effect of surface runoff) were more than 50% greater than the expected amounts. This heavy load fortunately was offset by the unusually high removal of suspended solids in the sedimentation tanks. In these tanks (290 ft long with unusually

NOTE.—This paper by George J. Schroepfer, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Messrs. F. C. Roberts, Jr., and Rolf Eliassen.

[7] Cons. Engr.; Head, Dept. of Civ. Eng., Univ. of Minnesota, Minneapolis, Minn.

[7a] Received by the Secretary March 3, 1941.

long overflow weirs) the average removal of 71.1% (detention period 1.5 hr) of suspended solids, which itself is greater than the performance of most plants, has been raised under average conditions over a period of several weeks to approximately 80% by the use of air flocculation at the trifling cost of 6 cents

TABLE 15.—OPERATING DATA FROM JANUARY 1, 1940, TO DECEMBER 31, 1940

Item	Description	Monthly			Expected at time of design
		Maximum	Minimum	Average	
1	Flow (mgd)	113.87	90.16	104.17	122
	Sewage Strength (ppm):				
2	Suspended solids	345	245	300	185
3	B.O.D.	240	160	210	200
4	Screenings (cu ft per million gal)	1.96	0.78	1.06	3.1
5	Grit (cu ft per million gal)	9.3	3.1	6.2	4.1
	Total Removal in Plant (%):				
6	Suspended solids	83.5	64.0	77.2	56
7	Settleable solids	96.5	84.1	91.5	....
8	B.O.D.	53.0	35.4	43.2	36
9	Filter cake produced (tons), average daily	340.2	264.7	308.3	235
10	Cake moisture (%)	68.1	61.1	64.8	68
11	Cake volatiles (%)	68.6	51.4	59.8	55
12	Ash from sludge (tons)	1,760	820	1,275	....
	Sludge Conditioning Chemicals (%):				
13	Ferric chloride	2.32	1.59	1.83	3.0
14	Lime	6.53	3.53	4.51	10.0

per million gal treated. The expected 56% removal of 185 ppm would have left 81 ppm in the effluent, whereas the actual performance of 80% removal of 300 ppm has left only 60 ppm in the effluent.

The B.O.D. removal was boosted by flocculation as well as by removal of suspended solids, the author's figures showing 49.8% with flocculation as against 44.2% without flocculation. The total capacity for removals of suspended solids and B.O.D. with the use of chemical precipitation will be awaited with much interest. From the performance of the plant already given, the maximum removals as expected by the author, in the tabulation preceding the heading "Plant Operation: Screen and Grit Removal," appear to be readily obtainable. The bacteriological results from the use of chlorine, although of short duration, are interesting.

The conclusions from this phase of the operation seem to extend the usefulness of this type of flexible plant design to many situations where a variable river flow exists. For such conditions, a careful study of quantity and quality of both river and sewage may result in economy of both first cost and operation.

The sludge disposal department of the plant appears to have been as successful as the sewage treatment department. Drawing sludge frequently from the sedimentation tanks to the concentration tanks and stopping withdrawal with thinning sludge have resulted in a heavier sludge. A quantity of sludge 35% higher than expected, offset by higher heat value, less moisture, higher volatiles, and unusually close control of sludge conditioning and filter operation, has led to a lower over-all cost of sludge disposal than was estimated. It is remarkable that practically no oil fuel is necessary. The costs given by the author compel attention.

The improvement of the river above the plant is very satisfactory in appearance, as the analytical results indicate. Improvement of conditions in the river below the plant can be appraised only after the installations of treatment plants for the large packing plants located there.

Supplementary data on cost supplied by the author permit the summary in Table 16 for the 12-month period ending December 31, 1940:

TABLE 16.—COST OF OPERATION AND MAINTENANCE, JANUARY 1, 1940, TO DECEMBER 31, 1940

Item	Description	Operation	Maintenance	Total	Percentage of total
1	Administration, engineering laboratory....	$74,637.24	$1,326.01	$75,963.25	25.6
2	Sewage treatment..........	43,843.65	8,917.08	52,760.73	17.8
3	Sludge disposal..........	112,580.62	18,218.60	130,799.22	44.1
4	Miscellaneous..........	33,508.90	3,376.68	36,885.58	12.5
	Total..........	$264,570.41	$31,838.37	$296,408.78	100.0

In this period a total of 38,038 million gal of sewage was treated at a cost of $7.80 per million gal for all operating and maintenance items. During this period the magnetite filters have not been in full operation and chemical precipitation has not been used. Consequently, over a sequence of years, which includes a considerable proportion of drought when all facilities of treatment are used, the costs may be somewhat higher. It might be concluded that the cost of administration, engineering, and laboratory, at about 25% of the total cost of operation and maintenance, is unduly high, but it is probable that every dollar thus spent is reflected in much greater savings in the other items.

It has been a pleasure to examine the detailed records of performance and accompanying cost data available in the office of the author. It will be a great public benefit when all sewage works are as carefully and economically managed as the one in Minneapolis and St. Paul; plant operators will be stimulated to greater interest when comparative figures are made available. These cities are indeed fortunate in having conceived and built a plant so well fitted to the existing conditions and so effectively and economically operated.

The emphasis in this paper upon performance, and particularly upon costs, deserves more than passing notice at this time. A sewage treatment plant is but one of the agencies of the promotion of health and cleanliness of a community; there are others of perhaps even greater importance, which need consideration and call for the expenditure of public funds. Since public financial resources are limited, the analysis of costs and relative value of various public works is an important duty resting upon engineers who are becoming increasingly responsible for the conservation of national resources.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

THE GRAND CENTRAL TERMINAL IN PERSPECTIVE

Discussion

BY MESSRS. JAY DOWNER, AND WILLIAM J. WILGUS

JAY DOWNER,[33] M. AM. SOC. C. E. (by letter).[33a]—All those who have been concerned with the development of Westchester County are enthusiastic in their appreciation of the important influence of the Grand Central Terminal on the growth and character of the suburban territory to the north of New York City. It has been recognized that in all respects this terminal is the best in the country, and its convenience as compared with other large stations has been particularly appreciated. Colonel Wilgus has presented a comprehensive and readable account of the inception and execution of this great undertaking, which eventuated from the foresight and ability displayed by him and his associates.

Without in any way detracting from the importance of this improvement, some exception may be taken to the assumption of full credit for the increase in assessed valuations in Weschester County, which rose from about \$172,000,000 in 1900 to \$1,828,000,000 in 1932. During the construction of the Bronx River Parkway, valuations rose from \$383,000,000 in 1913 to \$786,000,000 in 1923, after which, and during the development of the more comprehensive Westchester County Park System, the rise was accelerated to a high level of \$1,828,715,000 in 1932. During these nine years the county, with some aid from the state, had expended upwards of \$77,000,000 for its park and parkway system, which was in addition to the earlier Bronx River Improvement costing about \$17,000,000. In addition, trunk sewer systems and a county highway system helped to develop Westchester's suburban area.

Therefore, much as one may appreciate the fundamental importance and basic value of the Grand Central Terminal and the electrification of the three railroads leading through the county, one feels justified in asking the railroads

NOTE.—This paper by William J. Wilgus, Hon. M. Am. Soc. C. E., was published in October, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1940, by Messrs. F. Lavis, E. R. Hill, Alonzo J. Hammond, and H. L. Ripley; January, 1941, by Messrs. A. J. Meehan, and J. P. Hallihan; February, 1941, by Messrs. Arthur V. Sheridan, and C. E. Smith; and March, 1941, by Messrs. Harold M. Lewis, and Bion J. Arnold.

[33] Cons. Engr., New York, N. Y.

[33a] Received by the Secretary March 28, 1941.

to share with the far-flung parkway, trunk sewer, and highway systems some of the credit for the increase in values that were notably accelerated during the construction of these county improvements. At one period those who were instrumental in promoting the County Park System claimed credit for 75% of this increase. In Colonel Wilgus' paper it is assumed that the railroads may be credited with 100%. Each major improvement was responsible in greater or lesser degree for the phenomenal rise in values, and it would be impossible to apportion the credit accurately. Perhaps there is credit enough for all. Certainly the Grand Central Terminal and the electrification of the railroads contributed magnificently to the amenities of life in Westchester.

WILLIAM J. WILGUS,[34] HON. M. AM. SOC. C. E. (by letter).[34a]—In this closure to his four-dimensional paper, in which the "element" of time has played a part, the writer first would express his deep sense of appreciation to the Society in general for having given it a place in its publications, and in particular to those of his fellow members whose scholarly contributions have given to it life and color. Thus has the product of the spirit, brain and brawn of many distinguished men and their aides in engineering, architecture, contracting, law, finance, operation, and administration been given "its day in court."

Both Mr. Hammond and Colonel Arnold point to the example set by the Grand Central Terminal in the utilization of "air rights" under somewhat similar circumstances at Chicago and, as Colonel Arnold states, at Cleveland. In this connection it should be remarked that the first plan of the Grand Central Terminal showing a lofty station building and adjoining hotel was not that of Mr. Reed, as thought by Colonel Arnold, but that of Samuel Huckel, Jr., who prepared such a study to illustrate the writer's plan for the improvement based on the "air rights" idea submitted to Mr. Newman, President of the railroad, on December 22, 1902. Mr. Reed's brilliant conception of the Court of Honor and circumferential driveways came some months later, and brought to his firm its engagement as architects for the station building and later for the hotel. This has been explained and illustrated in some detail in the complete manuscript.[21]

It is indeed to be regretted, as expressed by Major Hallihan and Mr. Sheridan, that the Court of Honor in its original form was eliminated from the final plans. Had it been permitted to remain, the community would have profited from a separation of street grades all the way north to 49th Street, and from the obvious advantages of a plaza of majestic proportions such as are found in similar situations in Europe. It is not too much to believe that give-and-take negotiations between the city authorities and the railroad might have resulted in a just division of the financial burden and thus have made possible the retention of the Court-of-Honor plan.

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[34] Weathersfield, Ascutney P.O., Vt.

[34a] Received by the Secretary March 18, 1941.

[21] Copies of the manuscript in full have been filed for reference at Engineering Societies Library, 29 West 39th Street, New York, N. Y., at the New York Public Library, and at the Congressional Library, Washington, D. C.

Mr. Smith refers to the mezzanine which, if installed at the Grand Central Terminal between street level and track platform level, would have added very materially to the convenience of passengers. Undoubtedly this is true were it practicable to bring it about, but unfortunately this would have required the raising of street levels and the lowering of track levels which were not permissible.

FIG. 18.—FUTURE GRAND CENTRAL TERMINAL (IF AND WHEN A HIGH BUILDING IS ADDED SURROUNDING THE CONCOURSE)

Mr. Hammond refers to the wisdom of providing separate platforms for the handling of passengers and baggage. Unfortunately, as in the case of the mezzanine, this was not possible at the Grand Central Terminal because of physical limitations. Studies were made for baggage platforms equipped with mechanical conveyers between the running tracks; but this required the objectionable transfer of the building columns to the middle of the passenger platforms and the lessening in number or narrowing of the latter within the

inexpansible width of the terminal area. It was a matter of great regret to the writer that separate means for handling baggage could not be devised without an inadmissible sacrifice of passenger platform space. Mr. Hammond also mentions the happy effect of sunlight streaming through gable windows upon crowds of passengers assembled in the station concourse. It is to be expected that if and when the lofty building is added to the structure, as illustrated in Fig. 18, pains will be taken not to destroy this feature.

Mr. Meehan points to dangers incident to the two right-angled turns in each direction where the circumferential elevated driveways come together at 42d Street. It would be wise indeed to give earnest thought to his suggested remedy. When the structure was built the planners did not sufficiently visualize the speed and volume of street traffic that were to come in later years.

In respect to the change of motive power at the terminal and on its approaches, Mr. Smith considers that "If the New York Central Railroad were adopting a system of electrification today, it is doubtful if it would adopt its present system of third-rail, 600-volt direct current." Unquestionably his company, the New Haven, and the other roads he mentions, have, in the high tension alternating current system, an admirable one for their conditions as proved by the passage of time. His road is indeed to be congratulated on having led the way; but conditions on the New York Central were not of the same order. As is said by Colonel Arnold, at the time when the problem of changing its motive power from steam to electricity arose, there was "no electrification approaching the magnitude of this one." In the words of Mr. Hill, "electric traction was still in its formative stage." On top of this departure from precedent went the need, referred to by Messrs. Lavis, Hill, and Ripley, of keeping traffic moving—without delay or serious inconvenience to the public—at the terminal itself, where hundreds of steam-operated trains daily had to be kept moving "on time," coupled with innumerable horizontal and vertical changes of position of the tracks during two-level, underneath and overhead construction. This condition, if for no other reason, barred the use of exposed high tension wires supported from poles, as did the narrow clearance of only one inch above the top of steam locomotive smoke-stacks in the Park Avenue tunnel bar the use of high tension working conductors suspended from its roof. Apart from these physical obstacles there was the legal obstacle to the erection of pole-supported wires on the Park Avenue Viaduct and elsewhere within the city limits. Along with these prohibitory conditions went the state of the art which led the members of the Electric Traction Commission unanimously to decide that in recommending so revolutionary a change of motive power affecting the safety and reliability of a vast number of through and suburban train movements, they should adhere to what was then deemed to be least experimental. The adopted system, so far as the writer knows, has worked successfully since the running of the first electric train on September 30, 1906. Were the work to be done over again, and under present-day conditions, it is very possible, and even probable, that the New Haven method would receive serious consideration. The physical obstacles to its adoption in the first decade of the century have disappeared and perhaps those of a legal nature could be removed.

Mr. Smith gives some very interesting data showing the let-up in the rate of increase in his company's number of passengers entering and leaving New York City during and immediately following the depression from 1907 to 1915. With continued steam operation after 1907 it would have been impossible to have won much, if any, increase. The old Grand Central Station yard had reached the limit of its capacity, and the terrors of the smoke- and gas-infested Park Avenue tunnel had their deadening effect on further growth of Westchester County suburban communities. That the electrification, in the long run, did promote a marked upward trend in the traffic of both roads using the terminal is shown by the tabulated data in the paper.

As to planning for coming years, Mr. Hill very wisely asks who will say what the future holds? But is not all life a gamble? No one knows what the morrow may bring forth; and yet one may prophesy as best he can from day to day in preparation for what he believes may occur. Mr. Smith points to the decrease of some 20% in number of passengers handled at the terminal between 1930 and 1939 as indicating no need for material enlargement in its capacity and none for an auxiliary terminal at 149th Street in the Bronx (Mott Haven). As an auxiliary the writer agrees that, no doubt, a Mott Haven terminal is unnecessary; but with Mr. Sheridan and Mr. Lewis he considers it to be highly necessary to the community it would serve numbering a million and a half people. It would not only there afford a meeting place for rail, highway, and air carriers to and from nearby and distant regions, but also, in the improvement of the great Mott Haven open yard (now a deterrent to the progress of that community), it would do for the Bronx what the Grand Central Terminal has done for mid-Manhattan.

Mr. Sheridan and Mr. Lewis offer differing suggestions for an improved Park Avenue north of 96th Street and extending to the site of the proposed Bronx Terminal at 149th Street and beyond. It would seem clearly evident that the one of these to be approved by public authority, including the "grand-centralizing" of the Mott Haven yard, should have its details developed in full for immediate use when the United States again will be forced to make a frantic search, as in 1933, for worthy projects on which to engage its idle men, machinery, and materials otherwise going to waste.

Messrs. Downer and Lavis question if the building up of Westchester County can be attributed to the terminal improvement and electrification of its approaches to the extent indicated in the paper. In this they are joined by Mr. Sheridan, and as well by Mr. Smith, in respect of both the Grand Central Zone and the region north of New York City. The writer now realizes that he did not make sufficiently clear that, in designating the improvements in question as the "primary cause" of the increase in assessments, the "impulse" that started things going, the "spark" that started a train of events, he failed to dwell as he should on what followed, such as the erection of magnificent structures between Madison and Lexington avenues from 42d Street to 96th Street, and the initiation and creation of the remarkable parkway system and accompanying high-class residential development in Westchester County. The thought he intended to convey was that the beginning in these respects—and there always has to be a first cause—laid in the enterprise he has attempted to

describe, very much as the child is looked upon as the father of the man, the acorn as the oak's beginning, and the Erie Canal as the greatest single factor in bringing to the State and City of New York their preeminence in the nation during the first half of the nineteenth century. If in this he is deemed to have gone too far, it is, he hopes, not to be taken as a lack of attachment to the cause of truth.

The writer intended to establish the claim that the transformation in question eliminated deterrents to developments that followed. The limitations of the Grand Central Station on further train service, the dreadful dangers and discomforts of the Park Avenue tunnel, the dirt and noise of steam-operated trains, the hazards of grade crossings and the occupancy of city streets in such important communities as Mount Vernon, White Plains, and Yonkers—all these disappeared with the changes made financially feasible by the utilization of air rights which, until then, had been fallow.

Then came the realizations that Park Avenue could be made a fashionable thoroughfare of the first order and Westchester County a fruitful field for parks and parkways having no peer.

In conclusion the writer would indulge himself in a renewal of his thanks to his fellow members who have so generously and enlighteningly given of themselves in the discussion, and also to those who will have paid him the compliment of reading through this record to the end.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

FORT PECK SLIDE

Discussion

BY JOEL D. JUSTIN, M. AM. SOC. C. E.

JOEL D. JUSTIN,[11] M. AM. SOC. C. E. (by letter).[11a]—There is only one good thing about a slide such as occurred at Fort Peck. If it is thoroughly investigated and studied, the engineering profession is thereby better equipped to prevent similar slides in the future. Therefore, Mr. Middlebrooks' paper is worthy of careful study by all engineers interested in the design and construction of earth dams. It should help to clear away the misconceptions of many engineers who have not had the opportunity to study the investigation that followed. Thus, one engineer recently told the writer that he was opposed to using a hydraulic-fill dam at a certain location because hydraulic-fill dams had a habit of sliding upstream!

The investigation of the Fort Peck Slide was undoubtedly the most thorough and extensive that has been made in connection with any earth-dam slide during construction. The importance and size of the structure and the investment involved justified the extent to which the investigation was carried. Any one who takes the trouble to study the complete record of the investigations[12] thoroughly will reach the conclusion that the cause of the slide was insufficient shear strength in the foundation to resist the stresses imposed by the dam as originally designed at the section where failure occurred.

The main dam is approximately 10,000 ft long, but the slide was limited to the upstream part of the dam, approximately between stations 4 and 30. The investigation gave no indication that the foundation under the downstream part of the dam between stations 4 and 30 was any better than the foundation under the upstream part of the dam in this section, but there was no movement in the downstream part of the dam at any point. In this connection, it is significant to note that, whereas the average upstream slope in the section where the slide took place averaged 1 on 4, the slope of the downstream face in the same section averaged 1 on 8.5. It is also significant that immediately

NOTE.—This paper by T. A. Middlebrooks, Assoc. M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1941, by Jacob Feld, M. Am. Soc. C. E.

[11] Cons. Engr., Philadelphia, Pa.

[11a] Received by the Secretary March 6, 1941.

[12] See "Report on the Slide of a Portion of the Upstream Face of the Fort Peck Dam," Corps of Engineers, U. S. Army, July, 1939.

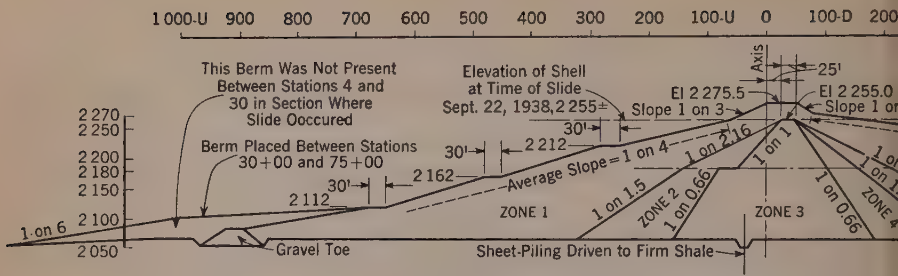


Fig. 13.—Typical Cross Section of Fort Peck Dam

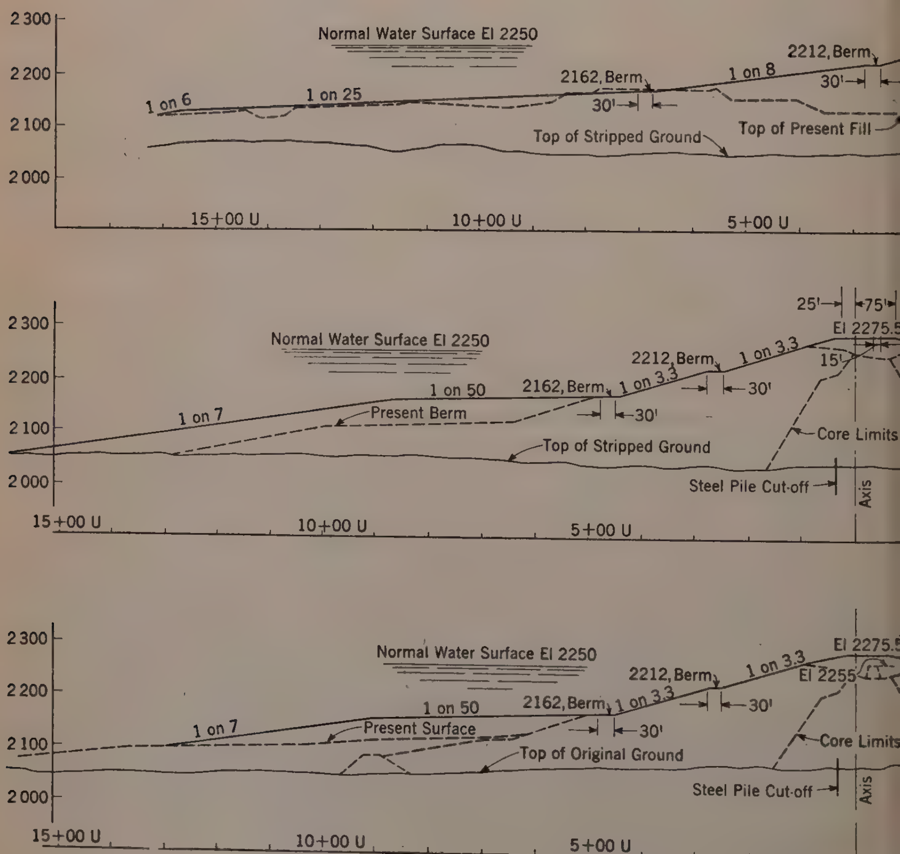
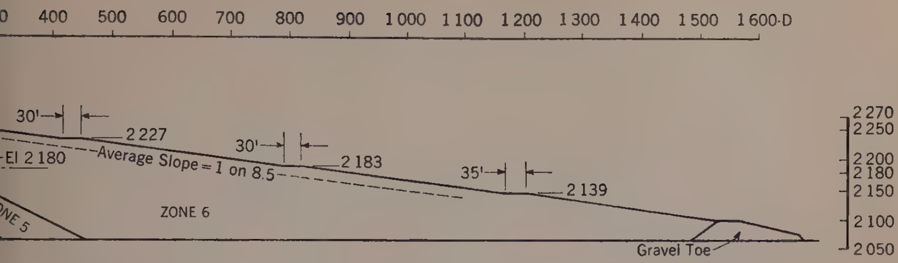
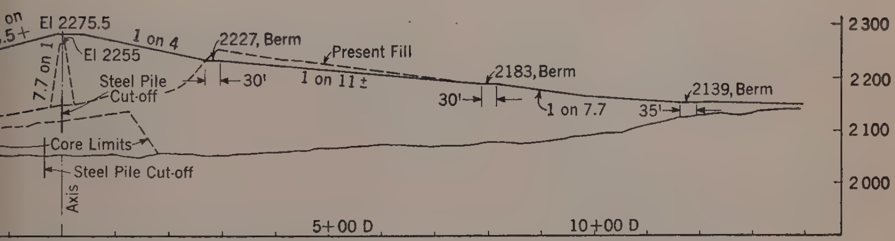
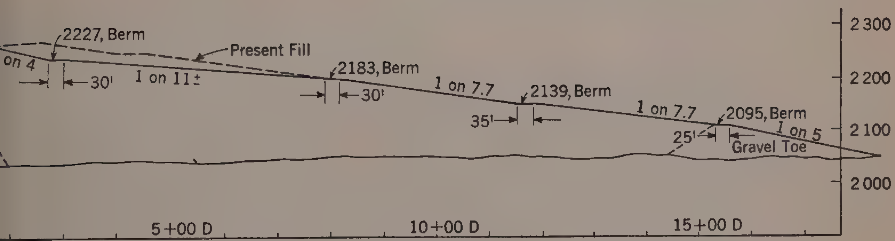


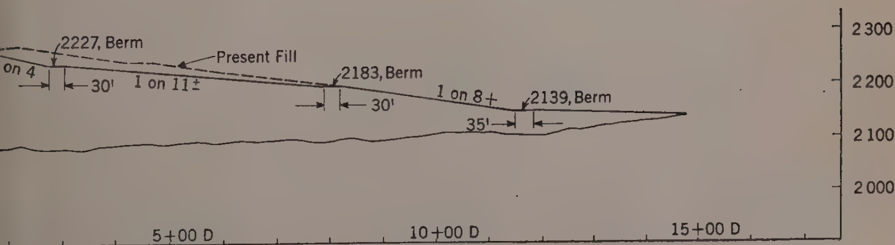
Fig. 14.—Typical Cross Sections of Fort Peck Dam



IGINALLY DESIGNED AND CONSTRUCTED PRIOR TO THE SLIDE







DESIGNED AND RECONSTRUCTED AFTER THE SLIDE

north of the slide area there was an extensive upstream berm at El. 2,112 extending from Station 30 to Station 75 (see Fig. 13).

The flatter downstream slope resulted in lower intensity of shear stress both in the foundation and at the base of the dam. Similarly on the upstream side, the berm north of Station 30 resulted in lower intensity of shear stress in the foundation just north of Station 30, where there was no movement, than in the foundation just to the south of Station 30 where the slide took place.

Thus from the foregoing it is evident that at the section where the slide took place, not only was the foundation weaker, as shown by Mr. Middlebrooks, but also the unit shearing stresses in the foundation were higher than in the adjoining section.

As a result of the investigations and studies conducted after the slide, and in accordance with the decisions of the Board of Consultants appointed for the purpose, the design was modified and the sections shown in Fig. 14 were adopted and executed for the reconstruction.[12]

Hydraulic-fill methods were used in placing the larger part of the additional shell material within the slide area and also for the extensive additional upstream berm throughout the remainder of the dam. The new core (which was a highly impervious glacial till from the north abutment) and the topping off of the dam, were constructed by rolled-fill methods. The upper part of the downstream face was steepened to 1 on 4, as indicated in Fig. 14, and the material thus salvaged was used in topping off the shell.

In effect, the slide provided a life-sized laboratory test for determining the shear resistance of the foundation. Consequently, the determination of the factor of safety for the redesigned section may be accepted with considerable confidence.

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DISCUSSIONS

RECOMMENDED PRACTICE AND STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

Discussion

By Messrs. Harold E. Wessman, N. T. Stadtfeld, C. A. Ellis, R. H. Sherlock, S. C. Hollister, Thomas K. A. Hendrick, Morris Berman, F. R. McMillan, and Meyer Hirschthal

Harold E. Wessman,[58] M. Am. Soc. C. E. (by letter).[58a]—The framers of any code of specifications have a rather difficult task in allowing enough latitude for the exercise of judgment on the part of an engineering designer, and at the same time providing enough specific information to avoid needless controversy in checking designs—in other words, in discussions between city building departments and designers.

The Joint Committee Specifications and Recommendations are an excellent piece of work in this respect. They allow latitude for judgment, but at the same time provide explicit rules that in general are much better expressed than preceding specifications. The development of new knowledge, of course, may result in changing these rules in the future, even as the requirements of earlier codes have been changed. In fact, probably the chief function, now, of the code committees of the various constituent organizations subscribing to the 1940 Joint Code is that of a continued critical study of these rules in the light of new research.

It seems likely that several city building codes could stand complete revision—a complete rephrasing—in the light of the Joint Committee Report. Although it is realized that procedures of amendment and change are slow, nevertheless it is to be hoped that the Report will be taken very seriously by specification writers in the future.

Note.—This Report was published in June, 1940, *Proceedings*, Part 2. Discussion on this Report has appeared in *Proceedings*, as follows: September, 1940, by L. J. Mensch, M. Am. Soc. C. E.; November, 1940, by Messrs. John C. Sprague, and Walter R. Hnot; December, 1940, by Edward C. Gould, Assoc. M. Am. Soc. C. E.; February, 1941, by O. G. Julian, M. Am. Soc. C. E.; and March, 1941, by Jacob Feld, M. Am. Soc. C. E.

[58] Chairman, Dept. of Civ. Eng., Prof., Structural Eng., Coll. of Eng., New York Univ., New York, N. Y.

[58a] Received by the Secretary March 13, 1941.

N. T. Stadtfeld,[59] M. Am. Soc. C. E. (by letter).[59a]—The "history" of the materials used in concrete that was included in the specifications is extremely important in appraising the Joint Report. The writer will confine his remarks to portland cement, although there is much that could be said about the history of the aggregates. The New York City Board of Water Supply (BWS), under its Specifications, inspected, tested, and shipped more than 3,000,000 bbl of cement since 1937 with the result that there are in existence thousands of kiln temperature charts and kiln rotation charts. On these records, again and again, occurs the notation, "Clinker discarded; not to be used for B.W.S. cement"; and each such notation may represent one hour of burning, ten hours of burning, or even two days of burning! For BWS work an improperly burned clinker is rejected; but as a cement plant never discards any clinker (being supposedly 100% efficient) one may wonder who buys the cement made from clinker rejected by BWS. It is the individuals, municipalities, state and federal agencies, because they ask no questions about the history of portland cement! They simply accept cement on the basis of a few laboratory tests that are in themselves no indication of durability.

Engineers worry considerably about the grading of the aggregates, the proper water-cement ratio, and strength to be obtained; and after laboratory tests and computations, they give the result the "high-falutin'" name of "concrete-mix design." Then they make changes in the field to obtain a good looking and workable concrete. By setting a cement factor, not cut down to the last thimbleful, but generous in the allowance for field difficulties, and by proportioning properly graded aggregates to obtain workability, the resultant mix is certain to be satisfactory in every respect. But all this is in vain, if no history of the materials is available, and especially that of the portland cement.

It can no longer be denied that concrete structures are going to pieces—for various reasons. True, many are standing up well; but many are not, as testified by the recent trouble in California,[60] and by experience at Parker Dam where the superstructure is cracking due to expansion.

In the California investigation it was found that the disintegration occurred only where a certain aggregate had been used—and only in those cases where this aggregate had been used with cements high in alkalis. In other words, it was determined that portland cement low in alkalis (the alkalis being the oxides or hydroxides of sodium and potassium) would stand up with this aggregate; but that concrete made with that aggregate and cement high in alkalis would go to pieces. This is a new development and of extreme importance.

The late Thaddeus Merriman, M. Am. Soc. C. E., former chief engineer, and consulting engineer of the Board of Water Supply, was an investigator of portland cement for many years. It was Mr. Merriman who wrote the present BWS Specifications. He maintained that hundreds of thousands of dollars were spent on the investigation of portland cement—all done on samples sent

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[59] Div. Engr., Board of Water Supply, New York, N. Y.

[59a] Received by the Secretary March 14, 1941.

[60] "Expansion of Concrete Through Reaction Between Cement and Aggregate," by Thomas E. Stanton, M. Am. Soc. C. E., *Proceedings*, Am. Soc. C. E., December, 1940, p. 1781.

in to the laboratory, or selected by the laboratory—with absolutely nothing known about the history of the clinker and cement represented by the samples.

Unless the engineer knows that the clinker was made at the proper temperature and left in the kilns for the proper length of time; unless he knows that the clinker was properly handled, and not left out in the rain for more than a year (as the writer has seen it); unless he knows how the flue dust was disposed of—whether it was discarded as it should have been in nearly all cases, or fed back to the raw mix, or even added to the finished product—unless he knows these facts about the cement he is buying or specifying, he knows very little about its essentials. The writer would like to have others enjoy the phenomenon of "no laitance" in concrete placed under the most difficult of circumstances. The fact should be brought to the attention of the engineering world that the history of a product is of the utmost importance.

C. A. Ellis,[61] M. Am. Soc. C. E. (by letter).[61a]—In comparing the old specification for the permissible load on a column[62] with the new (see Section 851 through Section 862), consider a column of, say, 13-in. diameter and a 10-in. core. Let the steel ratio be about 4%. The permissible load under the new specifications is found to be approximately twice the load formerly permitted. Not only is it stated as acceptable practice to include the 1.5 in. outside the core, but also to disregard the ratio n. It will take designers some time to become accustomed to that. In a few years it will become orthodox practice, accepted too casually, until some great disaster occurs in a concrete building to give the profession another jolt, and cause for further research.

R. H. Sherlock,[63] M. Am. Soc. C. E. (by letter).[63a]—It is suggested that the Committee describe, briefly, its attitude on allowable stresses in combined bending and direct stress, stating whether or not there was any rational basis for the formula in Section 861. The writer understands that this formula supplies merely a smooth transition from one stress value to another.

S. C. Hollister,[64] M. Am. Soc. C. E. (by letter).[64a]—The division on two-way slabs (Sections 809 through 815) is presented in its present form for the first time. Its purpose is to reduce to convenient coefficient-table form a member that is very difficult to design by the usual analytical processes. The basis of the coefficients in Table 5 (Section 812) is simply a combination of analytical results of a wide variety of pertinent cases and test data, based on theories presented by Dean Westergaard[5] in 1926.

The panel under consideration is divided into conventional middle strips and column strips. The coefficients to determine positive and negative

[61] Prof., Structural Eng., Purdue Univ., West Lafayette, Ind.

[61a] Received by the Secretary March 14, 1941.

[62] *Proceedings*, Am. Soc. C. E., October, 1924, Section *G*, p. 1203.

[63] Prof., Civ. Eng., Univ. of Michigan, Ann Arbor, Mich.

[63a] Received by the Secretary March 14, 1941.

[64] Dean, College of Eng., Cornell Univ., Ithaca, N. Y.

[64a] Received by the Secretary March 17, 1941.

[5] "Formulas for the Design of Rectangular Floor Slabs and Supporting Girders," by H. M. Westergaard; *Proceedings*, American Concrete Institute, 1926, p. 26.

moments for these two strips are then read from Table 5. Ratios are given for long-span to short-span directions over a considerable range. Suggestions are presented for balancing the coefficient across a support if two adjacent values do not seem in harmony.

One feature that is brought to the attention of the designer is the reinforcement of the corners of panels, especially when such corners are built into masonry supports or rigid supports, such as walls. Experience confirms the analytical treatment, which indicates that bending moments actually exist in such corners, and that, if suitable reinforcement is not provided, cracks will certainly appear.

The Report presents rules for determining the bending moments and shears for completing the remainder of the design.

THOMAS K. A. HENDRICK,[65] ASSOC. M. AM. C. E. (by letter).[65a]—In the ever-recurring crossword puzzle on the quality of concrete, the Report of the Joint Committee this year—as in the past—has failed to include the modest seven-letter word, h-i-s-t-o-r-y. The history of what? The answer is the history of the manufacture of the cement. There is no one who would maintain that the quality of the cementitious materials has little effect on the quality of the concrete, for the cement and water form the glue that makes the entire mass tend toward being monolithic.

The quality of the cement, moreover, is interlocked with the history of the cement. For instance, it is known that the speed of rotation of the rotary kiln, and the temperature gradient through which the ingredients of the cement pass, have a definite effect on the degree of burning of the clinker from which the cement is ground. Also, it is known that the use of water in cooling and grinding the clinker may have a deleterious effect on the later products of hydration. Furthermore, the manner of storing the clinker subsequent to its calcination has a functional relationship to the manner in which the resulting cement behaves later.

By carefully kept records, the engineer should be able to know—not guess at—the actual record (history) of the speed of rotation of the kiln. The engineer then would be in a position to explain the variations in the speed of kiln rotation and the causes for such variations during the manufacture of the cement. From the actual record (history), also, the engineer should be able to know the variations of the temperature gradients and the causes of such temperature variations during the formation of the clinker. How, where, under what conditions, and for how long a period of time of storage the clinker has been kept are pertinent items of specific information that should be available to the engineer at all times as an active historical record in the cement manufacture.

Many engineers do not agree with each other on the merits of various tests, such as the sodium sulphate test which the Committee has suggested (Section 209) for the accelerated soundness tests of aggregates, nor do they see eye to eye with each other on the sugar solubility test (not suggested by the Com-

[65] Asst. Engr. Designer, Board of Water Supply, New York, N. Y.

[65a] Received by the Secretary March 17, 1941.

mittee); but it is safe to say that they do concur in the opinion that the engineer should be responsible for the quality of the concrete.

F. R. McMillan, M. Am. Soc. C. E., has stated that the quality of the concrete is squarely up to the engineer. Of the ingredients required for concrete—namely, coarse aggregate, fine aggregate, water, and cement—the cement is the one that may be very fickle not only during its birth period but also during its later life. Lacking the pedigree of the cement, the engineer may be at a loss in accounting for its subsequent behavior. Important as the water-cement ratio is, important as physical and chemical tests are, these data by themselves, without the history of the cement manufacture, are insufficient for a correct diagnosis. Such phenomena, for example, as "laitance" cannot be explained without the history of the manufacture of the cement. Given a cement with a good birth certificate (accurately filled out and documented), together with a record of a healthy childhood and adult life period, then, with equal care on such matters of the other ingredients, one can confidently expect a healthy concrete, which the engineer wants to give his client.

The statement "the quality of the concrete is squarely up to the engineer" implies that the engineer knows the complete history of the coarse aggregate, the fine aggregate, the water, and the cement; and by complete history is meant all the pertinent facts which an engineer, in the exercise of diligence, care, and intelligence in the discharge of his responsibilities, would reasonably deem to be relevant, competent, material, and satisfactory. Therefore, in the interest of the science of concrete engineering, the Joint Committee Report in the future will do well to include, in an appropriate manner, something on the history of the manufacture of the cement.

Morris Berman,[66] Assoc. M. Am. Soc. C. E. (by letter).[66a]—Where concrete slabs or beams frame into walls, specifications should cover not only the necessary reinforcement of the ends of such slabs or beams for negative moment (as suggested by Section 809(*c*)), but should also contemplate the cracking of the walls through inability to resist such moment. This tendency to crack is to be found mainly on the upper stories, where the weight of the wall and other loads are insufficient to counterbalance the tension in the wall due to slab end moment. Cracking has been particularly noticeable at corners of building walls on the upper stories. Near a wall corner the flexibility of a wall, considered as a vertical beam, is reduced by the restraint of the cross wall, resulting in an increased moment approaching a fixed-end condition. Elwyn E. Seelye, M. Am. Soc. C. E., provided special beams and columns on upper stories for the Queensbridge housing project in New York City, partly to resist slab end moments, but mainly to resist thrust against walls due to temperature. If ends of slabs framing into walls are inadequately reinforced for negative moment, they may crack at the ends; conversely, if such negative reinforcement is introduced, the wall into which the slab frames may crack, so that the designer seems to be "between the devil and the deep blue sea" in either case. The introduction of

[66] New York, N. Y.

[66a] Received by the Secretary March 17, 1941.

framing, particularly at upper-story corners, to transmit slab moments as mentioned, or the provision of clear space over tops of slabs where they frame into walls by a special detail so as to permit end deflection and thus eliminate end moment altogether, are two of the possible solutions of the problem.

Referring to Section 805 of the recommendations pertaining to bending moments in frames, it should be noted that, ordinarily, such moments, when computed analytically, are obtained by methods which, although generally acceptable, are approximate in a degree. Thus, each story is considered as a separate unit, whereas moments developed in any story are actually transmitted, to a small degree, to adjoining stories across column points of inflection. Also, the effects of moving loads and the distribution of moments through torsion are most generally not considered in practice, although they can be by analytical methods already established. In addition, moments and shears due to wind are computed only approximately, particularly for irregularly shaped multistoried structures. Scaled structural models may be used to measure these moments, shears, and reactions precisely (provided the models reproduce the proposed structure in essential detail) so as to allow for transmission of moments between stories, moving loads, and other factors. Thus, if it is desired to determine the maximum unit stress in a column subjected to both direct load and bending due to moving loads on various floors, an influence line for reaction and bending in the column at any girder connection may be determined by measurements at such connection on a model. Each flange of the column should be considered separately for the purpose of adding positive and negative unit stresses due to a unit load along every girder in the bay or on the structure. It is proposed that moments, shears, and stresses be measured by one of the several mechanical devices available for such purposes.

F. R. McMILLAN,[67] M. AM. SOC. C. E. (by letter).[67a]—An ideal specification for concrete would be one in which only the requirements of the final product were stated. Such a specification, however, is impossible, due to the lack of precise definition of the qualities desired and the absence of adequate methods of test for completed concrete structures. Furthermore, there are so many factors contributing to the quality of concrete in the completed structure, including the important factor of workmanship (that is, the human element), that no engineer would have the courage to omit all reference to the various steps necessary in the production of concrete.

The present state of the art is such, therefore, that concrete specifications, like those for some other materials, represent compromises between the ideal mentioned, and the opposite extreme, in which every item is covered in minute detail.

In the sections of the Joint Committee Report concerned with proportioning, two such compromises are offered—each representing an approach to one of the two extremes. In Alternate A (Sections 301-SA to 309-SA), only certain outside limits are specified—maximum water-cement ratio, minimum strength, maximum and minimum slump, maximum size of aggregate, and the rather

[67] Director of Research, Portland Cement Assn., Chicago, Ill.

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general requirement that the concrete shall be plastic and workable. Within these limits, the contractor has considerable choice as to materials and mixtures.

This specification puts the responsibility for the quality of the concrete squarely up to the contractor. In return, however, it gives him a maximum of opportunity to utilize his special skill and experience in the selection of materials and the design of mixtures for the greatest economy.

Tests during the progress of the work to insure compliance with the strength requirement are made by the engineer at the expense of his client. These are based on standard laboratory procedure and laboratory control and curing in order to eliminate variations which might be introduced by variations in field curing.

In Alternate B (Sections 301-SB to 310-S), fixed proportions are specified in the following terms: A definite quantity of cement per cubic yard of concrete, maximum water per sack of cement, the maximum size of the aggregate, range in the percentage of fine aggregate, and a range in slump; also, approximate quantities of fine and coarse aggregate per sack of cement are indicated. Under this specification, the quality of the concrete is squarely up to the engineer.

Presumably on important work, the engineer will inform himself as to local materials, so that he can fix the proportions to give the quality desired, and at the same time insure to his clients the maximum possibilities as to economy in the available materials. It is for work of this character (largely controlled work, where the engineer has the opportunity for these tests) that this type of specification is best adapted.

The estimated strengths, which are to be filled in in the blanks indicated in Table B, are not a part of the specification; they are to be considered purely as information. If the strengths indicated are not achieved, it is the responsibility of the engineer to make such changes as he wants to secure the desired strengths; and he pays accordingly.

If this type of specification is used for work that does not justify elaborate tests in advance, the engineer would use arbitrary quantities in setting up his table of proportions. The Report of the Committee gives tables (Tables 3 and 4) from which appropriate values could be selected.

These recommendations have been criticized, but if they are carefully read and judiciously applied, it is believed that they will prove safe.

Obviously, blanket recommendations of this kind must be conservative, and therefore they cannot be depended upon to yield the maximum economy.

In both these alternate specifications, the principal emphasis is based upon the matter of workability. Deficiencies, both of over-set and too-dry mixes, are recognized and guarded against. In the sections of the Report covering ready-mixed concrete, care was taken to work them out to meet the special conditions encountered in this field. The curing requirements leave something to be desired, indicating certain reluctance on the part of the Committee to give full approval to some of the methods that are now commonly used. The Committee definitely leans toward an all-water curing.

The use of laboratory-cured specimens instead of field specimens, as a means of determining the compliance of the concrete with Alternate A, will be easily understood if it is remembered that the requirements for proportioning

and curing are to be separately enforced. What happens in the field in the nature of unfavorable temperatures, severe drying conditions, etc., should not be confused with the selection of materials or the proportions to be used. The provisions are drawn with that in mind.

To accompany this Alternate A in which the strength is the principal factor, there are clauses intended to cover cases in which the strength fails to meet the specified values. These represent almost the only remedies that could be presented.

Meyer Hirschthal,[68] M. Am. Soc. C. E. (by letter).[68a]—The 1940 regulations for flat slabs in the Joint Committee Report (Section 831 through Section 850) follow those of the 1924 Report[69] rather closely, as there have been few tests that would cause a change in analysis of flat slabs subject to uniform loads. The basic formula (Eq. 6) for total bending moment remains the same as in 1924,[70] giving effect to the reduction of 28% due to plate action, with the coefficient of 0.09 instead of the theoretical 0.125.

The latest Report contains a frank statement that the flat-slab design therein contained is based on empirical methods resulting from tests and modified by theoretical analysis. This Report differs from the 1924 Report somewhat in the arrangement of the material—in the order of the paragraphs, or sections.

There is only a slight difference in the statement of limitations for the application of the flat-slab design and a clarification of the design sections and the identification of the various panel strips.

The provisions for the design of flat slabs at, and adjacent to, discontinuous edges are more specific in the present regulations than in the 1924 Report, where they were covered by a general statement.

The tolerance in the use of moment coefficient is somewhat increased in the latest Report (from 3% to 6%). However, the greatest difference between the two reports is found in the provisions for slab thickness. The 1924 Report provided a rather complicated formula for the dropped panel, or for the slab when no dropped panel is used,[71] containing terms for the diameter of the column capital, the span lengths of the panel in both directions, the ratio of the bending moment at the section in question to that of the entire bending moment, and the width of the band, as well as the unit load.

When the recommended proportions for column capital diameter and width of band were applied to the ratio recommended for negative moment in the column strips, this expression resulted in a simple formula for the thickness of t_1 in terms of the span length and square root of the intensity of the uniform load. This had been arrived at by equating the ratio of the aforementioned negative bending moment (for the column strip) to the resisting moment of the section through the drop panel, or to the column section when no drops were

[68] Concrete Engr., D. L. & W. R. R., Hoboken, N. J.

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[69] *Proceedings*, Am. Soc. C. E., October, 1924, p. 1198.

[70] *Loc. cit.*, Eq. 36, p. 1198.

[71] *Loc. cit.*, Eq. 37, p. 1200.

used, based on unit stresses of 800 lb per sq in. (2,000-lb concrete) in compression and 18,000 lb per sq in. in tension for the reinforcement, which combination of stresses results in a resisting moment of 138 $b\ d^2$.

Instead of following the foregoing procedure, the 1940 Report permits the designer to select a strength of concrete determined upon by him and the allowable stress in the reinforcing steel suitable to the grade adopted; and, by selecting the proper ratio of bending moment in Table 6, he obtains a depth of slab to resist it. However, a modification required in the present specification is based on the consideration that the distribution of extreme fiber stresses across the wide band is not uniform, particularly for the compression side, resulting in a higher stress at the center of the band and curving down to a minimum at the ends. For that reason it is recommended that a factor of 1.2 be applied to the depth found theoretically, resulting in a 20% greater thickness than that found mathematically (Eq. 7).

In addition to this change, limiting (minimum) values are set for roof and floor slabs both in absolute dimensions and in the proportions of span or panel lengths for 2,500-lb concrete, together with provision for the modification for other strengths of concrete in proportion to their cube roots.

In both regulations, attention is called to the necessity of special analysis for types of flat slabs or arrangement of panels not specifically covered in the regulations. Neither the old nor the new Report provides for the condition of load concentrations such as occur in railroad or highway bridges or viaducts. The majority opinion of the Joint Committee was that these moving loads also were special conditions requiring specific analysis.

Concentrations of the character of locomotive loading are, of necessity, not subject to distribution among the panels of flat-slab construction in a manner similar to ordinary uniform loads (or individual concentrations) to produce maximum positive and maximum negative moments for continuous structures. Such conditions of loading involve unloaded panels that cannot obtain in the case of a locomotive, tender, and train load. A design for this type of loading involves conditions of locomotive and tender with, and without, train attached for the various positions, so located as to result in maximum moments at the sections required. Even when such resulting moments are resolved into equivalent uniform loads, these loads may not be arranged so as to result in unloaded panels in the desired locations for ordinary uniform loads on continuous spans. When the panels with their columns are treated as rigid frames, the various conditions of loading would still apply.

A detailed discussion of the treatment of locomotive loadings in flat-slab design, and the actual application of such designs in practice, have been presented elsewhere by the writer.[72]

Attention is called also to the provision in the 1940 Report for beams with girders framing into columns in interior panels of flat-slab design for special loads. At times there are concentrated loads on a flat-slab structure that require a greater depth of member for transmitting the load to the column than

[72] See *Engineering News-Record*, March 29, 1923, May 17, 1928, March 27, 1930, December 8, 1932, and September 21, 1933; also "Design of Girderless Flat Slab Structures for Railroad Locomotive Loadings," *Manual*, A. R. E. A., 1937, pp. 8–53.

the slab will give. It is then necessary to insert a beam and transfer the load of the beam to girders, which in turn transfer it to the columns. The Committee thought it advisable, in cases of that character, to frame the complete panel by girders to make more certain of the transfer in stress and moment in that particular panel, to the columns, rather than to permit them to be transferred beyond.